CORR 2001-66

A Simple Iterative Method for Linear Programming

Maria Gonzalez-Lima*, Henry Wolkowicz

Abstract This paper presents a new primal-dual interior/exterior-point method for linear programming. We begin with the usual perturbed primal-dual optimality equations $F_\mu(x, y, z) = 0$. Under nondegeneracy assumptions, this nonlinear system is well-conditioned, i.e. it has a nonsingular Jacobian at optimality and is not necessarily ill-conditioned as the iterates approach optimality. We apply preprocessing to obtain one single bilinear equation which is also well-conditioned. We then apply a preconditioned conjugate gradient method (PCG), within an inexact Newton framework, directly on the linearized equations. This is done without forming the *normal equations* system. The work of an iteration consists almost entirely in the (approximate) solution of this well-conditioned linearized system, using PCG. Therefore, improvements depend on efficient preconditioning. In the sparse case, the linearized system consists of a large sparse part and a small dense part. Primal and dual feasibility are 100% guaranteed throughout the iterations. Since the linearization is well conditioned, we can use affine scaling and not maintain positivity once we are close enough to the optimum. In addition, we identify some of the primal and dual variables which are converging to 0 and delete them. Therefore, we get smaller systems as the iterations progress. These techniques reduce the number and complexity of the iterations. We present numerical tests with both *diagonal* and *partial Cholesky* preconditioners.

Keywords Linear Programming, large spare problems, preconditioned conjugate gradients.