

C&O 666 Continuous Optimization

Fall 2022

Instructor: Levent Tunçel

We will study fundamentals of *continuous optimization* in Euclidean spaces. For most of the purposes of this course, a *continuous optimization problem* is the problem of minimizing (or maximizing) a given continuous function of finitely many real variables, subject to constraints described as equations and inequalities on some other given continuous functions. Our work will include mathematical foundations, design and analysis of algorithms to solve continuous optimization problems.

Topics Include:

- (1) *Introduction:* Notation, formulations, fundamental observations justifying the necessity of identification and exploitation of structure in continuous optimization problems, quick review of some of the relevant fundamental results from geometry, linear algebra, real analysis, and topology.
- (2) *Unconstrained Continuous Optimization:* Optimality conditions, obtaining derivatives, first and second order algorithms, steepest descent, Newton's Method, quasi-Newton Methods, conjugate gradient method, line search and trust region methods (and a mathematical study of convergence and other fundamental properties of these algorithms).
- (3) *Role of Convexity in Continuous Optimization:* Characterizations of convexity in functions, duality and convex conjugacy (fundamental properties), subgradients and subdifferentials, Lagrange multipliers.
- (4) *Constrained Continuous Optimization:* Existence and uniqueness of optima, first and second order optimality conditions, Karush-Kuhn-Tucker Theorem, local versus global optimization, penalty and barrier methods, gradient projection methods, sequential quadratic programming, augmented Lagrangeans.
- (5) *Complexity Analyses for First-Order and Second-Order Algorithms:* Modern first-order algorithms used in very large-scale problems such as those in data science and machine learning and their complexity analysis; modern interior-point algorithms and their complexity analysis.

Textbook: *Numerical Optimization* (by J. Nocedal and S. Wright), 2nd edition, Springer 2006 (pdf is available through uWaterloo Library)

Other References:

- (1) *Nonlinear Programming* (by D. Bertsekas), 3rd edition 2016 (available on reserve at DC Library: T57.8.B47x 2016)
- (2) *Convex Optimization* (by S. Boyd and L. Vandenberghe) 2004-... (7th printing, 2009, is available online; and a 2004 hardcopy on reserve at DC Library: QA402.5.B69 2004)
- (3) *Lectures on Convex Optimization* (by Y. Nesterov) 2nd edition 2018 (pdf is available through uWaterloo Library login)

Prerequisites: Students who have no prior exposure to continuous optimization should read Appendices A.1 and A.2 of the textbook (pages: 598–634).