CORR 2000-41

Quadratic Expansions of Spectral Functions

Adrian S. Lewis & Hristo S. Sendov

Abstract A function, F, on the space of $n \times n$ real symmetric matrices is called *spectral* if it depends only on the eigenvalues of its argument, that is $F(A) = F(UAU^T)$ for every orthogonal U and symmetric A in its domain. Spectral functions are in one-to-one correspondence with the symmetric functions on $\mathbb{R}^n$: those that are invariant under arbitrary swapping of their arguments. In this paper we show that a spectral function has a *quadratic expansion* around a point A if and only if its corresponding symmetric function has quadratic expansion around $\lambda(A)$ (the vector of eigenvalues). We also give a concise and easy to use formula for the 'Hessian' of the spectral function. In the case of convex functions we show that a positive definite 'Hessian' of f implies positive definiteness of the 'Hessian' of F.