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The Quadrangulation Conjecture for Orientable Surfaces

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Abstract By means of character theory and symmetric functions, Jackson and Visentin [5] proved the existence of certain bijections between the set of quadrangulations in orientable surfaces and decorated maps (with marked edges and coloured vertices) in orientable surfaces. the bijections preserve a weight function consisting of a pair (g, n) of integer parameters. For quadrangulations, g is the genus and n is the number of faces. For decorated maps, g is the genus plus half the number of white vertices and n is the number of edges.

The Quadrangulation Conjecture concerns the problem of finding a natural bijection of this type. Tutte's medial construction is a solution in the special case $g = 0$ of planar maps. We give a construction of a bijection $\tilde{\Xi}$ which both extends Tutte's medial construction to non-planar maps and preserves the parameter n of the Quadrangulation Conjecture. (The parameter g is not generally preserved, except when $g = 0$.) Non-orientable surfaces play an important part in the construction of $\tilde{\Xi}$.