

## Abstract

The existence of a non-negative integer-valued invariant, called the Map-Jack invariant, for rooted maps has been conjectured by Goulden and Jackson [4]. It has the defining property that, if it is marked by an indeterminate  $b$ , then the generating series for rooted maps in locally orientable surfaces is a particular series, involving the Jack symmetric function  $J_\lambda^{(b+1)}$ , that specialises to the generating series for rooted maps in orientable surfaces for  $b = 0$  (and clearly to all surfaces for  $b = 1$ ). We propose a candidate Map-Jack invariant  $\eta$  that is determined by depth first search and by the local topological effect of deleting edges from maps, and give results that support its candidacy. In particular, we prove its correctness up to face partition. We also show how the algorithm for determining  $\eta$  may be associated with the Jack symmetric functions. This is achieved by means of the Laplace-Beltrami operator.