Optimal Design of Solar PV Farms with Storage

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Abstract

In the context of a large-scale solar farm participating in an energy market, we consider the problem of allocating a capital budget to solar panels and storage to maximize expected revenue over multiple time periods. This problem is complex due to many factors. To begin with, solar energy production is stochastic, with a high peak-to-average ratio, thus the access link is typically provisioned at less than peak capacity, leading to the potential for waste of energy due to curtailment. The use of storage prevents power curtailment, but the allocation of capital to storage reduces the amount of energy produced. Moreover, energy storage devices are imperfect and their costs diminish over time. We mathematically model these constraints and demonstrate that the problem is still convex, allowing efficient solution. Numerical examples demonstrate the power of our model in doing a sensitivity analysis to various design assumptions. We find that it is typically optimal to invest 90-95% of the initial capital on solar panels and the rest on storage. Interestingly, it is best to defer investment on lead-acid batteries (but not Lithium-ion batteries) closer towards the end of lifetime of the PV panels.

I. Introduction

One of the defining features of the modern energy landscape is the rise of large-scale solar farms. Driven by financial incentives and the continuing exponential decrease in costs, these farms can produce hundreds of megawatts of peak power, matching conventional sources. Indeed, in

This work was done at the David R. Cheriton School of Computer Science and ECE Dept., University of Waterloo.

May 2012, more than 50% of the load in Germany was met from PV sources alone [9], a fact that would have been inconceivable a decade ago.

The energy generated by small solar farms is easily absorbed into the existing transmission grid. As farm sizes grow, however, this situation is likely to change. Large solar farm operators in many countries are already *dispatchable*, that is, they must curtail production when asked to do so. It is likely that future large solar farm operators will be asked to participate in the generation market, on a level playing-field with traditional generators, as described next.

Traditional large-scale generators of electrical power are paid for power generation by a "market maker" that buys electricity from generators and sells it to distributors. Market makers predict the demand for the next day or hour and enter into daily or hourly contracts with generators who can meet the estimated demand. Importantly, generators need to pre-commit to a certain *constant* power level; if they fail to meet their commitment, they must pay a fine.

Given the stochastic nature of solar generation, their participation in a market requiring constant power-level commitments is challenging. It is only feasible either with very conservative power commitments or the introduction of energy storage devices (ESDs) that smooth out variations in solar generation. The focus of our work is in the optimal allocation of a certain capital budget to solar panels and ESDs to maximize revenue from participation in the day-ahead or hour-ahead market over the investment time horizon.

To gain an insight into the problem, note that investing the entire budget in solar panels maximizes the peak output power of the solar farm. However, this peak output power is highly variable and the farm operator must therefore make conservative commitments, i.e., to a generation level that they are likely to meet with very high probability, leading to a loss of revenue from any generation that exceeds this commitment. Instead, it is better to allocate a fraction of the investment to ESDs to provide less-variable output power. Of course, this comes at the cost of a lower peak output power. Maximizing revenue requires a careful balance between these competing forces.

Note that the outputs from our solution include both a budget allocation to PV and ESD over the lifetime of the solar farm as well as guidelines for choosing the market commitment that should be made by the farm in each time period, a non-trivial task. Specifically, the key contributions of our work are:

- modelling the design of a solar PV farm that is participating in an electricity market as a convex optimization problem

- determining “good” target power commitments in each time period to maximize expected revenue
- gaining engineering insights into the problem through numerical examples using real irradiance traces.

We have tried to make our system model as realistic as possible. Specifically, we model the following: (a) the lifetime of an ESD is typically shorter than that of a solar panel, thus ESDs will need to be purchased several times over the lifetime of the farm, (b) ESDs are imperfect in that they exhibit conversion inefficiency and self-discharge and that their charge and discharge rates are finite, (c) large solar farms are usually sited in remote, unpopulated locations and connected to the grid over an access link of finite capacity, leading to potential curtailment at times of peak generation, (d) different ESDs exhibit different types of imperfections, and (e) ESD costs are anticipated to decline over time. To our knowledge, no prior work has modelled these real-world constraints all together.

Our work makes several assumptions. We assume that we know the access line capacity and the irradiance over the course of a year at the solar farm location. We assume we are given the prices for PV panels and ESDs as well as their price evolution over time. We also assume that we know the hourly electricity prices over the year. Of course, in practice, these quantities are unknowable and must be predicted. Therefore, our solution at present does not take into account the prediction errors.

Nevertheless, we gain many new interesting insights that are insensitive to our assumptions. For example, we find that it is typically optimal to invest 90-95% of the initial investment on solar panels and the rest on ESDs. Moreover, we find that it is better to respond to the diurnal variation in solar power by varying the power commitment once every hour, smoothing out high-frequency fluctuations with ESDs, rather than committing to a single power level for the whole day. As well, investment on the lead-acid batteries (but not Li-ion batteries) is best shifted towards the end of lifetime of PV panels, to account for the battery price decays over time.

The rest of the paper is organized as follows. We discuss the system model and notation in Section II. We formulate the problem, study it, and simplify it in Section III. We present our numerical examples in Section IV. We discuss the existing work on solar farm designs in Section V and conclude the paper in Section VI.

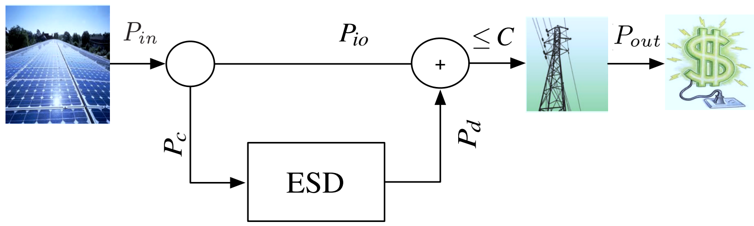

Fig. 1: **System model**

II. System model

Figure 1 illustrates our system, consisting of solar PV panels and an ESD. We assume a discrete-time model, where time is slotted; $0, T_u, 2T_u, \ldots$, with T_u being the time unit. To simplify notation, we define t to mean the time $t \times T_u$. We assume $t = 0$ is the time the PV farm system in Figure 1 is created. The available power from solar PV panels at any time t is $P_{in}(t)$. The actual output power from the solar PV farm, $P_{out}(t)$, is transmitted over an access line of capacity of C power units to the grid. The target committed output power is denoted $P_s(t)$ at any time t (and is not shown in the figure). Note that this commitment cannot exceed the access line capacity, thus

$$0 \leq P_s(t) \leq C \qquad \forall t \geq 0. \tag{1}$$

In our problem formulation, $P_s(t)$ for each time t is a control variable, so that this choice can be made in a way to maximize expected revenue.

