

**Abstract**

Two orthonormal bases  $B$  and  $B'$  of a  $d$ -dimensional complex inner-product space are called mutually unbiased if and only if  $|\langle b|b'\rangle|^2 = 1/d$  holds for all  $b \in B$  and  $b' \in B'$ . The size of any set containing pairwise mutually unbiased bases of  $\mathbb{C}^d$  cannot exceed  $d + 1$ . If  $d$  is a power of a prime, then extremal sets containing  $d + 1$  mutually unbiased bases are known to exist. We give a simplified proof of this fact based on the estimation of exponential sums. We discuss conjectures and open problems concerning the maximal number of mutually unbiased bases for arbitrary dimensions.