CO749 Spring 2019
Topological Graph Theory
Bruce Richter

This course will take a broad view of the interaction between point-set topology and graph theory. We will begin with a study of graphs embedded in surfaces, which will include some strictly point-set topological matters. (For example, we will prove that a graph G embeds in the plane if and only if it has a piecewise-linear embedding. This can be used to prove the Jordan Curve Theorem.) This topic connects directly with the Graph Minors Project; it having been the focus of a recent topics course, we will not dwell on GMP. We will discuss how to prove that each surface has only finitely many minor-minimal obstructions.

Related to graphs embedded in surfaces are crossing numbers of graphs. There are many interesting facets to this problem: given a family of graphs, can we determine their crossing numbers? what is the crossing number of the complete graph? What are the minimal graphs with crossing number at least k?

Infinite graphs often are described without topology, but there are interesting questions about embedding them in surfaces. As surfaces are compact, they automatically end up with limit points in the surface that are not in the graph. One can also intrinsically define their ends and compactify the graph without reference to an embedding. The interactions are interesting.

The study of infinite graphs led to a study of more general topological objects: graph-like spaces. We will study embeddings of graph-like graphs and their relation to Tutte's characterization of graphic matroids.

Grades will be determined by: participation in class; submissions to assigned homework; and a project consisting of a written report on a topic/paper and (depending on numbers) an oral presentation.