Guided Symbolic GPT

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Abstract

Symbolic regression aims to discover interpretable mathematical expressions that describe the relationship between input and target variables, a task traditionally performed manually in scientific research. In this work, we introduce *Guided Symbolic GPT*, an extension of Symbolic GPT that incorporates guiding tokens encoding mathematical properties such as symmetry and convergence. These tokens provide structural information that conditions generation and improves robustness and interpretability. **Our contribution is the use of guiding tokens to inject mathematical properties into language-model-based symbolic regression**, and **our results show that Guided Symbolic GPT consistently outperforms the baseline Symbolic GPT on symbolic regression benchmarks while using the same model architecture**.

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Chapter 1

Introduction

1.1 Symbolic Regression

Symbolic regression is the task of identifying a function that captures the relationship between one or more *input variables* and one or more *target variables*. This involves searching over the space of closed-form mathematical expressions composed of operators and functions such as $\sin, \cos, \log, \exp$, and others. The ideal expression must be sufficiently complex to accurately model the target variables given the input variables, yet simple enough to remain robust in the presence of noise.

The motivation for symbolic regression arises from the automated discovery of physical laws, a process historically carried out manually Koyré [2013]. Physical laws are typically characterized by a limited number of variables and constants, making them well-suited to symbolic regression. Importantly, symbolic regression differs fundamentally from neural network–based approaches: whereas neural networks provide black-box approximations, symbolic regression seeks interpretable, closed-form representations that can often yield mechanistic insights. Furthermore, symbolic regression can incorporate dimensional constraints of variables, enabling physically meaningful formulations. These advantages have already facilitated applications in diverse domains, including the discovery of ecological dynamics Chen et al. [2019], the analysis of material stability He and Zhang [2021], mortality prediction after major surgery Arina et al. [2025], and the identification of inconsistencies in established physical models Kaheman et al. [2019].

Formally, symbolic regression is the problem of finding a mathematical function f defined over the input variables $\mathbf{x} = [x_1, \ldots, x_d]$ given a dataset $D = (\mathbf{x_i}, y_i)_{i=1}^{n}$, such that $f(\mathbf{x}_i) = y_i$, or $f(\mathbf{x}_i) \approx y_i$ in the presence of noise. Moreover, the learned function f should generalize to a test dataset $D' = (\mathbf{x}j, y_j)j = 1^m$ generated by the same underlying process. A further desirable property is parsimony: f should be as simple as possible while still adequately fitting the data.

Unlike conventional machine learning methods, symbolic regression is inherently more

challenging. The task requires exploring an immense, partially discrete, and often hierarchically nested space of candidate functions. As a result, symbolic regression is known to be NP-hard Virgolin and Pissis [2022].

The key contributions of our approach are summarized as follows:

- Introduces **Guided Symbolic GPT**, extending the SymbolicGPT transformer framework with *guiding tokens* that encode mathematical properties such as symmetry and convergence.
- Injects *domain-informed meta-information* directly into the model's generative process, contrasting with scaling-focused approaches.
- Improves model **interpretability** and **robustness** without increasing model capacity.

In this thesis, **Chapter 2** reviews related work in transformer-based and genetic-algorithm–based approaches to symbolic regression. **Chapter 3** presents our methodology, detailing the architecture of Guided Symbolic GPT and the supporting technical components. **Chapter 4** reports experimental results evaluating the performance of Guided Symbolic GPT across a range of tasks. **Chapter 5** offers a discussion of the implications, limitations, and broader impact of our approach. Finally, **Chapter 6** concludes the thesis and highlights promising directions for future research.

Chapter 2

Related works

Symbolic regression (SR) aims to automatically discover interpretable mathematical expressions that describe relationships among variables. Early approaches were largely based on genetic programming (GP) McKay et al. [1995], Augusto and Barbosa [2000], Korns [2013], where candidate expressions evolve through operators such as mutation and crossover. A landmark study demonstrated that evolutionary SR can rediscover fundamental physical laws directly from data, establishing SR as a tool for automated scientific discovery Schmidt and Lipson [2009]. Subsequent GP variants explored probabilistic search strategies and hybrid optimization schemes Gong et al. [2022].

To address scalability and data efficiency, alternative formulations recast SR as a sparse regression problem. Methods such as SINDy identify parsimonious governing equations using compressed-sensing principles Kaiser et al. [2018]. Related techniques have been applied across physics and ecology to recover interpretable dynamical systems Chen et al. [2019], Quade et al. [2016]. In parallel, approaches incorporating physical priors and dimensional constraints, including AI Feynman and the Symbolic Physics Learner, improved accuracy and interpretability in low-data regimes Udrescu and Tegmark [2020], Sun et al. [2022]. Unit-consistency constraints were later integrated into SR frameworks to further guide physics-informed equation discovery Tenachi et al. [2023].

