## Abstract

We present a characterization of those Euclidean distance matrices $D$ which can be expressed as $D = \lambda(E - C)$ for some nonnegative scalar $\lambda$ and some correlation matrix $C$, where $E$ is the matrix of all ones. This shows that the cones

$$\text{cone } (E - \mathcal{E}_n) \neq \overline{\text{cone } (E - \mathcal{E}_n)} = \mathcal{D}_n,$$

where $\mathcal{E}_n$ is the elliptope (set of correlation matrices) and $\mathcal{D}_n$ is the (closed convex) cone of Euclidean distance matrices.

The characterization is given using the Gale transform of the points generating $D$. We also show that given points $p^1, p^2, \ldots, p^n \in \Re^r$, for any scalars $\lambda_1, \lambda_2, \ldots, \lambda_n$ such that

$$\sum_{j=1}^{n} \lambda_j \, p^j = 0, \qquad \sum_{j=1}^{n} \lambda_j = 0,$$

we have

$$\sum_{j=1}^{n} \lambda_j \, ||p^i - p^j||^2 = \alpha \text{ for all } i = 1, \ldots, n,$$

for some scalar $\alpha$ independent of $i$.