SIGGRAPH 2024
DENVER+ 28 JUL — 1 AUG

Velocity-Based Monte Carlo Fluids

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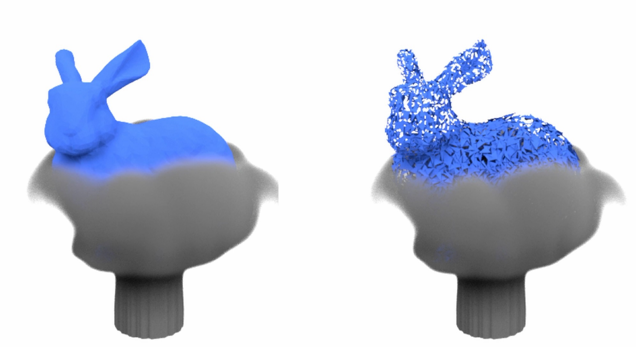
Problem & Motivation

We develop a velocity-based Monte Carlo fluid solver. The vorticity-based Monte Carlo fluid method [Rioux-Lavoie and Sugimoto et al. 2022] has showcased its advantages over conventional methods, such as the capability to solve problems with complex boundary geometry without relying on cut-cell or conforming mesh. However, vorticity-based methods cannot simulate harmonic velocity components, yielding incorrect results. Our new velocity-based method does not have such a problem. Moreover, our method can utilize the advancements from the computer animation literature on velocity-based techniques.



Vorticity-Based Monte Carlo Fluids
[Rioux-Lavoie and Sugimoto et al. 2022]

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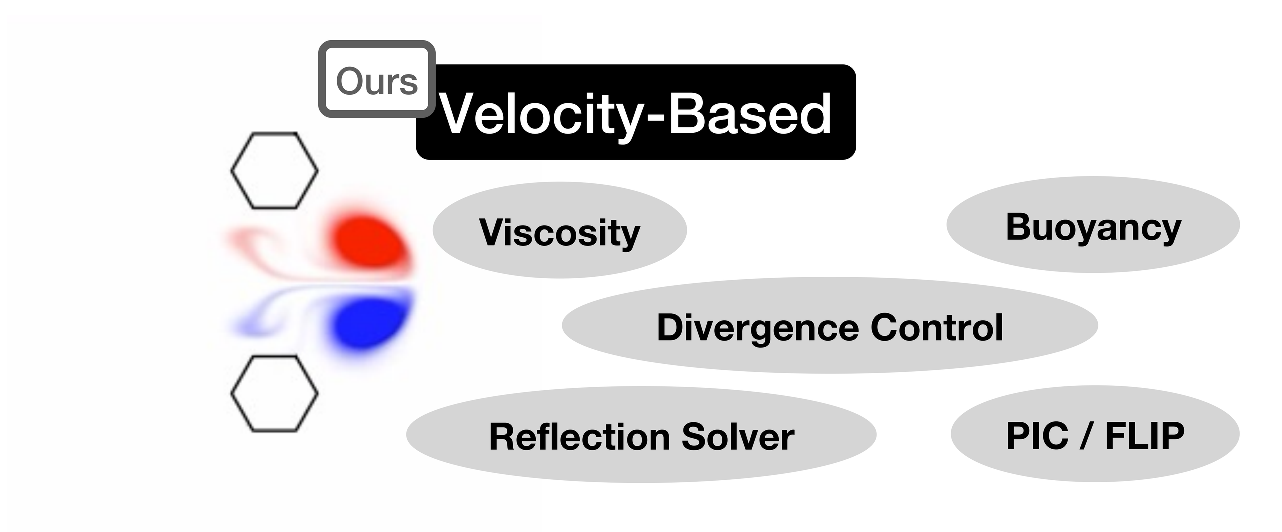
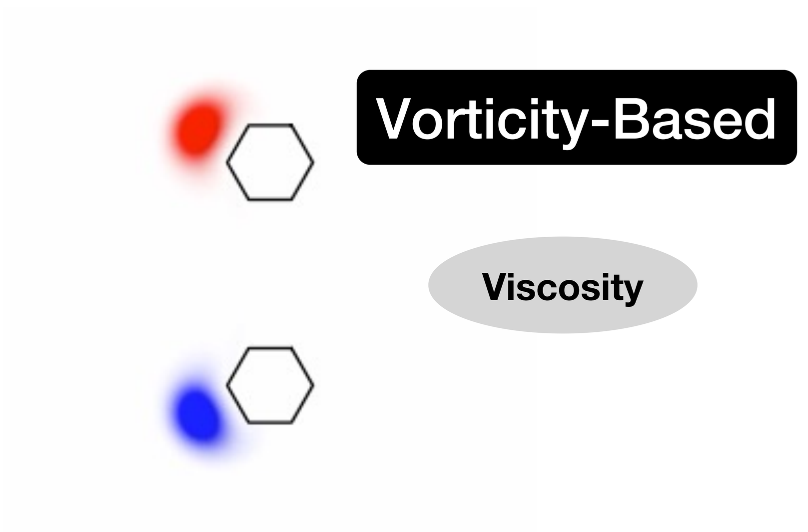




