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Further Results on mean Colour Numbers

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Abstract Let $P(G, \lambda)$ denote the chromatic polynomial of a graph G. Dong has proved that $(\lambda - 2)^{n-1}P(G, \lambda) - \lambda(\lambda - 1)^{n-2}P(G, \lambda - 1) \geq 0$ for $\lambda \geq n$ if G is connected, where n is the vertex number of G. This result implies that $P(G, n)/P(G, n - 1) \geq n^n/(n - 1)^n$, which was a conjecture proposed by Bartels and Welsh. In this paper, we give a different and simpler proof to Dong's result and further prove a stronger result that $(\lambda-3)^{n-2}P(G, \lambda)-\lambda(\lambda-2)^{n-3}P(G, \lambda-1) \geq 0$ for $\lambda \geq n$, where G is a graph whose vertex set has an ordering $x_1, x_2, \ldots, x_n$ such that x_i is contained in a clique K_3 of the subgraph $G[V_i]$ for $i = 3, 4, \ldots, n$ where $V_1 = \{x_1, x_2, \ldots, x_i\}$.

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