

## **Abstract**

We demonstrate an alternative method for calculating the asymptotic behaviour of the discrete one-coin quantum walk on the infinite line, via the Jacobi polynomials that arise in the path integral representation. We calculate the asymptotics using a method that is significantly easier to use than the Darboux method. It also provides a single integral representation for the wavefunction that works over the full range of positions,  $n$ , including throughout the transitional range where the behaviour changes from oscillatory to exponential. Previous analyses of this system have run into difficulties in the transitional range, because the approximations on which they were based break down here. The fact that there are two different kinds of approach to this problem (Path Integral vs. Schrödinger wave mechanics) is ultimately a manifestation of the equivalence between the path-integral formulation of quantum mechanics and the original formulation developed in the 1920s. We also discuss how and why our approach is related to the two methods that have already been used to analyse these systems.