We denote by $P_{io}(t)$ and $P_d(t)$, respectively, the portions of the output power that come directly from the input solar power and from the ESD. Thus, we can write

$$P_{out}(t) = P_{io}(t) + P_d(t) \leq P_s(t) \quad \forall t \geq 0, \tag{2}$$

where the last inequality implies that the entire system might fail to provide the target output power at certain times and it is never larger than the target output power.

Given, our notation, the system model in Figure 1 has the following constraints:

$$0 \leq P_d(t) + P_{io}(t) \leq P_s(t) \tag{3}$$

$$0 \leq P_c(t) + P_{io}(t) \leq P_{in}(t) \tag{4}$$

$$0 \leq P_c(t), P_{io}(t), P_d(t) \tag{5}$$

Besides these constraints, we also model ESD imperfections as follows. The charging (discharging) power must not exceed α_c (α_d) at any time. The ESD loses a fraction of $1-\eta_c$ ($1-\eta_d$) when charging (discharging), because of ESD charging (discharging) inefficiency, due to energy conversion losses. To achieve a reasonable ESD lifetime, only a DoD fraction≤ 1 of the entire ESD is allowed to be used. Finally, the stored energy is reduced by a fraction $1-\gamma \leq 1$ after each time unit it is kept in the ESD, due to self-discharge. In summary, if $b(t)$ is the state of charge evolution at time t, then we have

$$b(0) = 0 \tag{6}$$

$$b(t) = (1-\gamma)b(t-1) + \eta_c P_c(t)T_u - P_d(t)T_u/\eta_d; \;\; \forall t \geq 1 \tag{7}$$

$$0 \leq b(t) \leq DoD \times B \tag{8}$$

$$0 \leq P_d(t) \leq \alpha_d \tag{9}$$

$$0 \leq P_c(t) \leq \alpha_c, \tag{10}$$

where B is the size of the ESD.

The output power $P_{out}(t)$ is transmitted via the access line to be sold in an electricity market. We believe that the day-ahead electricity market is likely to use one of the two following policies to pay/charge solar PV owners. In the first policy, the supplier earns $\$c_1(t)$ for each energy unit it produces, and pays $\$p_1(t)$ for each energy unit it falls short at time t during the day of operation. Thus, the revenue in this policy is given by

$$Rev_1 = \sum_{t=1}^{T} \Big(c_1(t)P_{out}(t) - p_1(t)\big(P_s(t) - P_{out}(t)\big)\Big)T_u, \tag{11}$$

where $T \times T_u$ is the lifetime of the PV farm. In the second policy, everything is the same as before, except that the supplier is rewarded for its *target* output power $P_s(t)$ rather than its actual production P_{out}. In this case, the revenue is

$$Rev_2 = \sum_{t=1}^{T} \Big(c_2(t)P_s(t) - p_2(t)\big(P_s(t) - P_{out}(t)\big)\Big)T_u, \tag{12}$$

where $c_2(t)\$$ and $p_2(t)\$$ are, respectively, the per-energy unit reward and penalty. However, the revenue formulations in theses two policies from Eq. (11) and Eq. (12) are equivalent if we redefine the penalty price; i.e., by setting $c_2(t) = c_1(t)$ and $p_2(t) = c_1(t) + p_1(t)$, keeping in mind that $p_2(t) \geq c_1(t)$. Following existing work (e.g., [5], [12]), we choose Eq. (12) to be the objective function for our problem. We simplify notation by using $c(t)$ and $p(t)$ instead of $c_2(t)$ and $p_2(t)$. Combining Eq. (2) and Eq. (12) with some manipulations, yields

$$Rev = \sum_{t=1}^{T} \Big((c(t) - p(t)) P_s(t) + p(t) \big(P_d(t) + P_{io}(t) \big) \Big) T_u. \tag{13}$$

Given a total budget of $\$K$, our goal is to optimally size a solar PV farm (illustrated in Figure 1) with the maximum revenue (Eq. (13)) over its lifetime. The budget can be used to buy either solar PV panels or ESDs. In the next section, we formulate and discuss this problem in greater detail.

III. Problem formulation

We formulate two versions of the revenue maximization problem in this section. The first formulation takes P_s as a free variable in the optimization problem, thus allowing us to give precise guidance to the solar farm owner on the level of market commitment in each time period. However, our solution is highly dependent on precisely forecasting irradiance in the next few time periods, making it less robust in practice. In our second formulation, we pick a "reasonable" choice for P_s for each time period. This reduces the complexity of the problem, makes the solution more intuitive, as well as less sensitive to errors in the irradiance forecast. However, the formulation is only quasi-optimal and less comprehensive than the first formulation.

