

# Asymptotics of Smallest Component Sizes in Decomposable Combinatorial Structures of Alg-Log Type

Li Dong, Zhicheng Gao, and Daniel Panario  
School of Mathematics and Statistics  
Carleton University, Ottawa, Canada  
{ldong,zgao,daniel}@math.carleton.ca

Bruce Richmond  
Department of Combinatorics and Optimization,  
University of Waterloo, Waterloo, Canada  
lbrichmond@math.uwaterloo.ca

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## Abstract

A decomposable combinatorial structure consists of simpler objects called components which by themselves can not be further decomposed. We focus on the case when the component generating function  $C(z)$  is of alg-log type, that is,  $C(z)$  behaves like

$$c + d(1 - z/\rho)^\alpha \left( \ln \frac{1}{1 - z/\rho} \right)^\beta (1 + o(1))$$

when  $z$  is near the dominant singularity  $\rho$ . We provide asymptotic results about the size of the smallest components in random combinatorial structures for the cases  $0 < \alpha < 1$  and any  $\beta$ , and  $\alpha < 0$  and  $\beta = 0$ . The particular case  $\alpha = 0$  and  $\beta = 1$ , the so-called exp-log class, have been treated in previous papers. We also provide similar asymptotic estimates for combinatorial objects with a restricted pattern, that is, when part of its factorization pattern is known. We extend our results to include certain type of integers partitions.

*Keywords:* decomposable structures; restricted pattern; labeled and unlabeled structures; generating functions; alg-log type.