CO 452/652 - Integer Programming
Winter 2019

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1 Course outline

Integer Programming is a mathematical program that can be seen as a linear program with the additional requirement that some (or all) of the variables are required to take on integer values. While linear programming is already very useful in modeling problems and can be solved well using either interior point methods or the simplex method, **Integer programs, or IPs** have a much broader range of applications. Although no method exists that can guarantee an efficient solution to ALL IPs, Integer Programming research has gone quite far in the past 60 years, to the point that some IPs can effectively be used as subroutines in more complex problems.

In this course we will explore an upper-level undergraduate/basic graduate introduction to the topic of Integer Programming. The focus will be on structural properties and ideas, with several proofs. Some ideas of algorithms that are useful to solve IPs in practice will also be presented, but will not be the main focus.

A tentative list of topics to be covered include:

- Basic geometric definitions: Convex/affine/linear combinations. Convex/affine hull. Dimension.
- Basic polyhedral theory: Faces, facets, extreme points. Minimal representations. Edges and extreme rays. Double-description of polyhedra. Polarity.
- Integer programs: Meyer's theorem. Perfect formulations. Extended formulations and projections. Integral polyhedra and Total Dual Integrality. Union of polyhedra and representability.
- Cutting planes: Separating hyperplanes. Equivalence of optimization and separation. Gomory cuts and other cutting planes. Rank and closures.
- IP in fixed dimension. Basis reduction, LLL and Lenstra's algorithm.

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2 Books and Supplementary Material

The main textbook to follow is "Integer Programming, by M. Conforti, G. Cornuéjols and G. Zambelli." (CCZ) It is available online through UW library.

Other books that can be used as reference are

- Optimization over Integers, by D. Bertsimas and R. Weismantel.
- Integer and Combinatorial Optimization, by G. Nemhauser and L. Wolsey. Available at UW library.
- Theory of linear and integer programming, by A. Schrijver. Available at UW library.

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3 Prerequisite

Students are assumed to know linear algebra and the basics of linear programming, particularly:

- Some geometric definitions/results of polyhedra:
  - Convexity, convex combinations, convex sets, linear combinations, linear independence (CCZ, sections 3.4.1, 3.4.3; BT 2.1).
  - Extreme points, basic feasible solutions, vertices (and the equivalence of these three) (BT 2.2)
- Farkas' Lemma (CCZ, section 3.2; BT 4.6)
- LP weak and strong duality, complementary slackness (CCZ, section 3.3; BT 4.2,4.3)
- Simplex method (BT 3.1,3.2,3.5)

Note: Above, CCZ refers to Cornuéjols, Conforti, Zambelli and BT refers to Bertsimas and Tsitsiklis, Introduction to Linear Optimization