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Some consequences of the Linear Programming Bound for Designs

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Introduction There is a trivial upper bound, $\binom{v-t}{k-t}$, on λ_t of a simple design with v points, block size k and strength t. Now, what is the smallest positive integer λ_t can be? In [2], Delsarte has formulated a linear program with t equalities and $(k-t)$ inequalities which minimizes λ_t. For each value of t, v and k, the linear program can be computed and the lower bound on λ_t is often pretty strong compared to other known bounds. However, the linear program consists of k constraints and k variables, that is, its size grows very quickly. Moreover, we have to run the linear program once for every specific set of values for t, v and k.

In this paper, we use a system of $(t+1)$ equations resulting from simple counting argument together with the last two inequalities of Delsarte's linear program to derive an explicit formula of a lower bound on λ_t for several families of parameters. They are listed in the following table.

v	k	t	$t \geq \cdot$	$\lambda_t \geq \cdot$
$2t+6$	$t+3$	even	2	$\frac{(t-1)(t-4)(t-6)}{6(t+2)}$
$2t+7$	$t+3$	even	4	$\frac{(t-2)(t-5)(t-6)}{6(t+2)}$
$2t+7$	$t+3$	odd	7	$\frac{(t-5)(t-5)(t-6)}{6(t+1)}$
$2t+8$	$t+3$	even	20	$\frac{(t-7)(t-6)(t-7)}{6(t+1)}$
		odd	9	
$2t+9$	$t+3$	all	13	$\frac{(t-9)(t-7)(t-8)}{6(t+1)}$
$2t+10$	$t+3$	all	13	$\frac{(t-11)(t-8)(t-9)}{6(t+1)}$
$2t+9$	$t+4$	even	4	$\frac{(t-3)(t-6)(t-8)}{8(t+2)}$

We only illustrate the proof for $v = 2t+7$ and $k = t+3$ in this paper since the same method is applied to all the other families of parameters listed in the table.