A. *First formulation: General P_s*

In this section, we size the solar PV farm, while also optimizing over all choices of P_s. There are two design parameters:

- Optimal budget split between PV panels and ESD: **Introducing** θ^{pv}

Let us denote $\theta^{pv} \leq 1$ the ratio of the total budget invested for PV panels. Denote by $i(t)$ the available solar power at any time t per unit area at the given location. The available power from solar PV panels P_{in} is given by

$$P_{in}(t) = \alpha_{pv} A i(t), \tag{14}$$

where α_{pv} is the efficiency of the solar PV panels and A is the total surface area. We define $P_{max}(A)$ to be the peak power of a PV farm with total surface area A. Then, from Eq. (14), we have

$$P_{max}(A) = \max_t(i(t))\alpha_{pv}A. \tag{15}$$

Given a price per unit peak power u, a solar PV farm with total surface area A costs $P_{max}(A)u$. Thus, for a given θ^{pv} and K, the size of the PV panels is given by

$$A = \frac{K\theta^{pv}}{u \max_t(i(t))\alpha_{pv}}. \tag{16}$$

Inserting this value in Eq. (14), yields

$$\begin{aligned} P_{in}(t) &= \frac{K\theta^{pv}}{u \max_t(i(t))} i(t) \\ &= \theta^{pv} P^*_{in}(t), \end{aligned} \tag{17}$$

where $P^*_{in}(t)$ is defined to be

$$P^*_{in}(t) := \frac{K}{u \max_t(i(t))} i(t). \tag{18}$$

Given a θ^{pv}, we compute the PV panel size and P_{in}, respectively from Eq. (16) and Eq. (17). In the next section, we discuss how to compute the storage size.

- Optimal investment on ESDs: **Introducing** θ^B_l

To compute the ESD size for a given θ^{pv}, we note that the lifetime of an ESD in years, L_B, is typically much smaller than the lifetime of PV panels in years $(T \times T_u)/(24 \times 365)$. Thus, in order to have storage throughout the lifetime of PV panels, we need to buy ESDs $n \geq 1$ times, where

$$n = \left\lceil \frac{T \times T_u}{24 \times 365 \times L_B} \right\rceil. \tag{19}$$

Let $\theta^B_1, \ldots, \theta^B_n$ be the fractions of the total budget invested in ESD, in each of the n purchases. The overall budget is either invested in ESD purchase or in buying PV panels. Thus,

$$\theta^{pv} + \sum_{l=1}^{n} \theta^B_l = 1. \tag{20}$$

Due to the fast improvement of ESD technologies in recent years, ESD prices have an ongoing price decay, which is expected to continue. We assume an exponential price decay over time; we take the price in a subsequent year to be $(1-d) \leq 1$ of the price in the current year, where

d is the decay fraction of the storage price per year. Thus, if L_B is the ESD lifetime in years and θ_l^B is the budget ratio investment in ESD in its l'th purchase period, its corresponding size B_l in that period is given by:

$$\begin{aligned} B_l &= \frac{\theta_l^B K}{v \times (1-d)^{(l-1)L_B}} \qquad \forall l \in \{1, \ldots, n\} \\ &= B_l^* \theta_l^B \qquad\qquad\qquad \forall l \in \{1, \ldots, n\} \end{aligned} \tag{21}$$

where v is the price per unit of storage at $t = 0$ and B_l^* is defined to be

$$B_l^* := \frac{K}{v \times (1-d)^{(l-1)L_B}} \qquad \forall l \in \{1, \ldots, n\}. \tag{22}$$

Given the budget split for each ESD purchase θ_l^B at any purchase period l, we can obtain the ESD size B_l for that period l from Eq. (21).

It turns out that different ESD imperfection parameters scale differently with θ_l^B. This is because only charging and discharging rate limits are functions of storage size. To be more precise, suppose that (α_c^*, α_d^*, η_c^*, η_d^*, DoD^*, γ^*) are the ESD imperfection parameters when $\theta_l^B = 1$ (i.e., investing the entire budget K on buying storage in period l). Then, for any θ_l^B, we have

$$\alpha_c(\theta_l^B) = \theta_l^B \alpha_c^*; \quad \alpha_d(\theta_l^B) = \theta_l^B \alpha_d^*; \quad \eta_c(\theta_l^B) = \eta_c^* \tag{23}$$

$$\eta_d(\theta_l^B) = \eta_d^*; \quad DoD(\theta_l^B) = DoD^*; \quad \gamma(\theta_l^B) = \gamma^*. \tag{24}$$

Combining all of the above constraints, we can formulate the revenue maximization problem $\mathbf{P1}$, given K, C, T, L_B, T_u, $c(t)$, $p(t)$, $i(t)$, u, v, d, and ESD imperfections (α_c^*, α_d^*, η_c^*, η_d^*,

DoD^*, γ^*), as follows

$$\mathbf{P1}: \max_{\substack{\theta^{pv},(\theta_l^B)_{\forall l},P_s(t),\\ P_d(t),P_{io}(t),P_c(t)}} \sum_{t=1}^{T} \Big((c(t)-p(t))P_s(t) + p(t)(P_d(t)+P_{io}(t)) \Big) T_u \tag{25}$$

$s.t.$

$$0 \leq \theta^{pv}, \theta_1^B, \ldots, \theta_n^B \leq 1 \tag{26}$$

$$\theta^{pv} + \theta_1^B + \ldots + \theta_n^B \leq 1 \tag{27}$$

$$B_l = \theta_l^B B_l^* \qquad \forall l \tag{28}$$

$$b_l(0) = 0 \qquad \forall l \tag{29}$$

$$b_l(t) = (1-\gamma^*)b_l(t-1) + \eta_c^* P_c(t) T_u - P_d(t) T_u / \eta_d^* \quad \forall t, \forall l \tag{30}$$

$$0 \leq b_l(t) \leq DoD^* \times B_l \qquad \forall t, \forall l \tag{31}$$

$$0 \leq P_d(t) + P_{io}(t) \leq P_s(t) \qquad \forall t \tag{32}$$

$$0 \leq P_c(t) + P_{io}(t) \leq \theta^{pv} P_{in}^*(t) \qquad \forall t \tag{33}$$

$$0 \leq P_c(t), P_{io}(t), P_d(t) \qquad \forall t \tag{34}$$

$$0 \leq P_d(t) \leq \theta_l^B \alpha_d^* \qquad \forall t, \forall l \tag{35}$$

$$0 \leq P_c(t) \leq \theta_l^B \alpha_c^* \qquad \forall t, \forall l \tag{36}$$

$$0 \leq P_s(t) \leq C \qquad \forall t \tag{37}$$

Problem $\mathbf{P1}$ is LP and hence, is convex. However, solving it requires precise forecasts of future solar irradiance, something this is notoriously hard to predict. Hence, we propose a new (simpler) problem $\mathbf{P2}$ that yields a robust and quasi-optimal solution to $\mathbf{P1}$, while being more intuitive. We first discuss how to choose a reasonable profile for the market commitment value P_s.