Neural symbolic regression replaced heuristic search with learned inductive priors. Early methods used neural networks to bias symbolic search or parameterize expression trees Li et al. [2019], Mundhenk et al. [2021], Petersen et al. [2019]. Learned guidance was shown to accelerate convergence and improve generalization in neural-guided GP, deep symbolic regression models, and reinforcement-based systems Mundhenk et al. [2021], Xu et al. [2023], Tian et al. [2024]. Transformer architectures, originally introduced for sequence modeling Vaswani [2017], later enabled direct sequence generation of symbolic expressions from data in SR settings.

Large-scale generative transformers marked a turning point for SR. SymbolicGPT demonstrated that pretrained language models can generate compact and interpretable formulas without explicit evolutionary search Valipour et al. [2021]. Later work focused on

scaling behavior, expression validity, and sample efficiency Biggio et al. [2021], Kamienny et al. [2022], Vastl et al. [2024]. Alternative designs explored continuous optimization strategies for symbolic regression Liu et al. [2023], Holt et al. [2023], Scholl et al. [2024], while generative flow networks provided a complementary framework for structured symbolic generation Li et al. [2023]. Sequential decision-making was further incorporated through neural-guided dynamics models and transformer-based planning approaches Li et al. [2024], Shojaee et al. [2023].

Benchmarking and meta-learning efforts have shaped the modern SR landscape. Standardized datasets support reproducible evaluation of generalization Olson et al. [2017], Matsubara et al. [2022], while early benchmarking efforts emphasized interpretability and domain relevance Zhang et al. [2012], with recent extensions providing more systematic and reproducible evaluations de Franca et al. [2024]. Recent surveys highlight that interpretability, physical consistency, and computational scalability remain open challenges Makke and Chawla [2024].

This work builds on transformer-based symbolic regression by introducing **Guided Symbolic GPT**, which incorporates guiding tokens encoding mathematical properties such as symmetry and convergence. Rather than increasing model capacity or focusing solely on scaling, domain-informed meta-information is injected directly into the generative process. This design improves interpretability and robustness and aligns with emerging work on controllable and explainable neural symbolic regression Bendinelli et al. [2023], Li et al. [2022], Virgolin and Pissis [2022]. By combining symbolic reasoning with learned inductive priors, Guided Symbolic GPT advances reliable and interpretable automated scientific discovery.

Chapter 3

Methodology

3.1 Data

The performance of Symbolic GPT is directly influenced by the quality and diversity of its training data. To promote robustness across a wide range of input domains and to mitigate overfitting to specific sets of equations, we designed a specialized data generation pipeline.

3.1.1 Computation Graph

At the core of this pipeline lies the *Computation Graph*, which serves as the foundation for all symbolic representations. The graph is structured as a binary tree, where each node corresponds to either an operator (unary or binary), a variable placeholder, a constant placeholder, or an empty placeholder. Representing expressions as graphs, rather than sequences, enables richer analyses through algorithms such as breadth-first search (BFS) or depth-first search (DFS).

The binary tree structure provides a simple yet powerful and extensible framework, since most mathematical and physical functions can be decomposed into unary and binary operations. Notably, any unary operator can be reformulated as a binary operator by assigning an empty placeholder as its right child. This design choice ensures both generality and consistency in the representation of symbolic expressions.

3.1.2 Metadata Generation

In this component, we introduce *mathematical property tokens*, which are later utilized by Guided Symbolic GPT to produce more robust and reliable outputs. These properties are well established in the mathematical literature and are commonly used by experts to evaluate symbolic expressions. By incorporating such properties, the model gains additional structural information about the underlying datasets, enabling more principled

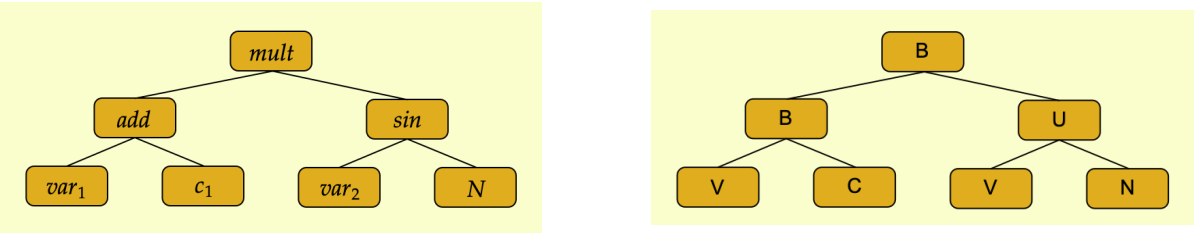


Figure 3.1: Left: Computation Graph of $(var_1 + C_1) \times sin(var_2)$. Right: Structure of the Computation Graph

assessments of the generated outputs.

