

ABSTRACT. Let  $w \geq 2$  be an integer and let  $D_w$  be the set of integers which includes zero and the odd integers with absolute value less than  $2^{w-1}$ . Every integer  $n$  can be represented as a finite sum of the form  $n = \sum a_i 2^i$ , with  $a_i \in D_w$ , such that of any  $w$  consecutive  $a_i$ 's at most one is nonzero. Such representations are called *width- $w$  nonadjacent forms* ( $w$ -NAFs). When  $w = 2$  these representations use the digits  $\{0, \pm 1\}$  and coincide with the well known *nonadjacent forms*. Width- $w$  nonadjacent forms are useful in efficiently implementing elliptic curve arithmetic for cryptographic applications. We provide some new results on the  $w$ -NAF. We show that  $w$ -NAFs have a minimal number of nonzero digits and we also give a new characterization of the  $w$ -NAF in terms of a lexicographical ordering. We also generalize a result on  $w$ -NAF and show that any base 2 representation of an integer, with digits in  $D_w$ , that has a minimal number of nonzero digits is at most one digit longer than its binary representation.