CO 749: Designs, Nibble and Absorbers

Course Outline for Winter 2023

Instructor: Luke Postle

This course is a topics course in graph theory and extremal combinatorics centering on a suite of some of the latest and most advanced methods.

Topics include:

- **Graph Designs:** Steiner systems, Latin squares, Erdős-Hanani conjecture, the Existence of Designs conjecture
- **Graph Edge Coloring:** Pippenger-Spencer Theorem for edge-coloring hypergraphs, Kahn's Theorem approximately solving the List Coloring Conjecture
- **Graph Vertex Coloring:** Kim's Theorem for coloring graphs of girth five; coloring hypergraphs of girth five and small codegrees
- **Rainbow Coloring:** Ryser-Brualdi-Stein conjecture, Aharoni-Berger conjecture, etc; namely studying if an edge-colored graph admits a *rainbow* matching of large size or in all the colors (rainbow = all edges different colors)
- **High Girth Designs/Coloring:** Finding high girth designs (i.e. locally sparse) or high girth edge colorings (for example, no bicolored cycles)

Techniques discussed include:

- **Probabilistic Methods:** Expectation, Lovász Local Lemma, etc.
- **Concentration Inequalities:** in particular, the latest version of Talagrand's inequality
- **Nibble:** Rödl's revolutionary technique for solving a problem via small iterative steps; used to find apporimxate solutions to many problems
- **Absorbers:** a recent revolutionary technique that turns approximate solutions into exact solutions; recently used to resolve many long-standing conjectures in graph theory, graph designs, and extremal combinatorics.

Prerequisites. The course will be mostly self-contained, but it is designed as a graduate topics course in graph theory. Hence it assumes mathematical maturity and familiarity with graph theory concepts/proofs. Namely some background in or exposure to graph coloring and the probabilistic method is desirable; for example CO 642. Please contact the instructor if there are questions about placement in the course or background materials.

Suggested reading/background reading:

Graph Theory, Fifth Edition, Diestel. Available on-line.

Probabilistic Method, Fifth Edition, Alon and Spencer.

Probabilistic Method in Graph Coloring, Molloy and Reed.