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Arrangements, Circular Arrangements and the Crossing Number of $C_7 \times C_n$

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Abstract Motivated by the problem of determining the crossing number of the Cartesian product $C_m \times C_n$ of two cycles, we introduce the notion of an (m,n)-arrangement, which is a set $\{S, T, C_1, C_2, \ldots, C_n\}$ of closed curves and a set $\{P_1, P_2, \ldots, P_m\}$ of paths in the plane, such that S and T are disjoint and in the same face of $C_1 \cup C_2 \cup \ldots \cup C_n$, each P_i joins a point on S to a point on T, and each P_i has a vertex $v_{i,j}$ on C_j so that in traversing P_i from S to T, the $v_{i,j}$ occur in the order $v_{i,1}, v_{i,2}, \ldots, v_{i,n}$. The main result is that every (m,n)-arrangement has at least $(m-2)n$ crossings. This is used to show that "(m,n)-circular arrangements" (no S and T and the P_i are closed curves) which can be broken up into disjoint arrangements have $(m-2)n$ crossings. This last fact implies that the crossing number of $C_7 \times C_n$ is $5n$, in agreement with the general conjecture that the crossing number of $C - m \times C_n$ is $(m-2)n$, for $3 \leq m \leq n$.