CORR 2002-15

Geometry of Homogeneous Convex Cones, Duality Mapping, and Optimal Self-Concordant Barriers

Van Anh Truong & Levent Tunçel

Abstract We study homogeneous convex cones. We first characterize the extreme rays of such cones in the context of their primal construction (due to Vinberg) and also in the context of their dual construction (due to Rothaus). Then, using these results, we prove that every homogeneous cone is facially exposed. We provide an alternative proof of a result of Güler and Tunçel that the Siegel rank of a symmetric cone is equal to its Carathéodory number. Our proof does not use the Jordan-vonNeuman-Wigner characterization of the symmetric cones but it easily follows from the primal construction of the homogeneous cones and our results on the geometry of homogeneous cones in primal dual forms. We study optimal self-concordant barriers in this context. We briefly discuss the duality mapping in the context of automorphisms of convex cones and prove, using numerical integration, that the duality mapping is not an involution on certain self-dual cones.

Keywords convex optimization, self-concordant barriers, homogeneous cones, symmetric cones, Siegel domains, facially exposed, Carathéodory number, interior-point methods

AMS Subject Classification 90C25, 90C51, 90C60, 90C05, 65Y20, 52A41, 49M37, 90C30