INDOCRYPT 2010, 13 December 2010

Getting a Few Things Right and Many Things Wrong

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Outline of Talk

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- Isogenies – common wisdom might (or might not) be wrong
- Survey of my own experiences being wrong most of the time
- History of embarrassing moments in “provable security”
- What is to be done? -- the keyword that unlocks the solution to this conundrum

Reference for the first part of the talk:

"ECC: The Serpentine Course of a Paradigm Shift"
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See especially Section 11 concerning the
security implications of isogeny walks.

Also see our videoabstract at
http://www.youtube.com
(search for "elliptic curve cryptography")

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- Smart houses and robotic butlers

- Controversy over whether or not Arizona's ancient Native American tribes were warlike

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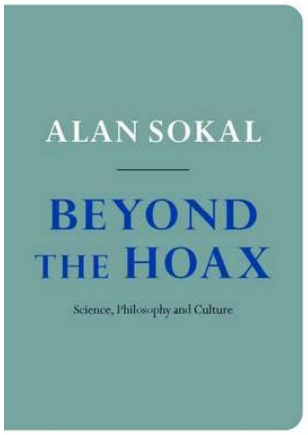
- The impact of Alan Sokal's "hermeneutics of quantum gravity"



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- In cryptography, for greatest security choose parameters as randomly as possible.
- In elliptic/hyperelliptic curve cryptography it's safest to choose the defining equation to have random coefficients.
- It's okay to use special curves for reasons of efficiency if you insist, but some day that choice might come back to bite you.

In 1991, I proposed the use of the non-supersingular $\mathbf{F}_2$ -curves (also called anomalous binary curves)

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because they seemed to have some efficiency advantages over random curves.

The U.S. National Security Agency (NSA) liked these curves, and at Crypto 1997 J. Solinas gave a talk presenting a thorough and definitive treatment of how to optimize ECC operations on these curves.

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Some people have been mistrustful of this family of curves, in part because of the "conventional wisdom" given above.

Another reason for mistrust is the syllogism:

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"NSA wants us to use these curves.

"We don't trust the NSA.

"Therefore we don't trust these curves."

My analysis is: NSA isn’t a monolith.

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Just because an NSA mathematician recommends a family of curves, it doesn't necessarily follow that they're no good.

WHITE HATS

BLACK HATS

N.S.A.

Mathematicians & Engineers

high intelligence, high ethics

Spies, Spooks & Bureaucrats

low intelligence, low ethics

WHITE HATS

BLACK HATS

Corporate world

Researchers & Engineers

high intelligence, high ethics

Marketing People & Executives

low intelligence, low ethics

WHITE HATS

BLACK HATS

University world

Professors

Administrators

high intelligence, high ethics

low intelligence, low ethics

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Actually, the empirical basis for that slide comes from the U.S., not from India.

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There are various scenarios in which someone who chooses ECC with a special curve might end up better off than someone else who chooses a random curve.

Some such scenarios are suggested by recent work on isogenies.

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Some such scenarios are suggested by recent work on isogenies. (For more details see Section 11 of the “serpentine course” paper.)

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(An isogeny between isomorphic curves is called an “endomorphism.”)

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By constructing isogenies from E to other curves in its isogeny class, we can transport its discrete log problem to equivalent discrete log problems on the other curves.

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Within the isogeny class of E, how widely different in size can two endomorphism rings be?

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A random curve is almost certain to have a discriminant with very small square part.

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By a result of Jao, Miller, and Venkatesan, one can use sequences of isogenies ("isogeny walks") to efficiently travel randomly and uniformly throughout the isogeny class of B-571, which consists of approximately 2^{285} curves.

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It has discriminant $-7(P_{22}P_{263})^2$ divisible by the square of a 22-bit prime times a 263-bit prime.

Most of the isogenous curves have very small endomorphism ring with index divisible by the 263-bit prime, and no one knows how to construct such a large degree isogeny from K-571 to such a curve.

At the very most, one can map K-571 to the curves whose endomorphism ring has index divisible at most by the 22-bit prime.

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There are approximately 2^{22} – that is, 40 lakhs – such curves.

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Let's suppose that an algorithm were found that solves the elliptic curve discrete log problem (ECDLP) in time T_1 in a proportion ε (a very small but not negligible proportion) of all elliptic curves over $\mathbf{F}_q$, where the property of being a "weak" curve is independent of isogeny and endomorphism class.

Let's also suppose that the "weak" property can be spotted quickly once we get to the curve.

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Then a discrete log on E can be found in time roughly $T_1 + T_2/\varepsilon$, where T_2 denotes the time for constructing an isogeny that moves the ECDLP to another curve –

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Then a discrete log on E can be found in time roughly $T_1 + T_2/\varepsilon$, where T_2 denotes the time for constructing an isogeny that moves the ECDLP to another curve – assuming, of course, that the class of curves to which isogenies from E can be constructed in time less than T_2 contains more than $1/\varepsilon$ curves.

