Abstract

We present a method to solve in an efficient way the problem of constructing the curves given by Torelli's theorem in dimension 3 over the complex numbers: For an undecomposable principally polarized abelian threefold A over $\mathbb{C}$ given by its period matrix Ω, compute a model of the curve of genus three (unique up to isomorphism) whose Jacobian, equipped with its canonical polarization, is isomorphic to A as a principally polarized abelian variety. We use this method to describe the non-hyperelliptic modular Jacobians of dimension 3. We also present another method to find $\mathbb{Q}$-rational equations of non-hyperelliptic modular curves of genus 3. We investigate all the non-hyperelliptic new modular curves C_f of genus 3 with $\operatorname{Jac}(C_f) \sim_{\mathbb{Q}} A_f$, where $f \in S_2^{\mathrm{new}}(X_0(N))$, $N \leq 4000$.

Key words: modular curves, modular Jacobians, non-hyperelliptic curves of genus 3, algebraic curve cryptography, discrete logarithm problem, Torelli's theorem.