CORR 99-55

A Strengthened SDP Relaxation via a Second Lifting for the Max-Cut Problem

Miguel F. Anjos, Henry Wolkowicz

Abstract We present a strengthened semidefinite programming, SDP, relaxation for the Max-cut problem, MC, and for the general quadratic boolean maximization problem. The well-known SDP relaxation can be obtained via Lagrangian relaxation and results in an SDP with variable $X \in \mathcal{S}^n$, the space of $n \times n$ symmetric matrices, and n constraints, $\mathrm{diag}(X) = e$, where e is the vector of ones. The strengthened bound is based on applying a *lifting procedure* to this well-known semidefinite relaxation after adding the nonlinear constraints $X^2 - nX = 0$ and $X \circ X = E$. The lifting procedure is again done via Lagrangian relaxation and results in an SDP with variable $Y \in \mathcal{S}^{t(n-1)+1}$, where $t(r) = r(r+1)/2$, and $2t(n-1)+1$ constraints. It is shown that the new bound obtained this way strictly improves the previous SDP bound, both empirically and theoretically.

Keywords Max-cut problem, semidefinite relaxations.