CO 452/652 - Integer Programming Winter 2021

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1 Course outline

Integer Programming is a mathematical program that can be seen as a linear program with the additional requirement that some (or all) of the variables are required to take on integer values. While linear programming is already very useful in modeling problems and can be solved well using either interior point methods or the simplex method, **Integer programs**, **or IPs** have a much broader range of applications. Although no method exists that can guarantee an efficient solution to ALL IPs, Integer Programming research has gone quite far in the past 60 years, to the point that some IPs can effectively be used as subroutines in more complex problems.

In this course we will explore an upper-level undergraduate/basic graduate introduction to the topic of Integer Programming. The focus will be on structural properties and ideas, with several proofs. Some ideas of algorithms that are useful to solve IPs in practice will also be presented, but will not be the main focus.

A tentative list of topics to be covered include:

- Basic geometric definitions: Convex/affine/linear combinations. Convex/affine hull. Dimension.
- Basic polyhedral theory: Faces, facets, extreme points. Minimal representations. Edges and extreme rays. Double-description of polyhedra. Polarity.
- Integer programs: Meyer's theorem. Perfect formulations. Extended formulations and projections. Integral polyhedra and Total Dual Integrality. Union of polyhedra and representability.
- Cutting planes: Separating hyperplanes. Equivalence of optimization and separation. Gomory cuts and other cutting planes. Rank and closures.
- IP in fixed dimension. Basis reduction, LLL and Lenstra's algorithm.

2 Books and Supplementary Material

The main textbook to follow is "Integer Programming, by M. Conforti, G. Cornuéjols and G. Zambelli." (CCZ) It is available online through UW library.

Other books that can be used as reference are

- Optimization over Integers, by D. Bertsimas and R. Weismantel.
- Integer and Combinatorial Optimization, by G. Nemhauser and L. Wolsey. Available at UW library.
- Theory of linear and integer programming, by A. Schrijver. Available at UW library.

3 Prerequisite

Students are assumed to know linear algebra and the basics of linear programming, particularly:

- Some geometric definitions/results of polyhedra:
 - Convexity, convex combinations, convex sets, linear combinations, linear independence (CCZ, sections 3.4.1, 3.4.3; BT 2.1).
 - Extreme points, basic feasible solutions, vertices (and the equivalence of these three) (BT 2.2)
- Farkas' Lemma (CCZ, section 3.2; BT 4.6)
- LP weak and strong duality, complementary slackness (CCZ, section 3.3; BT 4.2,4.3)
- Simplex method (BT 3.1,3.2,3.5)

Note: Above, CCZ refers to Cornuéjols, Conforti, Zambelli and BT refers to Bertsimas and Tsitsiklis, Introduction to Linear Optimization

Students that have taken CO255 or have a grade of at least 85% on CO250 should have enough background.

Web Sites:

We will be using two websites for the course:

• Piazza -

The main website of the course will be the Piazza website.

Throughout the semester, I will also post in-class topics, readings, and other course materials (handouts, homework, solutions, etc.) on the piazza course web site. Besides regular posting of class material, Piazza allows for class discussion and Questions/ Answers in a collaborative manner so that everyone has access and can help. The system is highly catered to getting you help fast and efficiently from classmates, the TA, and myself. Rather than emailing questions to the teaching staff, I encourage you to post your questions on Piazza. Note that piazza allows for private questions to be asked too if needed. Please only use email to the teaching staff for issues that are unique to you (like an illness, a regrade request, a special circumstance, etc.).

If you were enrolled in the class at the beginning of the term, you should already be enrolled in the Piazza website. If not, you may sign up directly at the above webpage. A note on the site usage: please do not ask questions like "What is the answer to question 1?". I encourage you to ask questions related directly to course content and/or questions like typos/extra assumptions needed in homeworks. Asking for hints is also ok. If in doubt, please use the private question feature in piazza and if the instructor team feels it is appropriate, we will convert it to a public note.

• D2L - http://learn.uwaterloo.ca

The D2L website will be used mostly for its gradebook functionality. You should get automatic access to the course website using your WATIam account/password.

Grading

Course grades will be calculated according to the following formula:

Note: Students must pass the final exam to pass the course. If a student fails the final exam, the maximum final course grade that such student can earn is 5 points below the passing grade.

The grading for CO 452 will typically be done by a subset of the questions for CO 652.

Note on late assignments: Homework assignments handed in late will incur a late penalty of 50% per late day. Please take into account potential technical difficulties when submitting assignments and try to submit assignments well in advance of the deadline. Note that crowdmark allows for updating your submission, that is, if you want to submit something early on and then want to edit it later, you are allowed to do so (up until the deadline).

Homeworks

There will be one homework every 2 weeks (released/due Thursday), with the first one out on January 14, 2021.

Midterm

The midterm will be held on Wednesday, February 24, 2021. It will be a take-home exam, where students will get the exam from crowdmark and will have a limited time to do it on their own and submit their answers. Crowdmark supports the feature of individual timing, so that each person can download the exam at the time they find more convenient (within a 24h window) and their deadline will be adjusted accordingly automatically by Crowdmark.