In a general set up let $f : R^d \to R$ and the input to the function is $\mathbf{x} = (x_1, x_2, ..., x_d)$, and for all except i (x_i) we use $\mathbf{x}_{-i}$.

1. Symmetric

$$f(\mathbf{x}_{-i}, x_i) = f(\mathbf{x}_{-i}, -x_i) \quad \text{for all admissible } (\mathbf{x}_{-i}, x_i).$$

2. Odd Symmetric

$$f(\mathbf{x}_{-i}, x_i) = -f(\mathbf{x}_{-i}, -x_i) \quad \text{for all admissible } (\mathbf{x}_{-i}, x_i).$$

3. Linear Relation

$$f(\mathbf{x}_{-i}, x_i) = a(\mathbf{x}_{-i})\, x_i \quad \text{for some function } a$$

4. Convergence to Zero at Infinity

$$\lim_{\|x_i\| \to \infty} f(x) = 0.$$

5. Monotonic increasing magnitude: There exists $R \geq 0$ such that for $|x_i| \geq R$, $|f|$ is nondecreasing in $|x_i|$ with $\mathbf{x}_{-i}$ fixed.

6. Variable Separability

$$f(\mathbf{x}_{-i}, x_i) = g(\mathbf{x}_{-i})\, h(x_i) \quad \text{for some functions } g \text{ and } h.$$

7. Swapping Variable

$$f(x_1, \ldots, x_i, \ldots, x_j, \ldots, x_d) = f(x_1, \ldots, x_j, \ldots, x_i, \ldots, x_d)$$

Following algorithms demonstrate how we generate these meta-information based on the equations.

Algorithm 1 Symmetric_Check (multi-variable pseudocode summary)

Require: Function $f(x_1, x_2, \ldots, x_n)$, number of variables n_var, constants
Ensure: List of Booleans indicating if f is symmetric in each variable
1: Initialize $diff\_threshold \leftarrow 10^{-3}$, $n\_check \leftarrow 30$, $max\_try \leftarrow 50$
2: **for** each variable var_i **do**
3: Collect up to n_check valid samples by randomly assigning inputs and evaluating f
4: For each sample, negate var_i and evaluate f, then compare outputs within $diff\_threshold$
5: If all samples pass, mark $check\_list[i] \leftarrow True$
6: **end for**
7: **return** $check\_list$

Algorithm 2 Odd_Function_Check (multi-variable pseudocode summary)

Require: Function $f(x_1, x_2, \ldots, x_n)$, number of variables n_var, constants
Ensure: List of Booleans indicating if f is odd in each variable
1: Initialize $diff\_threshold \leftarrow 10^{-3}$, $n\_check \leftarrow 30$, $max\_try \leftarrow 50$
2: **for** each variable var_i **do**
3: Collect up to n_check valid samples by randomly assigning inputs and evaluating f
4: For each sample, negate var_i, evaluate f, and check $out \cdot counter\_out \leq 0$ and $|out + counter\_out| < diff\_threshold$
5: If all samples pass, mark $check\_list[i] \leftarrow True$
6: **end for**
7: **return** $check\_list$

Algorithm 3 Linearity_in_Variable_Check (No Intercept)

Require: Function $f(x_1, \ldots, x_n)$, number of variables n_{var}
Ensure: List `isLinear`[1:n_{var}] indicating whether f is linear in each variable (i.e., $f(x_{-i}, x_i) = a(x_{-i})x_i$)

Set $m \leftarrow 30$ {number of random draws of x_{-i}}
Set $k \leftarrow 5$ {number of grid points for x_i}
Set $R2_{\text{threshold}} \leftarrow 0.995$
for $i = 1$ to n_{var} **do**
  Initialize empty list $R2\_values$
  for $j = 1$ to m **do**
    Randomly sample x_{-i} from the domain
    Sample a small grid $\{x_i^{(1)}, \ldots, x_i^{(k)}\}$ from the domain
    for $t = 1$ to k **do**
      $y_t \leftarrow f(x_{-i}, x_i^{(t)})$
    end for
    Fit least-squares slope $\hat{a}$ to $y_t \approx \hat{a}\, x_i^{(t)}$ **(no intercept)**
    Compute and record R^2 for the no-intercept fit; append to $R2\_values$
  end for
  $\tilde{R}2 \leftarrow \text{median}(R2\_values)$
  if $\tilde{R}2 \geq R2_{\text{threshold}}$ **then**
    `isLinear`[i] $\leftarrow$ `True`
  else
    `isLinear`[i] $\leftarrow$ `False`
  end if
end for
return `isLinear`

