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Critical Configurations of the Dollar Game and Parking Functions

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Abstract Critical configurations are integer-valued functions defined on the vertex-set of a graph that arise in the context of the dollar game. We give a bijection between critical configurations and spanning trees that carries index to external activity, yielding a bijective proof of a result of López. We also exhibit connections between critical configurations and parking functions, beyond the fact that critical configurations on complete graphs are complements of parking functions. On the one hand, our bijection generalizes the one of Beissinger and Peled between trees and complements of parking functions. On the other hand, we obtain a recurrence for the generating function of critical configurations that generalizes known results on parking functions (Kreweras) and k-parking functions (Yan). We also prove that 3-connected graphs are determined by their critical configurations.