CORR 2001-20

Active Sets, Nonsmoothness and Sensitivity

A.S. Lewis

Abstract Nonsmoothness pervades optimization, but the way it typically arises is highly structured. Nonsmooth behaviour of an objective function is usually associated, locally, with an *active manifold:* on this manifold the function is smooth, whereas in normal directions it is "vee-shaped." Active set ideas in optimization depend heavily on this structure. Important examples of such functions include the pointwise maximum of some smooth functions, and the maximum eigenvalue of a parametrized symmetric matrix. Among possible foundations for practical nonsmooth optimization, this broad class of "partly smooth" functions seems a promising candidate, enjoying a powerful calculus and sensitivity theory. In particular, we show under a natural regularity condition that critical points of partly smooth functions are stable: small perturbations to the function cause small movements of the critical point on the active manifold.

Keywords active set, nonsmooth analysis, subdifferential, generalized gradient, sensitivity, $\mathcal{U}$-Lagrangian, eigenvalue optimization, spectral abscissa, identifiable surface

AMS 2000 Subject Classification
Primary: 90C31, 49K40
Secondary: 65K10, 15A42