Algorithm 4 Converge_to_Zero_Check (pseudocode summary)

Require: Function $f(x_1, x_2, \ldots, x_n)$, number of variables n_var, constants
Ensure: List of Booleans indicating if f converges to zero for each variable

Initialize parameters: step sizes, limits, $n\_check \leftarrow 30$, $max\_try \leftarrow 50$, threshold
for each variable var_i **do**
  Collect up to n_check valid samples by adding initial large step to var_i and evaluating f
  For each sample, take multiple incremental steps, record $(|var_i|, |f|)$, and check monotonic decrease
  Check a final large step to ensure f is below threshold
  If all samples pass, mark $check\_list[i] \leftarrow True$
end for
return $check\_list$

Algorithm 5 Monotonic_Increasing_Magnitude_Check (pseudocode summary)

Require: Function $f(x_1, x_2, \ldots, x_n)$, number of variables n_var, constants
Ensure: List of Booleans indicating if $|f|$ increases monotonically with each variable
1: Initialize parameters: step range $[1, 3]$, $lim\_n\_steps \leftarrow 10$, $n\_check \leftarrow 30$, $max\_try \leftarrow 50$
2: **for** each variable var_i **do**
3: Collect up to n_check valid samples by randomly initializing inputs and evaluating f
4: For each sample, take multiple increasing steps in var_i, record $(|var_i|, |f|)$ pairs
5: Sort values by $|var_i|$ and check that $|f|$ strictly increases, allowing small fluctuations near zero
6: If all samples confirm monotonic increase, mark $check\_list[i] \leftarrow True$
7: **end for**
8: **return** *check_list*

Algorithm 6 Variable_Separability_Check (pseudocode summary)

Require: Function $f(x_1, x_2, \ldots, x_n)$, number of variables n_var, constants
Ensure: List of Booleans indicating if each variable is separable
1: Initialize thresholds: $zero\_threshold \leftarrow 10^{-2}$, $d\_sep\_threshold \leftarrow 2 \times 10^{-3}$, $n\_check \leftarrow 30$, $max\_try \leftarrow 50$
2: **for** each variable var_i **do**
3: Collect up to n_check valid samples by generating two random input sets
4: For each sample, swap the values of var_i between the two inputs and evaluate all combinations
5: Compute $d\_sep = |out_1 - out\_comb_1 \cdot out\_comb_2/out_2|/|out_1|$ and compare with $d\_sep\_threshold$
6: If all samples satisfy the threshold, mark $check\_list[i] \leftarrow True$
7: **end for**
8: **return** *check_list*

Algorithm 7 Swap_Variable_Check (pseudocode summary)

Require: Function $f(x_1, x_2, \ldots, x_n)$, number of variables n_var, constants
Ensure: Dictionary indicating if swapping each pair of variables preserves f
1: Initialize $n\_check \leftarrow 30$, $max\_try \leftarrow 50$, $zero\_threshold \leftarrow 10^{-3}$
2: Initialize $swap\_dict[i, j] \leftarrow False$ for all $i < j$
3: **for** each variable pair (var_i, var_j) with $i < j$ **do**
4: Collect up to n_check valid samples by generating random inputs
5: For each sample, swap var_i and var_j and evaluate f before and after swap
6: Compare outputs using $zero\_threshold$; record True if all samples satisfy threshold
7: If all samples pass, set $swap\_dict[i, j] \leftarrow True$
8: **end for**
9: **return** *swap_dict*

3.1.3 Huge Random Formula Set Generation Pipeline

This part outlines the process for generating a large set of random computation graphs.

1. **Structure Generation and Organization**: We define graph structures that serve as templates for computation graphs. Nodes within these structures act as placeholders for binary and unary operators, variable placeholders, constant placeholders, and null placeholders.

2. **Structure Selection**: To ensure a balanced dataset, we constrain sampling based on the number of variable placeholders and the depth of the binary tree, maintaining an upper threshold for each combination.