B. *Second Formulation: A reasonable choice for P_s:* **Introducing Δ**

An electricity market requires generators to make a constant power commitment for each *market time slot* (see Figure 2) and penalizes any violation. The length of these market time slots, denoted $T_s \times T_u$, is chosen by the market operator and is of the order of about one hour.

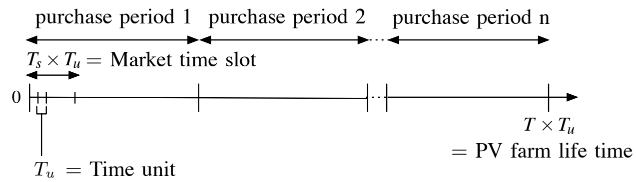

Fig. 2: **Illustrating the time unit definitions.**

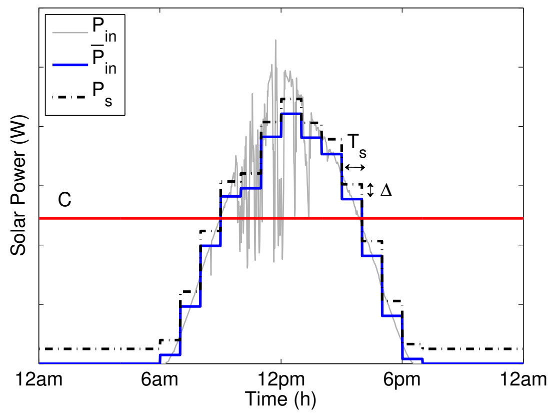

Fig. 3: **Illustrating the choice of P_s in a typical day.**

Thus, when a solar farm participates in an electricity market, its target output power P_s is very likely to be a step-wise constant function with the size of each step being T_s. Note that this type of target output power automatically captures the diurnal fluctuations in solar power, leaving only the short-term fluctuations to be captured by ESDs. To be more precise, any high-frequency fluctuation with time correlation smaller than the market time slot T_s can only be absorbed by ESDs, and low frequency fluctuations that last longer than T_s are best captured by changing the value of the target power commitment in the electricity market.

These observations suggest that P_s in any market time slot should more or less follow the diurnal variations in solar power, that is, the average expected solar power production during each market time slot[1]. In fact, in our work, we assume that the commitment P_s is the *average* of P_{in} during each time slot of size T_s, *but shifted by a value* Δ_l during purchase period l (see Figure 3). Introducing this shift of Δ_l helps in two ways. First, limited access line capacity

[1]Note that this is also much easier than to forecast the exact time series of future irradiance values.

might limit the output power in a market time slot. Therefore, surplus power generated during a time slot must be carried over to a subsequent slot, so that the target committed power of that subsequent market time slot should then be larger than its actual average (favouring $\Delta_l > 0$). The second reason is that converting a variable input power to its average over any time slot is only possible if we have some non-zero amount of stored energy available at the start of the time period, to cope with a shortfall during the first part of the time period. Thus, an ESD cannot convert the PV input power to its actual average unless $\Delta_l < 0$ in some of the previous time period. For these two real-world reasons, we have introduced this additional complexity in our problem formulation.

Let $\overline{P}_{in}(t)$ at any time t to be the average solar power over the market time slot j that t belongs to (i.e., $jT_s + 1 \leq t \leq (j+1)T_s$). This is

$$\overline{P}_{in}(t) = \frac{\sum_{\tau=jT_s+1}^{(j+1)T_s} P_{in}(\tau)}{T_s}; \qquad jT_s + 1 \leq t \leq (j+1)T_s \tag{38}$$

Assuming that t belongs to purchase period l, we set the target output power to be

$$P_s(t) = \min\left(C,\ \overline{P}_{in}(t) + \Delta_l\right), \qquad \forall t \geq 0 \tag{39}$$

where $\Delta_l \geq 0$ is a constant during purchase period l and C is the access line constraint. Combining Eq. (17)-(38)-(39), we have

$$P_s(t) = \min\left(C,\ \theta^{pv}\overline{P}^*_{in}(t) + \Delta_l\right), \qquad \forall t \geq 0 \tag{40}$$

where $\overline{P}^*_{in}$ is defined to be

$$\overline{P}^*_{in}(t) := \frac{\sum_{\tau=jT_s+1}^{(j+1)T_s} \frac{Ki(\tau)}{u \max_t(i(\tau))}}{T_s}; \quad jT_s + 1 \leq t \leq (j+1)T_s \tag{41}$$

Clearly, replacing P_s with Eq. (39) makes the problem non-linear. However, with the following transformation we can convert the problem to a Linear-Integer Programming (LIP).

We introduce three new variables: an integer vector $j(t) \in \{0,1\}$ for any t, and real-value vectors $X(t)$ and $w(t)$ as

$$X(t) := \theta^{pv}\overline{P}^*_{in}(t) + \Delta_l \tag{42}$$

and

$$w(t) := j(t)X(t). \tag{43}$$

Define M to be a constant upper bound on $X(t)$. This upper bound exists because $X(t)$ is finite. For instance, a candidate could be the maximum power of a PV farm with $\theta^{pv} = 1$. This is

$$M = P_{max}(A)|_{\theta^{pv}=1} \tag{44}$$

The following two constraints are equivalent to the equality in Eq. (43):

$$0 \leq w(t) \leq j(t)M \tag{45}$$

$$X(t) - (1 - j(t))M \leq w(t) \leq X(t). \tag{46}$$

To see this, consider the two possible choices of $j(t) = \{0, 1\}$. Moreover, the constraints on Eqs. (45)-(46) together with the following constraints ensure that $P_s(t)$ is equal to its value from Eq. (40):

$$P_s(t) = (1 - j(t))C + w(t) \tag{47}$$

$$w(t) \leq C \tag{48}$$

$$(1 - j(t))C \leq \theta^{pv}\overline{P}^*_{in}(t) + \Delta_l \tag{49}$$

This is shown by setting the two possible values of $j(t)$ and observing that the minimum operation in Eq. (40) is mimicked by enforcing infeasible cases.

