CORR 2000-17

Delsarte systems: combinatorial characterizations of Delsarte $\mathcal{T}$-designs via partially ordered sets

William J. Martin*

Abstract A number of important design-theoretic structures, such as combinatorial block designs and orthogonal arrays, can be characterized as Delsarte $\mathcal{T}$-designs in cometric association schemes. Several recent papers extend this theory to more exotic types of designs; however, the language of partially ordered sets can be used to describe the designs and to establish the connection to association schemes. In this paper, we introduce "Delsarte systems" as a tool for describing all of the known examples in a uniform fashion. A Delsarte system consists of an association scheme with a partial order on its eigenspaces together with a second partially ordered set having the vertices of the scheme as its maximal elements. The key axiom for these systems ties the incidence matrices of this partial order to the eigenspaces of the corresponding association scheme.

The results of the paper are as follows. First, given a Delsarte system, we prove an equivalence between certain families of Delsarte $\mathcal{T}$-designs and "designs" in the attached partially ordered set. Next, we give several ways of obtaining new Delsarte systems from known ones. Two general bounds, on on the size of a $\mathcal{T}$-design and the other on the degree of a $\mathcal{T}$-design, are established. A tight design in a Delsarte system is shown to induce a new Delsarte system of its own. We conclude by exploring some of the examples, including ordered orthogonal arrays and λ-trasitive sets of permutations.