

Abstract

The Steiner tree problem is a classical -hard optimization problem with a wide range of practical applications. In an instance of this problem, we are given an undirected graph $G = (V, E)$, a set of *terminals* $R \subseteq V$, and non-negative costs c_e for all edges $e \in E$. The goal is to find a minimum-cost tree T in G that connects all terminals in R .

The best known approximation algorithm known for the Steiner tree problem is due to Robins and Zelikovsky (SIAM J. Discrete Math, 2005) and achieves a performance ratio of 1.55. Robins and Zelikovsky's algorithm is a greedy algorithm. The best known LP-based algorithm for general graphs is a 2-approximation due to Agrawal, Klein and Ravi (SIAM J. Computing, 1995). The analysis of this algorithm is tight as the underlying undirected cut relaxation for the Steiner tree problem has an integrality gap of nearly 2. Rajagopalan and Vazirani (SODA, 2000) have recently proposed a new primal-dual $(1.5 + \epsilon)$ -approximation algorithm for the Steiner tree problem in quasi-bipartite graphs; these are graphs in which no two Steiner vertices are connected by an edge. Their algorithm is based on the so called *bidirected cut relaxation* for the Steiner tree problem.

Motivated by the result of Rajagopalan and Vazirani, most recent efforts on finding better LP-based approximation algorithms have focused on the bidirected cut relaxation. In this paper, we propose a new undirected formulation, and we show that Robins' and Zelikovsky's 1.55-approximation can be interpreted as a primal-dual algorithm using this relaxation. We also show that this relaxation has a gap of at most 1.5 in the special case of quasi-bipartite graphs.