Using the above definitions and notation and given K, C, T, L_B, T_u, $c(t)$, $p(t)$, $i(t)$, u, v, d, and ESD imperfections (α^*_c, α^*_d, η^*_c, η^*_d, DoD^*, γ^*), we can write the solar PV farm design optimization problem $\mathbf{P2}$ as follows:

$$\mathbf{P2}: \max_{\substack{\theta^{pv},(\theta_l^B)_{\forall l},(\Delta_l)_{\forall l},\\ P_d(t),P_{io}(t),P_c(t),j(t)}} \sum_{t=1}^{T} \Big((c(t)-p(t))P_s(t) + p(t)(P_d(t)+P_{io}(t)) \Big) T_u \tag{50}$$

$s.t.$

Constraints in Eqs. (26) to (36)

$$P_s(t) = (1-j(t))C + w(t) \qquad \forall t \tag{51}$$

$$w(t) \le C \qquad \forall t \tag{52}$$

$$(1-j(t))C \le \theta^{pv}\overline{P}^{*}_{in}(t) + \Delta_l \qquad \forall t, \forall l \tag{53}$$

$$X(t) = \theta^{pv}\overline{P}^{*}_{in}(t) + \Delta_l \qquad \forall t, \forall l \tag{54}$$

$$0 \le w(t) \le j(t)M \qquad \forall t \tag{55}$$

$$X(t) - (1-j(t))M \le w(t) \le X(t) \qquad \forall t \tag{56}$$

Note that in the above optimization problem, we have replaced the large vector P_s of size T in $\mathbf{P1}$ with the vector $(\Delta_l)_{\forall l}$ with much fewer elements, i.e., $n \ll T$. The above problem is an LIP and hence allows the use of the simple hill climbing method.

In the next section, we show that the problems $\mathbf{P1}$ and $\mathbf{P2}$ can be simplified for the case of a static market (where the reward and penalty prices are fixed). This simplification makes the problems more intuitive and is obtained by proving that some of the free parameters in $\mathbf{P1}$ and $\mathbf{P2}$ can be expressed in closed forms and excluded from the set of free parameters.

C. Simplification for a static market

Consider a static market, in which the reward c and penalty p prices are fixed, i.e., not a function of time. For a static market, we show that the optimal charging/discharging strategy is independent of the solar power trace and other conditions and this substantially simplifies the problem. In other words, both our problem formulations ($\mathbf{P1}$ and $\mathbf{P2}$) search for the optimal control strategy of charging/discharging by optimizing the revenue over P_c, P_d, and P_{io} among other parameters. In this section, however, we show that the optimal control strategy for a static market can be provided in closed-form and hence, P_c, P_d, and P_{io} can be excluded from the set of free variables.

Lemma 1 (Optimal control strategy for a static market)**.** *Given the system model described in Section II and the constraints in Eq. (2-10), the optimal values of P_c, P_d, and P_{io} for a static market can be expressed in closed-form. In other words, the following values maximize the revenue in Eq. (13) when $c(t)$ and $p(t)$ are not functions of time*

$$P_{io}(t) = \min(P_s(t), P_{in}(t)) \tag{57}$$

$$\begin{aligned} P_c(t) = \min([&P_s(t) - P_{in}]_+, \alpha_c, \\ &[DoD \times B - (1-\gamma)b(t-1)]_+/(\eta_c T_u)) \end{aligned} \tag{58}$$

$$P_d(t) = \min([P_s(t) - P_{in}(t)]_+, \alpha_d, (1-\gamma)b(t-1)\eta_d/T_u), \tag{59}$$

where $[x]_+ = \max(0, x)$ for any x.

Proof. For a given $P_s(t)$, we need to maximize P_{io} and P_d to maximize the revenue in Eq. (13). Observe that it is always the best to maximize P_{in} first, regardless of P_d and P_c. To see this, note that P_d is bounded by the existing charge in the ESD, $b(t)$. For each unit of energy that we move from P_{in} to P_d, we can only expect an energy return of only a fraction of $\eta_d\eta_c \leq 1$, whereas P_{io} is delivered to the output directly with no loss. Thus, we should maximize P_{io} regardless of P_d and P_c and find the values of P_d and P_c accordingly.

From Eqs. (3)-(4), the maximum value of P_{io} is given by

$$P_{io}(t) = \min(P_s(t), P_{in}(t)), \tag{60}$$

which matches the lemma statement. Given this value for P_{io} and using Eq. (3) and Eq. (4), we find that at any time t only one the variables P_c and P_d can be non-zero and the other must be zero. We also know that, given P_s and P_{io}, the values of P_c and P_d must be maximized. Thus, from Eqs. (4)-(7)-(8)-(10), the optimal value of P_c is given by

$$\begin{aligned} P_c(t) &= \min([P_s(t) - P_{io}]_+, \alpha_c, \\ &\qquad [DoD \times B - (1-\gamma)b(t-1)]_+/(\eta_c T_u)) \qquad (61) \\ &= \min([P_s(t) - P_{in}]_+, \alpha_c, \\ &\qquad [DoD \times B - (1-\gamma)b(t-1)]_+/(\eta_c T_u)), \qquad (62) \end{aligned}$$

where in the second line we inserted the value of P_{io} from Eq. (60).

