UNIVERSITY OF WATERLOO | FACULTY OF MATHEMATICS
Department of Combinatorics and Optimization

C&O Comprehensive Exam Syllabus for Continuous Optimization

The motivation of the comprehensive exam in continuous optimization is multifold: (a) To provide the students with a broad review of the theoretical background in the area of continuous optimization, and adequately strengthen their knowledge to pursue their PhD studies in this field; (b) To demonstrate and enhance the student's ability to work independently, and learn advanced topics on their own; and hence prepare them for high quality research projects; (c) As a byproduct, we expect that comprehensive exams material prepares PhD students to better perform the teaching duties related to their Teaching Assistantships (TA) assignments.

Suggested References:

[BV] S. Boyd and L. Vandenberghe, *Convex Optimization*, [QA402.5.B69 2004] (seventh printing, 2009, is available online)

[NW] J. Nocedal and S. Wright, *Numerical Optimization*, 2nd Edition, 2006 (available online, through uWaterloo Library).

[RW] R.T. Rockafellar and R.J.-B. Wets, *Variational Analysis*, 1998, Springer-Verlag (available online).

[Rock] R.T. Rockafellar, *Convex Analysis*, 1970, Princeton University Press, Princeton, NJ (available online).

Outline of Topics:

1. Linear programming, simplex method, duality (NW §§13.1–13.5 or refer to CO250 or CO255 lecture notes).
2. Optimality conditions: first and second order conditions for unconstrained and constrained optimization; in particular, KKT conditions for constrained optimization (NW §2.1, NW §§12.1–12.3); Lagrangian duality, minmax theory (BV §§5.1–5.5, Rock §§36–37; RW §11H).
3. Convex sets and supporting hyperplanes (BV §§2.1–2.3, 2.5; Rock §§2–3; RW §2A, §2E, §2H), affine hulls, relative interiors, tangent and normal cones (Rock §§6–8; RW §§6A–6B).
4. Convex functions (BV §3.1; Rock §4), sublinear functions and norms (RW §3.C), support functions (Rock §13)), differentiability and subgradient calculus (Rock §23, §25), Fenchel-Legendre conjugates and duality (BV §3.3; Rock §26, §31; RW §11H, §11A).
5. Canonical forms of convex optimization problems (BV §§4.1–4.4, 4.6).
6. Unconstrained optimization: basic first and second order algorithms including steepest descent and Newton's method, quasi-Newton methods (no memorization of formulae of updates is required), conjugate gradient methods; sufficient decrease criteria convergence rates; convergence theorems for Newton's method, line search and trust region methods (global convergence analysis) (NW pp. 21–23, §§3.1–3.3, §§4.1–4.2, Ch. 5, §§6.1–6.2).

7. Algorithms for convex optimization: Interior point methods; ellipsoid method and computational complexity of convex optimization (Refer to Y. Nesterov, *Lectures on Convex Optimization*, Vol. I, §3.2.8, available online.)
8. Complexity Theory: The classes P and NP, NP-completeness. (Kleinberg and Tardos, *Algorithm Design*, 2014, pages 451–530).