Abstract

Given two sets $\mathcal{A}$, $\mathcal{B} \subseteq \mathbb{F}_d^q$, the set of d dimensional vectors over the finite field $\mathbb{F}_q$ with q elements, we show that the sum-set

$$\mathcal{A} + \mathcal{B} = \{\mathbf{a} + \mathbf{b} \mid \mathbf{a} \in \mathcal{A}, \mathbf{b} \in \mathcal{B}\}$$

contains a k-term "geometric progression" of the form $\mathbf{v}\Lambda^j$, where $j = 0, . . . , k$ - 1, with a nonzero vector $\mathbf{v} \in \mathbb{F}_d^q$ and a nonsingular d x d matrix Λ whenever

$$\#\mathcal{A}\#\mathcal{B} \geq 20\mathrm{q}^{2d-2/k}.$$

We also consider some modifications of this problem including the question of the existence of elements of sumsets on algebraic varieties.