Similarly, the optimal value of $P_d(t)$ using Eqs. (3)-(7)-(8)-(9), is given by

$$\begin{aligned} P_d(t) &= \min([P_s(t) - P_{io}(t)]_+, \alpha_d, (1-\gamma)b(t-1)\eta_d/T_u) \\ &= \min([P_s(t) - P_{in}(t)]_+, \alpha_d, (1-\gamma)b(t-1)\eta_d/T_u). \end{aligned}$$

Thus, Eqs. (57-59) are the optimal values of P_{io}, P_c, and P_d, when optimizing for the revenue. □

Based on the above lemma, the optimal control strategy in our problem follows these straightforward static rules: The input power $P_{in}(t)$ is primarily used to serve the target output power, delivering $\min(P_{in}(t), P_s(t))$ to the output line. The leftover (if any) $[P_{in}(t) - P_s(t)]_+$ is stored. The energy is stored to the ESD with power $P_c(t)$ at any time t. If, at a given time t, the available solar power is insufficient (i.e., $P_{in}(t) < P_s(t)$), the energy stored in the ESD, if any, can be used to make up the difference.

In a *static* market, storing energy in the ESD when we have a chance to sell it, is always harmful because: (1) there is no gain in terms of revenue to postpone selling energy (2) we might lose some revenue because the ESD may become full, and (3) we lose stored energy due to self-discharge. This static control strategy, however, is not always optimal in a *dynamic* market, because it might be more beneficial to retain energy in the ESD if we know that the market price will soon increase.

Lemma 1 shows that we can exclude P_c, P_{io} and P_d from the set of free parameters in the optimization problem in Eqs. (50-56). Moreover, we know that $w(t)$ and $j(t)$ are defined only to enforce $P_s(t)$ to be the closed-form stated in Eqs. (40-41). This leaves us with only three free parameters in the optimization problem: θ^{pv}, θ_l^B, and Δ_l.

θ^{pv} controls the trade-off between larger input power and the reliability of the output power. Increasing θ^{pv} increases the output power P_{out} (and hence P_s), but makes it more bursty, leading to a potential increase in the penalty. Thus, it can potentially increase both the reward and the penalty.

$(\theta_l^B)_{\forall l}$ control the trade-off between investing in larger ESDs and how much value they add earlier in the lifetime of the farm, given the anticipated price decay of ESDs. Increasing θ_l^B for small values of l gives us a higher chance to make the input power smoother and reduce the penalty. However, we have a chance to buy much larger ESDs later when the ESD prices drop.

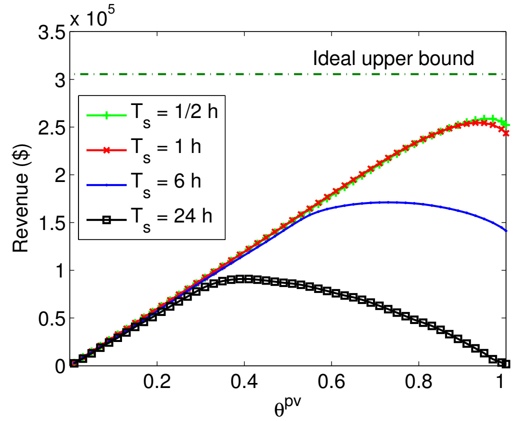

Fig. 4: The impact of the size of budget allocation to PVs (θ^{pv}) and market time slot duration (T_s) on annualized revenue.

It is also important to note that increasing the size of the ESD does not add much to the revenue after a certain threshold, when all input variations have been evened out.

Finally, Δ_l, for any purchase period l, optimizes the target output power. Increasing Δ_l decreases P_s and the potential reward, but may decrease the penalty by facilitating the output power flattening for a smaller target power.

IV. Numerical examples

We illustrate the use of our model by using it to design a solar PV farm with storage at a given location characterized by its irradiance trace. We use our second problem formulation ($\mathbf{P2}$) in Eqs. (50-56), while exploiting Lemma 1, assuming a static market with the reward and penalty prices, respectively, set to $c = \$291$/MWh and $p = 2 * c$, unless otherwise stated. The price (including hardware and installation) and the lifetime of a PV panel is, respectively, set to $u = 1.63$ \$/Watt and 20 years, which are the regular values in June 2014 [1]. We assume that our total initial budget is enough to build a 1MW solar farm with no storage; thus $K = \$1,630,000$. We use the solar irradiance dataset (i.e., $i(t)$ in our notation) from the atmospheric radiation measurement website [2] from the C1 station in the Southern Great Plains permanent site with a 1-minute time resolution. The yearly storage price decay factor d is set to 0.05. Unless otherwise stated, the access line capacity is set to $C = 0.5$MW. Although our problem formulation can be applied to a large set of ESDs, for simplicity, we only assume two widely-used storage

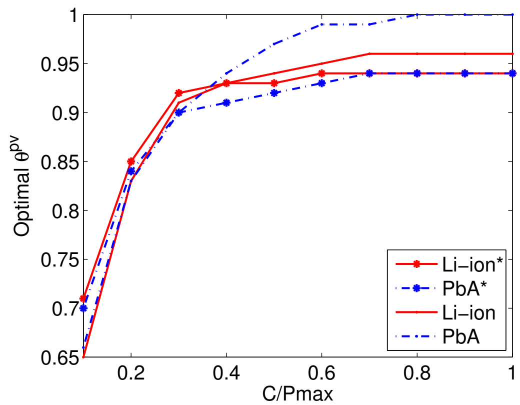

Fig. 5: Optimal θ^{pv} as a function of C. For the lines tagged with star, the battery price is halved and the penalty is increased to $p = 3c$.

technologies for our numerical examples; Lithium-ion (Li-ion) and Lead-acid (PbA) batteries. Their characteristics are given in Table I.

A. The impact of budget allocation to PVs and market time slot on revenue

The overall revenue is greatly affected by the budget allocation to panels versus ESDs. Figure 4 shows how the annualized revenue (total revenue divided by the solar farm lifetime in years) is influenced by varying the fraction of the budget allocated to PVs and the duration of the market time slot (in hours). The line marked as the 'ideal upper bound' is the maximum possible revenue with no penalty or curtailment (i.e., $p = 0$ and $C = \infty$), where it is optimal to invest the entire budget in solar PV panels (i.e., $\theta^{pv} = 1$). When there is a penalty, it is necessary to invest some part of the budget in an ESD. Nevertheless, because the ESD size is never adequate to fully capture all input variations, the achieved revenue is always less than the optimal.