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In other words, it is the possibility of random isogeny walks through an endomorphism class that under certain circumstances might make a random curve less secure than a special curve.

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Despite the reluctance of many cryptographic researchers to admit it, in fact cryptography is as much an art as a science.

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(i) $p = A^2 + B^2$ is prime;

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is prime.

Then the elliptic curve E over $\mathbf{F}_p$ defined (for suitable a in $\mathbf{F}_p$) by $y^2 = x^3 - ax$ is isolated, in the sense that it's infeasible to construct isogenies to other curves.

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However, the only NIST-recommended curves over a prime field are random ones.

Maybe they're right to do that…

Or maybe not.

PART II:

PART II: AN ABBREVIATED HISTORY OF MY MISJUDGMENTS AND ERRONEOUS BELIEFS DURING THE PAST 25 YEARS

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First major one:

In the late 1980's it seemed (to me at least) that any elliptic curve group would be secure as long as its order is prime or almost prime.

With that condition, I thought, all curves were created equal, and were endowed with an intractable ECDLP.

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So for pedagogical reasons why not use the simplest possible curves? And this is what I often did -- in my introductory book published in 1987 and in my articles and talks in the 1980's.

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Just choose p so that $(p+1)/4$ or $(p+1)/6$ is prime, and ECC is secure...

Or so I thought.

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And for supersingular curves, such as the two written above, the extension degree is very small. Usually it's 2, as in the above cases.

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What surprised me as much as the MOV attack that killed supersingular curves in 1991 was that 10 years later they made a roaring comeback from the grave – when pairing-based crypto took the research community by storm.

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In 1989, when I first proposed Hyperelliptic Curve Cryptography, if you'd asked me I would have explained what I saw as the main potential advantage of HCC over ECC by speculating that most likely the higher the genus, the more security you'd get.

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That's what I thought…

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But that turned out to be exactly the opposite of what happened.

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My favorite illustrative example, which was a genus-191 curve over $\mathbf{F}_2$, immediately became totally insecure.

After subsequent work by Gaudry, Diem, and others, it now seems that anything bigger than genus 2 is less secure than genus 1 or 2.

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I couldn’t have been more wrong in my intuition about the greater security of high genus!

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In the early 1990's, Mike Fellows and I became captivated by the notion that, despite the fiasco with knapsacks, good cryptosystems could in fact be constructed from NP-hard combinatorial problems.

We even wrote a paper with the exuberant title "Combinatorial Cryptosystems Galore!"

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"The most attractive feature of our cryptosystem was the name that Mike thought up for it: *Polly Cracker.*

"It was very inefficient, and before long some papers were published that indeed cracked the code."

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But otherwise you could use whatever field you most enjoy working with, and security is unaffected by that choice.

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His idea was to transport the ECDLP
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Fortunately, other people (such as Scott Vanstone) had had better instincts than I had, and all commercial implementations and all ECC standards used prime fields or prime-degree extensions of $\mathbf{F}_2$.

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In early 1998 I published *Algebraic Aspects of Cryptography.* In a section titled “Cultural Background” I discussed the Birch and Swinnerton-Dyer Conjecture, after which I essentially apologized to my readers for taking up their valuable time with something that, while mathematically important, has no relevance for cryptography.

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A mere 8 months later I was eating those words, after I received an email from J. Silverman outlining a striking new approach to the ECDLP.

It was a variant – somewhat backwards – version of index calculus, and for that reason Silverman called it “xedni calculus.”

What was most alarming for ECC people was that Silverman used the heuristics of the BSD Conjecture (and an analytic rank formula of Mestre) to boost the likelihood of a successful attack on the ECDLP.

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So the exact same mathematics that 8 months earlier I had been dismissing as irrelevant to cryptography turned out to be at the center of our fears about xedni.

After a lot of initial worry about xedni (fueled by our concern that RSA would use xedni as a weapon in their public relations battle with ECC, which was still going strong in 1998), I found that we could use the height function to show that xedni wouldn't work.

I was so thrilled about this success in defending ECC that I gave a talk at ECC 2000 titled

"Miracles of the Height Function: A Golden Shield Protecting ECC"

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At ECC 2007, Silverman made a similar analysis for all 4 ways one could try index or xedni with liftings to global fields.

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For example, Gaudry and Diem found subexponential index calculus algorithms for the ECDLP on elliptic curves defined over the degree-m extension of $\mathbf{F}_q$ as m and q grow suitably.

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I've learned the hard way that it's foolish to go around talking about "Golden Shields" that protect security of ECC protocols or anything else. And I don't believe in miracles anymore.

In each case of misjudgment, I thought
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It likes to play jokes on us.

I learned that, much as we might wish to convey an impression of self-confidence and mathematical certainty to the outside world, to our colleagues, and to ourselves, such self-confidence is rarely justified.