3. **Skeleton Sampling**: For a given depth and number of variable placeholders, we generate up to 100 skeletons by mapping binary (B) and unary (U) placeholders to corresponding operators.

4. **Variable Assignment**: Each structure is instantiated into a formula of length n by replacing variable (V) placeholders with variables, ensuring all variables appear at least once. Any remaining V placeholders are randomly assigned variables.

5. **Redundant Tree Filtering**: We eliminate redundant computation graphs that contain trivial or simplifiable substructures, such as:

$$\log(\exp(\cdot)), \quad \text{pow}_2(\sqrt{\cdot}), \quad \text{neg}(\text{neg}(\cdot)), \ldots$$

3.2 Model

This model consists of two primary components: GPT and PointNet. The PointNet module transforms the tabular dataset into a point embedding that encapsulates the information contained within the original data. This embedding is then combined with the embedding generated by GPT from the previous tokens. The resulting representation is used by the model to produce the logits for the next token. Finally, by sampling from these logits, the model generates the most probable next token.

As illustrated in Figure 3.3, the tabular dataset is processed such that each row is transformed into an embedding through a neural network. Subsequently, a Global Max layer is applied to capture the maximum values across all embeddings, resulting in a unified representation of the dataset.

Guided Symbolic GPT builds upon Symbolic GPT Valipour et al. [2021], with the key distinction being the introduction of *guiding tokens* alongside equation tokens. These guiding tokens encode additional mathematical information, which enhances the model's ability to generate more accurate and reliable predictions.

As illustrated in Figure 3.4 and 3.2, the architecture remains largely the same, with the guiding tokens being the only major addition. These tokens represent mathematical properties—such as symmetry, convergence to zero at infinity, and others—that serve as inductive biases. By explicitly incorporating such properties into the tokenization process, Guided Symbolic GPT achieves more robust and interpretable predictions compared to its predecessor.

The tokenizer used in this model is trainable and is trained specifically on the dataset employed in this study. In addition, mathematical property tokens are incorporated as part of the token set. Overall, the model's vocabulary comprises approximately 3,434 tokens.

As shown below, each input sequence is constructed with a fixed length of 40 tokens. The sequence starts with a guiding segment consisting of 20 tokens, which includes the guiding tokens encoding mathematical properties as well as padding tokens when necessary. This guiding segment is followed by the ground-truth equation tokens and their corresponding padding, occupying the remaining 20 tokens of the sequence.

Example:

```
[META\_START] [EVEN:x1] [ODD:x2] [SEP:x1] [SWAP:x1\_x3] [LIN:x2]
[META\_END] [Gpad] [Gpad] [EQUATION\_START] [sin] [x1] [+] [cos] [x2] [*] [x3]
[EQUATION\_END] [pad] [pad] [pad]
```

