

CORR 2002-22

A Survey of the Trust Region Subproblem Within a Semidefinite Programming Framework

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Abstract The trust region subproblem (the minimization of a quadratic objective subject to one quadratic constraint and denoted TRS) has many applications in diverse areas, e.g. function minimization, sequential quadratic programming, regularization, ridge regression, and discrete optimization. In particular, it determines the step in trust region algorithms for function minimization. Trust region algorithms are popular for their strong convergence properties. However, a draw-back has been the inability to exploit sparsity as well as the difficulty in dealing with the so-called hard case. These concerns have been addressed by recent advances in the theory and algorithmic development.

This paper provides an in depth study of TRS and its properties as well as a survey of recent advances. We emphasize large scale problems and robustness. This is done using semidefinite programming (SDP) and the modern primal-dual approaches as a unifying framework. The SDP framework solves TRS efficiently; and it shows that TRS is always a well-posed problem, i.e. the optimal value and an optimum can be calculated to a given tolerance. This contrary to statements in the literature which label TRS ill-posed or degenerate, if the so-called hard case holds. We provide both theoretical and empirical evidence to illustrate the strength of the SDP and duality approach. In particular, this includes new insights and techniques for handling the hard case.

Keywords Trust Regions, Semidefinite programming, Duality, Unconstrained Minimization.