CORR 2000-10

Exact largest and smallest size of components in decomposable structures

Daniel Panario* & Bruce Richmond

Abstract Golomb & Gaal [14] study the number of permutations on n objects with largest cycle length equal to k. They give explicit expressions on ranges $n/(i+1) < k \leq n/i$ for $i = 1, 2, \ldots$, derive a general recurrence for the number of permutations of size n with largest cycle length equal to k, and provide the contribution of the ranges $(n/(i+1), n/i]$ for $i = 1, 2, \ldots$, to the expected length of the largest cycle.

We view a cycle of a permutation as a component. We provide exact counts for the number of decomposable combinatorial structures with largest and smallest components of a given size. These structures include permutations, polynomials over finite fields, and graphs among many others (in both the labelled and unlabelled cases). The contribution of the ranges $(n/(i+1), n/i]$ for $i = 1, 2, \ldots$, to the expected length of the smallest and largest component is also studied.

Keywords largest and smallest components, random decomposable combinatorial structures, exponential class.