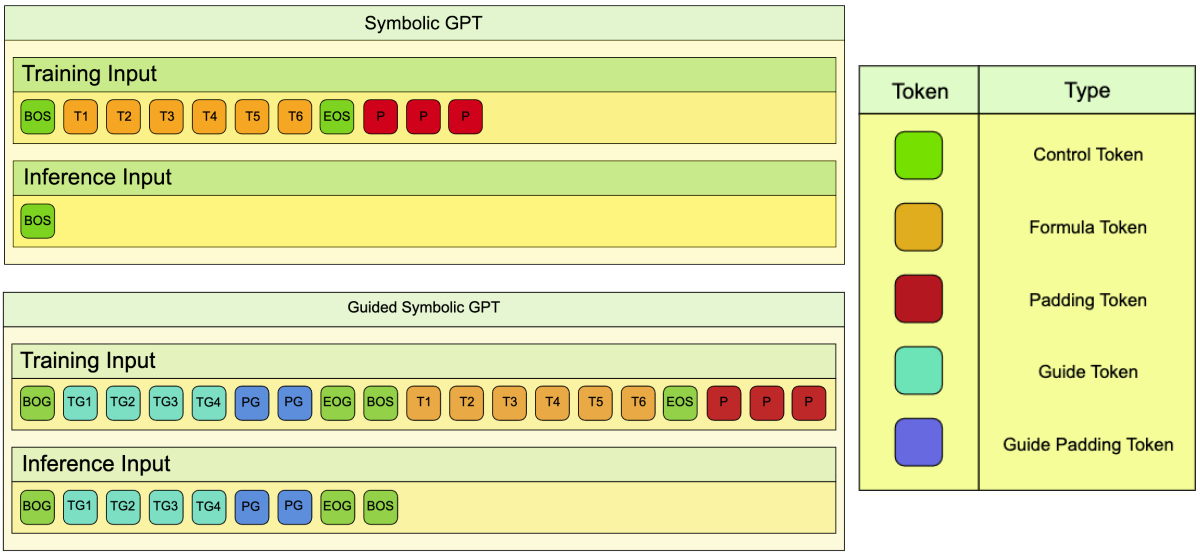
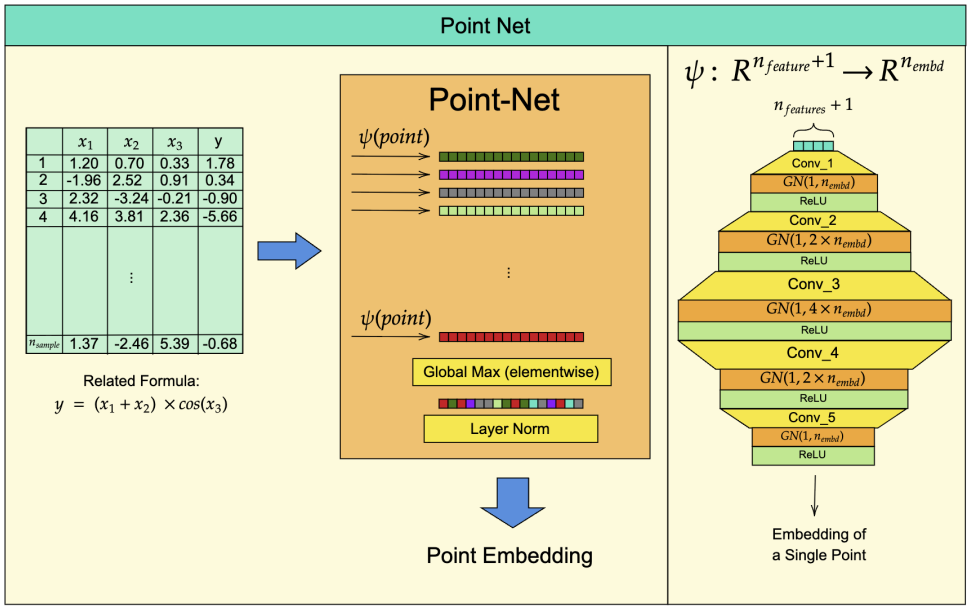


Figure 3.2: Token-level comparison between Symbolic GPT and Guided Symbolic GPT.

3.2.1 Training Details

To train the model, we employed a neural architecture comprising approximately 45 million parameters. The embedding dimension was set 512. The transformer component



	x_1	x_2	x_3	y
1	1.20	0.70	0.33	1.78
2	-1.96	2.52	0.91	0.34
3	2.32	-3.24	-0.21	-0.90
4	4.16	3.81	2.36	-5.66
		⋮		
n_{sample}	1.37	-2.46	5.39	-0.68

Figure 3.3: Point Net

consisted of 8 layers and 8 attention heads, with a dropout rate of 0.2. The attention block size was configured to 40 tokens, divided equally between 20 guiding tokens and 20 equation tokens. The model was trained for 10 epochs using a learning rate of 0.0001. Find the details in table 3.1

Operator set	$\{+, -, \times, \div, \sin, \cos, \exp, \log, gaussian, sqrt, inverse\}$
Variables	$d \in \{1, 2, 3\}$
Tree depth	max depth = 5
Constants	uniform in $[-3, 3]$
Input sampling	each $x_i \sim \mathcal{U}[-5, 5]$
Noise	none / Gaussian $\mathcal{N}(0, \sigma^2)$, $\sigma = 0.01$
#expressions (train/val/test)	70k / 15k / 15k
Seeds	data seed = 42, init seed = 42
Batch & optimizer	batch = 512, Adam (lr = 1×10^{-4})
Epochs	10
Embedding dimension	512
#Layer & #Attention head	8,8
Points per expression	500 points
Token budget	20 meta + 20 equation; truncation policy as above

Table 3.1: Summary of the dataset generation and training configuration used in our experiments.