The larger the size of the market time slot, the greater the role of ESDs in removing within time-slot fluctuations in power production, and hence the smaller the optimal budget allocation to PVs. We see that when T_s = 24h, the optimal PV investment is 40%, whereas with a T_s = 1h, the optimal PV investment rises to more than 90%.

Figure 4 also shows that the overall revenue increases as T_s decreases. This is because the diurnal variation in solar production is best removed by changing the target power at each time slot than using ESDs. The shorter the time slot, the easier it is to prevent overflow and underflow in the ESD. There is significant improvement in the maximum achievable revenue as the time slot decreases from 24 hours to 1 hour, but there is negligible improvement from $T_s = 1$h to $T_s = 1/2$h.

B. The role of access link capacity

A limited access link capacity can cause curtailment of power from the solar farm, making storage necessary to avoid revenue loss from wasted power. Figure 5 illustrates the optimal value of θ^{pv} as a function of the access line capacity C. We have also added curves, tagged with a star in the legends, that correspond to lowering the per-unit battery prices (in Table I) by 50% and simultaneously increasing the penalty ($p = 3c$). These are meant to characterize a possible future scenario enabled by changes in battery technology and the evolution of electricity markets.

To begin with, Figure 5 shows that the optimal investment split between PV panels and storage is highly dependent on C. The optimal allocation of budget to storage (smaller values of θ^{pv}) is greater as C decreases. This is because, for large values of the access line capacity, storage is used only to mitigate the sub-hourly fluctuations of the incoming solar power. As the access line capacity decreases and becomes a meaningful constraint, the ESD must also compensate for the power curtailment due to limited C and has to store energy across time slot boundaries. Thus, a larger storage is needed and more gain is expected to be obtained by investing more on ESDs. As can be observed in Figure 5 , the monotone increase of θ^{pv} vs. C has a saturation point. This is the point at which C is not a constraint anymore and storage is only used to mitigate sub-hourly fluctuations. Finally, the curves corresponding to the hypothetical prices show that investing in storage is much more appealing when storage prices decrease and penalties increase. These trends hold for both battery technologies.

Figure 6 shows the relative revenue gain obtained by adding storage, $($Revenue with storage $-$ Revenue without storage$)/$Revenue with storage, as a function of the access line capacity. We compute and use the optimal θ^{pv} to calculate the revenue for each value of C. Once again, this figure confirms that the smaller the access line capacity, the more pronounced the role of storage as discussed above. We also observe that the relative gain monotonically decreases as C increases until a saturation point after which increasing C has a negligible impact on revenue. Finally, Figure 6 shows that Li-ion batteries perform better than PbA batteries for all values of C.

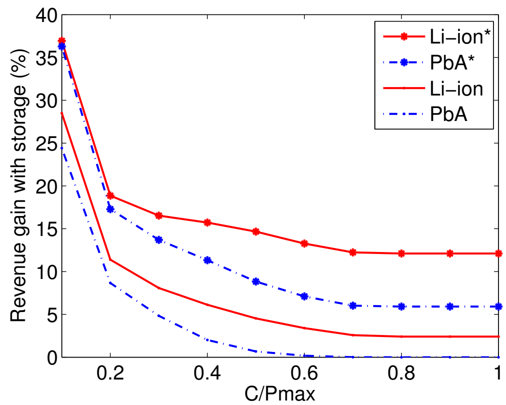

Fig. 6: Percentage gain in revenue as a function of C. For the lines tagged with star, the battery price is halved and the penalty is increased to $p = 3c$.

C. *Optimal investment spread on ESDs (θ_l^B)*

Unlike prior work, our model allows us to give insightful guidance to solar farm operators on the right time to buy different types of energy storage. Figure 7 shows the optimal allocation of budget to storage during each purchase period for various values of C and choice of battery technologies. We use the optimal θ^{pv} to create each curve. For large values of C, when storage is not critically needed to avoid curtailment, due to the imperfections of PbA batteries, it is best to buy them only when their price has decayed enough to outweigh these imperfections. Thus, most of the purchase of PbA batteries is towards the end of the investment period. Li-ion batteries, with fewer imperfections, in contrast, should be bought nearly uniformly in each time period. In contrast, for small value of C, when storage is critically needed, it is best to spread the investment on ESDs almost uniformly across all purchase periods for both Li-ion and PbA batteries. This is because sacrifices in the earlier periods are not justified by the better performance of later periods.

V. Related work

There is extensive work on sizing or analyzing the performance of battery-PV systems. Prior work can be categorized into two main classes: *stand-alone* problems and *grid-connected* problems (please see [11], [17], [21] for an extensive review of existing related work).

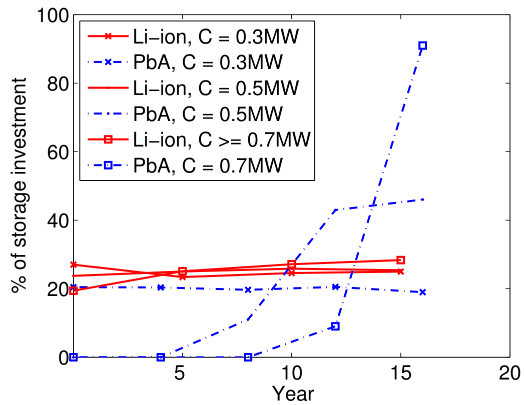

Fig. 7: The optimal allocation of budget to storage over the lifetime of PV system as a function of battery technology and access line capacity.

Stand-alone problems are those in which the system can only rely on solar power and storage to meet the demand power. Several papers studied the optimal sizing and cost analysis of stand-alone photovoltaic systems [10], [11], [16], [18]. The objective in stand alone systems is to minimize the cost of battery-PV system, while still meeting the power demand with a target loss of load probability. Cost minimization is either in terms of minimizing the initial capital cost of the system [19], [22] or the annualized cost of the system accounting for different lifetime of batteries and PV panels [8], [23]. Annualized cost minimization is further extended to a general target output power in [7].