Lecture participation mark

- 1. On the Monday of each week, I will release videos covering the material of the week.
- 2. On those videos, some results will have their proofs skipped and others will have details left out I will explicitly mark this with a green "Show" mark. The results that are skipped you don't need to go over the proofs, but you need to know the results you may need to apply them in other settings.
- 3. You are supposed to figure out the details on your own of the "Show" parts after you watch the videos.
- 4. There will be a weekly synchronous session (SS). It will be held on Mondays, at 10:30am, Waterloo time (note that this time slot is the Tutorial time slot in your schedule).
- 5. The class will be divided in groups of 2/3 students each.
- 6. In each (SS), a responsible group will be assigned to show to the whole class the details of the proofs with a "Show" mark for that given week.
- 7. The group will record a video (\leq 30mins long) showing those detailed proofs and send me the day before the (SS).
- 8. During the (SS), I will play the video and everyone can ask questions, which will be addressed by someone from the group.
- 9. A grade of up to 8% will be given for the whole responsible group. I will not evaluate video editing quality, but correctness and clarity in explaining the concepts.
- 10. A grade of up to 7% will be given for students that attend the (SS) as follows:
 - Attending 1-5 (SS): 1% per (SS) attended
 - Attending 6-8 (SS): 6%
 - Attending 9+ (SS): 7%
- 11. Attendance will be taken by polls done at the time of the (SS).

12. Students that feel they will be unable to complete the requirements for the lecture participation mark should contact me as soon as possible to make alternative arrangements.

Reviewing: Every student has the right to request a review of their homeworks and exams if they feel that there was a mistake in the correction and/or grading. The request must be done in writing. Students must send an email to both myself and the TA, stating the question to be reviewed and the reason you believe that the correction/grading was incorrect. The deadline for requesting a review is 1 (one) week after the graded work was handed out to the whole class.

Collaboration and acceptable sources of help:

For homework assignments, students are allowed to collaborate. Note that it is **NOT AC-CEPTABLE** to share your solutions with other students. Collaboration in homeworks is allowed in the sense that students are allowed to discuss with each other ideas on how to solve the problem, explain concepts, even point out details that should be taken care of, or pitfalls to avoid. Each student must, however, write their own solution independently. Copying someone else's work will be considered a violation of academic integrity. You also must state explicitly the names of the students you collaborated with.

Obviously for the participation mark, you are allowed to collaborate with your group freely.

For all other work, no collaboration is allowed.

Any violation of academic integrity will yield a mark of 0 in the corresponding assessment and will be reported to the appropriate Associate Dean.

On the INC Mark:

A grade of INC will only be awarded to students who cannot write the final exam for exceptional and justifiable reasons. In addition such students need to be in good standing prior to the final exam. To be in good standing a student must have a passing average in the assignments and the midterm. If you are not in good standing and/or miss the final exam due to a reason considered to be non-justifiable, then you will receive a grade of DNW.

Mental Health Support:

The Faculty of Math encourages students to seek out mental health support if needed. On-campus Resources:

- Campus Wellness https://uwaterloo.ca/campus-wellness/
- Counselling Services: counselling.services@uwaterloo.ca/ 519-888-4567 ext 32655
- MATES: one-to-one peer support program offered by Federation of Students (FEDS) and Counselling Services: mates@uwaterloo.ca

• Health Services: located across the creek from the Student Life Centre, 519-888-4096.

Off-campus Resources:

- Good2Talk (24/7): Free confidential help line for post-secondary students. Phone: 1-866-925-5454
- Here 24/7: Mental Health and Crisis Service Team. Phone: 1-844-437-3247
- OK2BME: set of support services for lesbian, gay, bisexual, transgender or questioning teens in Waterloo. Phone: 519-884-0000 extension 213

Diversity: It is our intent that students from all diverse backgrounds and perspectives be well served by this course, and that students' learning needs be addressed both in and out of class. We recognize the immense value of the diversity in identities, perspectives, and contributions that students bring, and the benefit it has on our educational environment. Your suggestions are encouraged and appreciated. Please let us know ways to improve the effectiveness of the course for you personally or for other students or student groups. In particular:

- We will gladly honour your request to address you by an alternate/preferred name or gender pronoun. Please advise us of this preference early in the semester so we may make appropriate changes to our records.
- We will honour your religious holidays and celebrations. Please inform of us these at the start of the course.
- We will follow AccessAbility Services guidelines and protocols on how to best support students with different learning needs.

Academic Integrity: In order to maintain a culture of academic integrity, members of the University of Waterloo community are expected to promote honesty, trust, fairness, respect and responsibility.

[Check https://uwaterloo.ca/academic-integrity/ for more information.]

Grievance: A student who believes that a decision affecting some aspect of his/her university life has been unfair or unreasonable may have grounds for initiating a grievance. Read Policy 70, Student Petitions and Grievances, Section 4,

http://www.adm.uwaterloo.ca/infosec/Policies/policy70.htm.

When in doubt please be certain to contact the department's administrative assistant who will provide further assistance.

Discipline: A student is expected to know what constitutes academic integrity to avoid committing academic offenses and to take responsibility for his/her actions. A student who is unsure whether an action constitutes an offense, or who needs help in learning how to

avoid offenses (e.g., plagiarism, cheating) or about "rules" for group work/collaboration should seek guidance from the course professor, academic advisor, or the undergraduate associate dean. For information on categories of offenses and types of penalties, students should refer to Policy 71, Student Discipline,

http://www.adm.uwaterloo.ca/infosec/Policies/policy71.htm For typical penalties check Guidelines for the Assessment of Penalties, http://www.adm.uwaterloo.ca/infosec/guidelines/penaltyguidelines.htm

Appeals: A decision made or penalty imposed under Policy 70, Student Petitions and Grievances (other than a petition) or Policy 71, Student Discipline may be appealed if there is a ground. A student who believes he/she has a ground for an appeal should refer to Policy 72, Student Appeals,

http://www.adm.uwaterloo.ca/infosec/Policies/policy72.htm.

Note for students with disabilities: AccessAbility Services, located in Needles Hall, Room 1401, collaborates with all academic departments to arrange appropriate accommodations for students with disabilities without compromising the academic integrity of the curriculum. If you require academic accommodations to lessen the impact of your disability, please register with AccessAbility Services at the beginning of each academic term.