

C&O 250 INTRODUCTION TO OPTIMIZATION

INSTRUCTORS: WALAA MOURSI AND LEVENT TUNÇEL

1. OVERVIEW

Suppose that the owner of a factory wants to maximize its production for the next 30 days. There is a limit on the resources available. Resources may include raw materials, labour, machine capacities, etc. This is an example of an optimization problem. The function that we are trying to maximize is the *objective function*, and the conditions imposed by the available resources are the *constraints* of the problem. Optimization problems are classified according to the type of objective function and the type of constraints.

One of the simpler mathematical models are *Linear Programming Problems* where both the constraints and the objective functions are linear. Even though this may appear at a first glance to be overly restrictive, linear programming algorithms are used very widely. Indeed, a survey of Fortune 500 companies shows that 85% of all respondents use such algorithms in their operations. It is not hard however, to imagine applications for which fractional variable values are not desirable. For instance a variable may indicate the number of employees to hire, or a variable may be restricted to values 0 or 1 to indicate one of two possible options (e.g., build a factory in Waterloo or do not). In these cases, we would like to add the condition that some variables in our linear programming problem take only integer values. These mathematical models are known as *Integer Programming Problems*. Finally, in certain instances, such as portfolio optimization (in financial mathematics), the natural way of formulating the optimization problem may require the use of non-linear constraints, or a non-linear objective function. This leads to *Nonlinear Optimization Problems*.

Note however, that the applications of Optimization are not restricted to problems of business. The same techniques have been very successful in Engineering as well as Physical and Social Sciences. The theory of optimization is utilized in mathematical research in many other mathematical research areas to lead to new advances, to discover and prove new theorems and to discover and design new algorithms.

In the first part of the course, we will illustrate these various models with examples that arise from real problems. The later part of the course addresses the subject of how to solve the aforementioned problems. The *Simplex Method* to solve Linear Programming problems will be discussed in some detail and general-purpose Integer Programming techniques such as *Branch-and-Bound* and *Cutting Planes* will also be described. These algorithms while guaranteed to terminate, may in the worst case (and often do in practice) take a prohibitively long time. No fast general algorithm is known for integer programs and none is believed to exist. However, there are efficient algorithms for many important special cases such as the *Maximum Weight Matching* problem. An indispensable tool for the design of such fast algorithms is the *Theory of Duality*, which will be a main focus of this course. We will move towards the conclusion of the course with a review of the various techniques used to solve linear and integer programs and by providing a geometric interpretation of these algorithms. This discussion will lead us to non-linear convex optimization problems. Also included in this last part of the material are approximation algorithms. These are the algorithms that are guaranteed to be computationally efficient, at the cost of finding solutions which are not necessarily optimal, but their objective values are within a guaranteed small factor of the best possible (optimal) objective value.

Course Homepage: ... is on uWaterloo-LEARN

Prerequisites: One of MATH 106 with a grade of at least 70%, MATH 115, 136, 146; Cumulative overall average of at least 60%.

2. OFFICE HOURS

Starting on September 14, 2020 and ending on December 7, 2020:

Day	Time
Monday	7:00p.m.–8:00p.m. (EST)
Thursday	11:00a.m.–noon (EST)

Office hours will be the appropriate forum for all questions pertaining to the course material and to assignments. Students are encouraged to attend and/or submit written questions ahead of time. Office hours will be held over Zoom. Students are encouraged to submit their questions for office hours ahead of time, using the corresponding office hours folder on piazza. There will be a summary/record of each office hour posted on the course website shortly thereafter.

3. CONTACT INFORMATION

Please use only your uWaterloo email account to send us email messages.

Name	Responsibility	email address
Walaa Moursi	Instructor	walaa.moursi@uwaterloo.ca
Levent Tunçel	Instructor and coordinator	levent.tuncel@uwaterloo.ca
Jonathan Chang	TA	ja2chang@uwaterloo.ca
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Questions regarding the way the assignments were marked should be directed to the Teaching Assistant listed responsible for that assignment. For questions pertaining to the current assignment please use the bulletin/discussion board which is setup on piazza.

3.1. Online Discussion Forum (piazza). Our discussion forum will be hosted by <https://piazza.com/>. Questions pertaining to the material or the logistics of the course need to be posted on piazza and should not be directed to your instructors by email.

4. TEXTBOOK

A Gentle Introduction to Optimization, Cambridge University Press, 2014 (by B. Guenin, J. Könemann and L.T.).

5. EVALUATION AND EXPECTATIONS

After completing the class, the students are expected to master at least the following:

- formulate simple real life problems as linear, integer, or continuous (non-linear) optimization problem,
- understand and be able to execute all the steps of various algorithms and their variants such as the Simplex Method and its variants. Moreover, be able to explain rigorously the derivation as well as all the steps of these algorithms.
- formulate the dual of various linear programming problems,
- understand and be able to explain how duality theory is used to develop the shortest path algorithm, the minimum cost perfect-matching algorithm in bipartite graphs and optimality conditions for linear, integer and non-linear optimization problems.
- develop the ability to utilize the main proof techniques and logical deduction skills in the course as well as the ability of proving simple related concepts independently,
- being able to rigorously explain and apply the geometric interpretation of the various algorithms and theorems covered such as those involving optimality conditions and infeasibility certificates.

5.1. Assignments. There will be nine homework assignments. Assignments are to be submitted via Crowdmark. You are responsible for the readability of the file you submit. Late submissions will not be accepted.

The first assignment is called Assignment 0 and is an assessment of your linear algebra background relevant to co250. The weight of Assignment 0 in the computation of final course grade is zero.