Grid-connected scenarios themselves are divided into two classes: residential installations and solar PV farms. Residential installations of PV systems have the option to serve the demand from PV panels, storage, or the grid. The price of buying/selling electricity to the grid is a function of the time of the day and season. Residential installations mostly aim at selling their excess power to the grid and buying their power shortage from the grid. These options create many challenging problems with different objective functions. Barra et al. [4] optimally size PV panels and storage such that a minimum target fraction of the total demand is guaranteed to be met by the battery-PV system and the cost of energy is minimized. Azzopardi and Mutale minimize the annual net cost, using a case study of a residential installation where energy can be stored, used, or sold [3]. They consider fluctuations in time of use pricing and use mixed integer programming to find the optimal size of each system component. Using a similar system model, Ru et al. [20] provide an optimization problem to determine the critical size of the battery after which an increase in size gives no performance benefit. Other work maximizes the benefit minus cost of a grid-connected solar PV panel with no storage [14], [15]. In a similar line of research, we find grid-connected wind farms design problems. However, designing a solar PV farm is very different from a wind farm due to many factors, including the different stochastic nature of solar and wind and different hardware constraints. For instance, the quantum of solar PV panels is quite small (one panel) which makes them flexible to be sized with high resolution, whereas wind turbines come in few different sizes, making the set of choices limited and integer [13].

Our work is different from the listed related work, because: 1) No prior work studied designing a farm aiming at maximizing its overall revenue throughout its lifetime. They rather design it such that the load is guaranteed to be met, or maximizing revenue for a given pre-designed renewable farm [5], [12]. Thus, in those problems, meeting the load with some target allowable uncertainty is the objective or a constraint, whereas we are optimizing over the revenue with no constraint on meeting a demand load. It is worth noting that in our system, the demand power is mapped to the target output power, which is itself a free variable in the optimization problem. In contrast, the demand power in the problem formulations found in the literature is fixed and given. 2) The transmission line constraint has never been considered in prior designs. This is because, the previous work mostly either considers a residential or a stand-alone system for which there is no need for power transmission. 3) The optimization problem formulation is difficult. In prior work, AI techniques such as genetic algorithm, particle swarm optimization and simulated annealing [17], [21] are used to provide only suboptimal solutions. However, we manage to prove the convexity (thus, facilitating hill climbing) and even further use a more simplifying lemma which together lead to an extremely fast implementation.

VI. CONCLUSIONS

Our work studies the optimal allocation of a certain capital budget to solar panels and ESDs to maximize revenue from the day-ahead or hour-ahead market over the investment time horizon. Unlike prior work, we have carefully modelled several real-world constraints, yet have formulated a convex problem that can be solved quickly using straightforward hill-climbing. Numerical evaluation using real irradiation traces show that it is typically optimal to invest 90-95% of the initial investment on solar panels and the rest on ESDs. We find that varying the power commitment level every hour is the best way to account for diurnal variations in solar power, rather than committing to a single power level for the whole day. Moreover, investment on the lead-acid batteries (but not Li-ion batteries) is best shifted towards the end of lifetime of PV panels, to account for the battery price decays over time.

The primary limitation of our work is that it requires long-term, fine-grained traces of solar irradiation and energy prices, something that may not be available for all potential solar farm locations. Moreover, our numerical examples assume a constant energy and penalty price, rather than the time-varying prices typical of the market today. We also assume a fixed line capacity C whose size cannot be increased through additional investments. Finally, we study only two types of ESDs. We hope to address these limitations in our future work.

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Name	*Description (units)*
K	Total available budget ($)
θ^{pv}	The fraction of the budget for PV panels
θ_l^B	The fraction of the budget for ESD's l'th purchase
C	Access capacity (MW)
Δ_l	Vertical shift used in P_s in purchase period l (MW)
$P_s(t)$	The target output power at time t (MW)
$P_{in}(t)$	The available power from PV panels at time t (MW)
$P_c(t)$	The storage charging power at time t (MW)
$P_d(t)$	The share of output power from storage at time t (MW)
$P_{io}(t)$	The share of output power from input at time t (MW)
$P_{out}(t)$	The output power from the solar PV farm at time t (MW)
$i(t)$	Solar irradiance at time t (MW/m^2)
$t = 0$	When we plan to build the farm
T	Lifetime of the PV farm in number of time units T_u
L_B	Lifetime of the ESD (years)
T_u	The size of the time unit (h)
T_s	Market time slot in number of time units T_u
$P_{max}(A)$	$\max_t P_{in}(t)$ of PV panels of size A (MW)
B	ESD size (MWh)
A	Total surface area of solar PV panels (m^2)
p_B	Storage price per unit of energy ($/MW)
$\overline{P}_{in}(t)$	Hourly average of P_{in} at time t (MW)
α_{pv}	The efficiency of solar PV panels
$\alpha_c(\alpha_d)$	ESD charging (discharging) power limit (MW)
$\eta_c(\eta_d)$	ESD charging (discharging) efficiency
γ	ESD leakage rate
DoD	ESD depth of discharge
$c(t)$	Revenue for each energy unit at time t ($/MWh)
$p(t)$	Penalty for each imbalanced energy unit at time t ($/MWh)
d	ESD price decay rate per year
v	Price per unit of storage size at time $t = 0$ ($/MWh)
u	Price per unit of 1MW PV panel at time $t = 0$ ($/MWh)
n	Total number of ESD purchase opportunities

	Li-ion	PbA
DoD	0.8	0.8
Round-trip efficiency ($\eta_c\eta_d$)	0.85	0.75
Charge time (B/α_c)	3h	12h
Discharge/Charge rate ratio (α_c/α_d)	5	10
Self-discharge (γ)	≈ 0	≈ 0
Lifespan (L_B)	5 years	4 years
Per-unit price (v)	400$/KWh	200$/KWh

TABLE I: ESD characteristics at room temperature and averaged over its lifetime [6], [24].