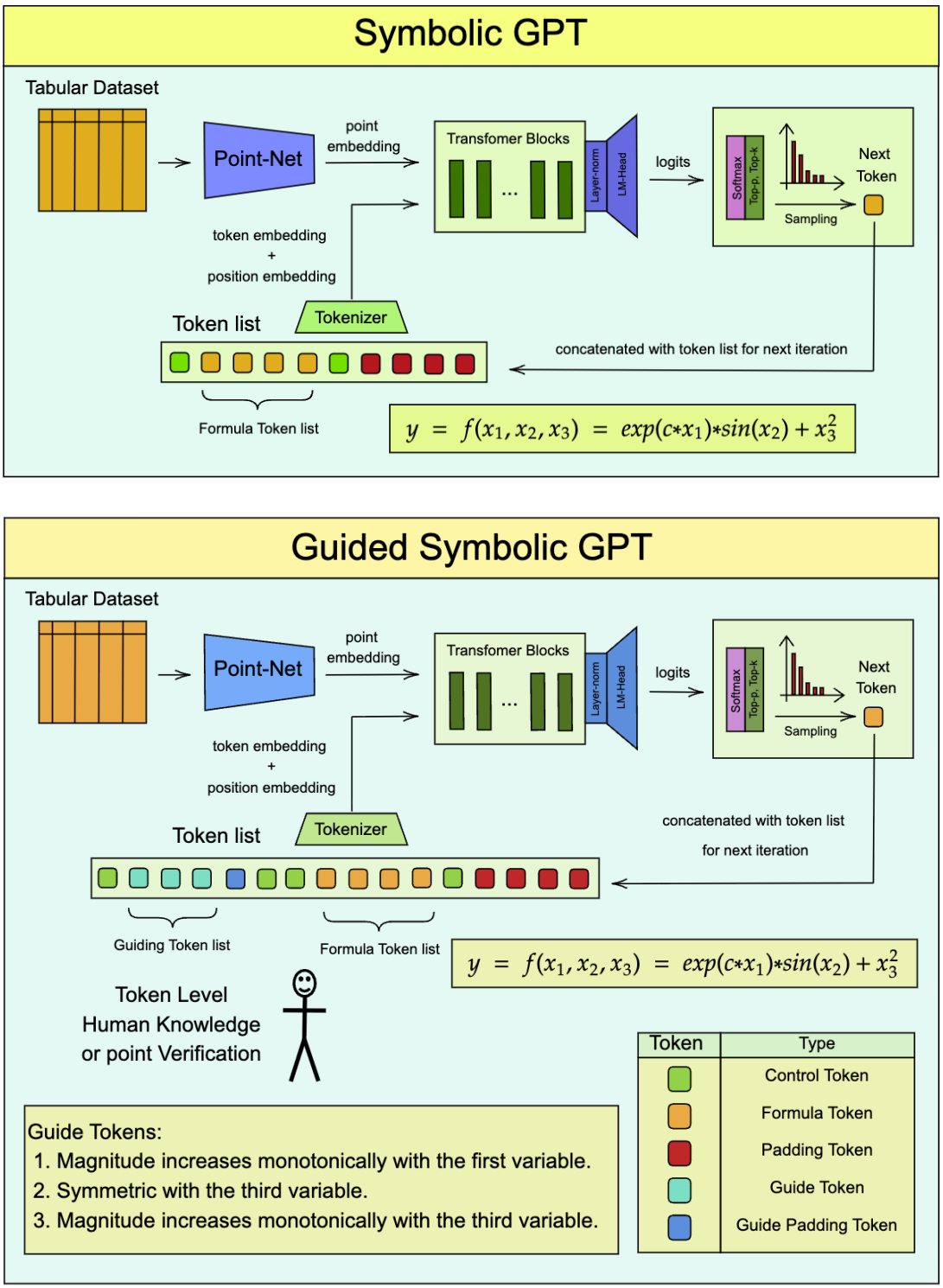


Figure 3.4: Symbolic GPT vs Guided Symbolic GPT

Chapter 4

Results

Our experiments demonstrate that *Guided Symbolic GPT* consistently outperforms *Symbolic GPT* under fair and controlled conditions. Model performance is evaluated using the coefficient of determination (R^2-score), defined as

$$R^2 = 1 - \frac{\sum_{i=1}^{n}(y_i - \hat{y}_i)^2}{\sum_{i=1}^{n}(y_i - \bar{y})^2},$$

where

- y_i denotes the true values,
- $\hat{y}_i$ represents the predicted values, and
- $\bar{y} = \frac{1}{n}\sum_{i=1}^{n} y_i$ is the mean of the true values.

To ensure a fair comparison, both models used in the experiments were constrained to the same scale, each containing approximately 45 million parameters. This design choice guarantees that improvements in predictive accuracy can be attributed to the inclusion of guiding tokens, rather than differences in model capacity or computational power.