PART III:

PART III: SOME SCANDALS IN THE HISTORY OF "PROVABLE SECURITY"

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For more details, please see the series of "Another Look" papers by Menezes and me on the eprint.iacr.org website.

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Soon after, MasterCard and Visa included OAEP in their SET electronic payment standard.

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This type of thing causes a credibility problem.

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In February of that year, Hugo Krawczyk – one of IBM's top cryptographers – submitted a paper to Crypto 2005 in which he claimed to have found flaws in the Menezes—Qu—Vanstone (MQV) key agreement protocol.

A case that I think is even more scandalous occurred in 2005.

In February of that year, Hugo Krawczyk – one of IBM's top cryptographers – submitted a paper to Crypto 2005 in which he claimed to have found flaws in the Menezes—Qu—Vanstone (MQV) key agreement protocol.

He especially criticized MQV for lacking a "proof of security."

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If his claims had been valid, this would have been a major embarrassment not only to Menezes and his coauthors, but also to the NSA, which had licensed MQV (and 25 other patented protocols) from Certicom and whose experts had studied it closely.

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When Menezes was finally sent a copy – after the Program Committee had already accepted it – he saw that, first of all, Krawczyk's objections to MQV were without foundation.

Moreover, he discovered that the paper's main argument ("proof") was fallacious.

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Menezes quickly found not only that the proof was fallacious, but that certain of the HMQV protocols succumb to the same attacks as MQV would have if those checks had not been put in it.

Top experts such as Krawczyk – and the members of the Crypto 2005 Program Committee – would not have made this blunder if they had not been mesmerized by the so-called proof.

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So-called “proofs of security” often do more harm than good because they lull people into a false sense of security and cause them to take leave of common sense.

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In particular, they highlight their reductionist security argument (that reduces a supposedly intractable problem to a successful attack of a specified sort) using terminology designed to convince the reader that their protocols have been “proved” to be secure.

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> "…permits savings on bandwidth and storage… substantially improves computational efficiency and scalability over any existing scheme with suitable functionality…

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They also broke the corresponding protocol.

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The amusing thing about this example is that about a year later Huang, Lee, and Yung showed that a crucial security proof in this paper was fallacious.

They also broke the corresponding protocol. (The flaw was catastrophic – details are given in my March 2010 AMS Notices article with Menezes.)

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> “This is the essence of modern cryptography, and what has transformed cryptography from an art to a science. The importance of this idea cannot be over-emphasized.”

Meanwhile, anyone who's dismayed by the large number of fallacious proofs in the provable security literature is supposed to be consoled by the prospect that advances in "theorem-proving" software will soon make it possible to prove the security of our protocols automatically

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Human mistakes and failings will supposedly disappear from the process of establishing guarantees of security.

I discuss this naïve and dubious notion in
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And anyone who's bewildered by the exotic nature of some of the cryptographic assumptions that underlie security proofs for many of the pairing-based protocols is supposed to be reassured by Boyen's exuberant explanation at the Pairings-2008 conference:

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"…in comparison to the admittedly quite simpler algebraic structures of twentieth-century public-key cryptography… the new 'bilinear' groups offer a much richer palette of cryptographically useful trapdoors than their 'unidimensional' counterparts…"

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Indeed, some of the assumptions used by leading researchers turned out to be significantly weaker than they had thought.

In our March 2010 article in the AMS Notices, Menezes and I explain why we do not share Boyen's enthusiasm for the bold and exotic assumptions that populate the landscape in pairing-based cryptography.

Indeed, some of the assumptions used by leading researchers turned out to be significantly weaker than they had thought.

And the jury is still out on most of the other assumptions, since hardly any of them have been investigated thoroughly.

Thus, on the one hand, we see the trend of bold and boastful writing by cryptographic researchers.

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How can we reconcile the disciplinary culture of our field with reality?

To paraphrase Lenin,

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What is to be done?

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Should we retreat into despair and cynicism?

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That is not the message of my talk.

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That is not the message of my talk.

There is a better answer to this conundrum.

There is a keyword that unlocks the puzzle!

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In Chapter 13, Verses 8-12 of the Bhagavad Gita, the all-knowing one, Sri Krishna, describes the qualities that are necessary for knowledge.

Here is the list, first in Sanskrit,
then in transliteration, then in English.

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then in transliteration, then in English.

Note what the very first quality is.

अमानित्वमदम्भित्वमहिंसा क्षान्तिरार्जवम् ।
आचार्योपासनं शौचं स्थैर्यमात्मविनिग्रहः ॥ ७ ॥

Amānitvam-adambhitvam-ahimsā kṣāntir-ārjavam
Ācaryopāsanam śaucam sthairyam-ātma-vinigrahaḥ

7. Humility; unpretentiousness; non-violence; forbearance; uprighteness; service of the teacher; purity; steadiness; self-control;

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Amaanitvam.