Asymptotics of Permutations with Nearly Periodic Patterns of Rises and Falls

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Abstract

Ehrenborg obtained asymptotic results for nearly alternating permutations and conjectured an asymptotic formula for the number of permutations that have a nearly periodic run pattern. We prove a generalization of this conjecture, rederive the fact that the asymptotic number of permutations with periodic run pattern has the form $Cr^{-n}n!$, and show how to compute the various constants. A reformulation in terms of id random variables leads to an eigenvalue problem for a Fredholm integral equation. Tools from functional analysis establish the necessary properties.