Model	$\mathbf{R^2 \geq 0.99}$	$\mathbf{R^2 \geq 0.95}$	$\mathbf{R^2 \geq 0.90}$
Baseline (no tokens)	59 ± 1	61 ± 1	66 ± 2
Guided (all tokens)	77 ± 1	79 ± 2	84 ± 2
Guided (random tokens)	70 ± 2	74 ± 3	80 ± 3
Guided (noisy tokens)	64 ± 2	70 ± 3	74 ± 4

Table 4.1: Hit rates by R^2 threshold (mean±SD over seeds). Random/noisy tokens test robustness.

To illustrate the effectiveness of the proposed approach, we present several representative examples. Table 4 compares the performance of the baseline Symbolic GPT model with that of the proposed **Guided Symbolic GPT**, which incorporates mathematical property tokens as additional inputs. These guiding tokens encode structural information

such as symmetry and convergence, enabling the model to generate expressions that are not only more accurate but also more consistent with underlying mathematical principles. The results demonstrate that incorporating these tokens leads to measurable improvements in predictive accuracy and interpretability.

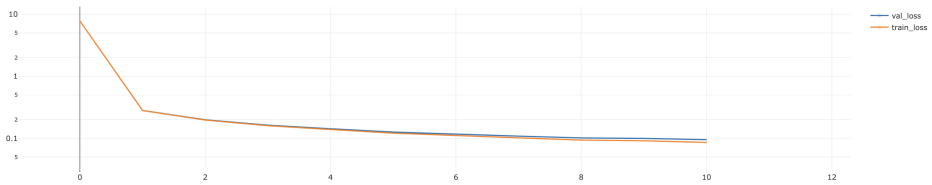


Figure 4.1: Train and validation loss in each step

Target Formula	**Math Properties**	**Symbolic GPT**	**Guided Symbolic GPT**
$x_1x_2x_3$	Linear in x_1, x_2, x_3	$x_1x_2\sin(x_3)$	$x_1x_2x_3$
$\frac{x_1x_2^2}{x_3}$	Convergent to zero in x_3, Linear in x_1, Symmetric in x_2	$x_1\cos(x_2) - x_3$	$\frac{x_1x_2^2}{x_3}$
$x_1x_3(x_1 + x_2)$	Linear in x_3	$x_1x_2 + x_1x_3$	$x_1x_3(x_1 + x_2)$
$\cos(x_1)x_2 + c$	Symmetric in x_1	$\sin(x_1)x_2 + c$	$\cos(x_1)x_2 + c$

Table 4.2: Comparison between Symbolic GPT and Guided Symbolic GPT

Chapter 5

Discussion

Our results demonstrate that incorporating guiding tokens significantly improves symbolic regression performance by enabling the model to retain and exploit meta-information during generation. By encoding mathematical properties directly into the input sequence, guiding tokens introduce an explicit structural bias that improves robustness, consistency, and interpretability of the generated expressions. This added structure is particularly valuable in scientific settings, where interpretability is as critical as predictive accuracy. Since scientists and domain experts often possess prior knowledge about expected model behavior, the proposed approach provides a practical mechanism for injecting such expertise into large language models, leading to more reliable and scientifically meaningful predictions.

Despite these advantages, the current implementation is limited in scope. Only seven guiding tokens are supported, a restriction imposed by the available datasets and the data-generation pipeline used in this work. As a result, the model is evaluated on a relatively narrow set of mathematical properties, and the generality of the approach beyond these constraints remains to be explored.

Several concrete directions for future work emerge from these limitations. First, expanding the set of guiding tokens to represent a broader range of mathematical and physical properties would allow the framework to capture richer forms of prior knowledge. Second, developing automated or data-driven methods for constructing meta-information could reduce reliance on handcrafted guidance and improve scalability. Finally, extending the framework to support post-hoc or iterative refinement of generated expressions based on meta-information would enable adaptive correction and further improve reliability in scientific discovery tasks.

Chapter 6

Conclusion

In this work, we introduced Guided Symbolic GPT, an extension of Symbolic GPT that incorporates explicit mathematical meta-information through guiding tokens. By encoding properties such as symmetry and convergence directly into the input representation, the proposed framework enables language-model-based symbolic regression to generate expressions that are more robust, interpretable, and consistent with known structural constraints. Experimental results on symbolic regression benchmarks demonstrate that this guidance consistently improves performance over the unguided baseline while preserving the original model structure. While the current implementation is limited to a small, predefined set of guiding tokens and does not support iterative refinement based on meta-information, the results establish guided symbolic generation as an effective and extensible approach. Overall, this work highlights the value of integrating expert-informed meta-information into neural symbolic regression and points toward guided generation as a promising direction for reliable and interpretable scientific modeling.

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