You are expected to do the assignments on your own. The only sources allowed for doing the assignments are:

- all of the material on the co250 Fall 2020 course website,
- the textbook,
- all of the material on the co250 Fall 2020 Piazza website,
- your Instructors and TAs.

Usage of any other source (dead or alive) in doing the homework assignments is against the academic integrity rules for co250 in the Fall 2020 term.

Teaching staff for the course actively look for evidence of academic offences when evaluating assignment papers. By uWaterloo policy, academic integrity violations by a student in assignments will result in a mark of zero in that assignment and will lead to a 5% deduction from the Final Course Grade. In addition, all academic offences are reported to the Associate Dean for Undergraduate Studies and are recorded in the student's file (this may lead to further, more severe consequences).

A missed homework assignment will be treated the same as a mark of zero unless the cause is illness (a medical note is necessary), or a similar good reason acceptable to the instructor, given promptly in writing, in which case the corresponding weight will normally be distributed to other assignments.

5.2. Personalized Assessments. There will be two personalized assessments (please see the course schedule). There are many potential pitfalls in remote education: lack of one-on-one interaction, risk of procrastination, lack of timely feedback. To remedy some of these potential drawbacks, teaching staff will put a lot of effort in this direction. Among these efforts, Personalized Assessments are one-on-one online meetings (approximately 15 minutes for each session) with a member of the teaching staff for co250. These meetings will provide one-on-one interaction, and help assess students' personal work on the assignments (including reading assignments). It will also provide an opportunity for students to illustrate how they have learned from the assignments (even on those parts they did not get completely right initially). Further details, logistics and examples will be described on the course website by October 1, 2020.

5.3. Final Grade.

Assignments 1–8	80%
Personal Assessment 1	6%
Personal Assessment 2	9%
Piazza Participation	5%

5.4. Copyright. All material you will have access to, including the videos, slides and assignments, are copyrighted by uWaterloo and your instructors. You are not allowed to share any of this material with anyone who is not currently enrolled in the course. In particular, you are not allowed to share or post this material online.

6. ACCESS ABILITY SERVICES

<https://uwaterloo.ca/accessability-services/>

University of Waterloo has a long standing commitment to support the participation and access to university programs, services, and facilities by persons with disabilities. Access Ability Services office (AAS) is located in Needles Hall, Room 1401, collaborates with all academic departments to arrange appropriate accommodations for students with disabilities without compromising the academic integrity of the curriculum. If you require academic accommodations to lessen the impact of your disability, please register with AccessAbility Services at the beginning of each academic term.

7. ACADEMIC INTEGRITY

In order to maintain a culture of academic integrity, members of the University of Waterloo community are expected to promote honesty, trust, fairness, respect and responsibility. Check

<http://www.uwaterloo.ca/academicintegrity/> for more information.

Grievance. A student who believes that a decision affecting some aspect of his/her university life has been unfair or unreasonable may have grounds for initiating a grievance. Read Policy 70, Student Petitions and Grievances, Section 4, <http://www.adm.uwaterloo.ca/infosec/Policies/policy70.htm>. When in doubt please be certain to contact the department's administrative assistant who will provide further assistance.

Discipline. A student is expected to know what constitutes academic integrity to avoid committing academic offences and to take responsibility for his/her actions. A student who is unsure whether an action constitutes an offence, or who needs help in learning how to avoid offences (e.g., plagiarism, cheating) or about "rules" for group work/collaboration should seek guidance from the course professor, academic advisor, or the undergraduate associate dean. For information on categories of offences and types of penalties, students should refer to Policy 71, Student Discipline, <http://www.adm.uwaterloo.ca/infosec/Policies/policy71.htm>. For typical penalties check Guidelines for the Assessment of Penalties, <http://www.adm.uwaterloo.ca/infosec/guidelines/penaltyguidelines.htm>.

Appeals. A decision made or penalty imposed under Policy 70, Student Petitions and Grievances (other than a petition) or Policy 71, Student Discipline may be appealed if there is a ground. A student who believes he/she has a ground for an appeal should refer to Policy 72, Student Appeals, <http://www.adm.uwaterloo.ca/infosec/Policies/policy72.htm>.

8. DIVERSITY

It is our intent that students from all diverse backgrounds and perspectives be well served by this course, and that students' learning needs be addressed both in and out of class. We recognize the immense value of the diversity in identities, perspectives, and contributions that students bring, and the benefit it has on our educational environment. Your suggestions are encouraged and appreciated. Please let us know ways to improve the effectiveness of the course for you personally or for other students or student groups. In particular:

- We will gladly honour your request to address you by an alternate/preferred name or gender pronoun. Please advise us of this preference early in the semester so we may make appropriate changes to our records.
- We will honour your religious holidays and celebrations. Please inform of us these at the start of the course.
- We will follow AccessAbility Services guidelines and protocols on how to best support students with different learning needs.

9. MENTAL HEALTH SUPPORT

The Faculty of Math encourages students to seek out mental health support if needed.

- On-campus Resources:
 - Campus Wellness <https://uwaterloo.ca/campus-wellness/>
 - Counselling Services: counselling.services@uwaterloo.ca/ 519-888-4567 ext 32655
 - MATES: one-to-one peer support program offered by Federation of Students (FEDS) and Counselling Services: mates@uwaterloo.ca
 - Health Services: located across the creek from the Student Life Centre, 519-888-4096.
- Off-campus Resources:
 - Good2Talk (24/7): Free confidential help line for post-secondary students. Phone: 1-866-925-5454
 - Here 24/7: Mental Health and Crisis Service Team. Phone: 1-844-437-3247
 - OK2BME: set of support services for lesbian, gay, bisexual, transgender or questioning teens in Waterloo. Phone: 519-884-0000 extension 213