Abstract

In an incomplete market, it is generally impossible to replicate an option exactly. In this case, total risk minimization chooses an optimal self-financing strategy that best approximates the option payoff by its final value. Total risk minimization is a dynamic stochastic programming problem, which is generally very challenging to solve; a direct approach may lead to very expensive computations.

We investigate total risk minimization using a piecewise linear criterion. We describe a method for computing the optimal hedging strategies for this stochastic programming problem using Monte Carlo simulation and spline approximations. We illustrate this method in the Black-Scholes and the stochastic volatility frameworks. We also compare the hedging performance of the strategies based on piecewise linear risk minimization, the traditional, quadratic risk minimizing strategies and the shortfall risk minimizing strategies. The numerical results show that piecewise linear risk minimization may lead to smaller hedging cost and significantly different, possibly better, hedging strategies. The values of the shortfall risk for the piecewise linear total risk minimizing strategies suggest that these strategies typically underhedge the options.