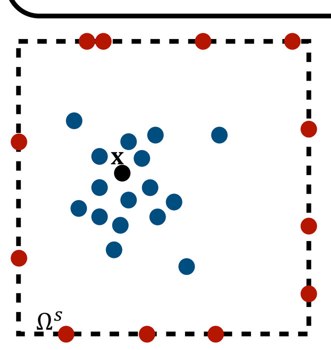
Method

We solve the incompressible Navier-Stokes equations via operator splitting [Stam 1999].
For each substep, we develop a pointwise solver with Monte Carlo.

Advection $\frac{\partial \mathbf{u}}{\partial t} = -(\mathbf{u} \cdot \nabla)\mathbf{u}$ — Semi-Lagrangian advection.
We update the velocity field by tracing back the velocity trajectory.

Projection $\frac{\partial \mathbf{u}}{\partial t} = -\nabla p$ such that $\nabla \cdot \mathbf{u} = 0$

without solid boundaries

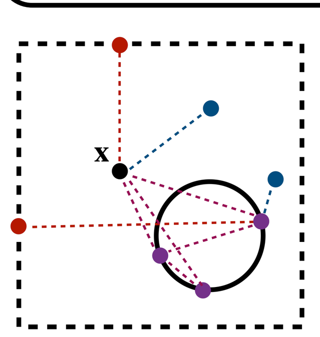


$\mathbf{u} \leftarrow \mathbf{u}^* - \nabla p$ where $\Delta p = \nabla \cdot \mathbf{u}^*$.

With the fundamental solution G, we get an integral over a finite simulation domain without dependency on the velocity divergence. Use Monte Carlo integration.

$$\nabla_{\mathbf{x}} p(\mathbf{x}) = -\int_{\Omega^s} \nabla_{\mathbf{x}}^2 G(\mathbf{x},\mathbf{y})\mathbf{u}^*(\mathbf{y})\mathrm{d}V(\mathbf{y}) + \int_{\partial\Omega^s} \{\nabla_{\mathbf{x}} G(\mathbf{x},\mathbf{y})\}\{\mathbf{n}_{\mathbf{y}} \cdot \mathbf{u}^*(\mathbf{y})\}\mathrm{d}A(\mathbf{y})$$

with solid boundaries



We get a boundary integral equation to compute the solid boundary effects.
This equation is similar to the rendering equation for light transport simulation, and we solve it with a ray-tracing-style solver, the Walk-on-Boundary method [Sugimoto et al. 2023].

Diffusion $\frac{\partial \mathbf{u}}{\partial t} = \nu \nabla^2 \mathbf{u}$

without solid boundaries — Evaluate Gaussian blur with Monte Carlo.

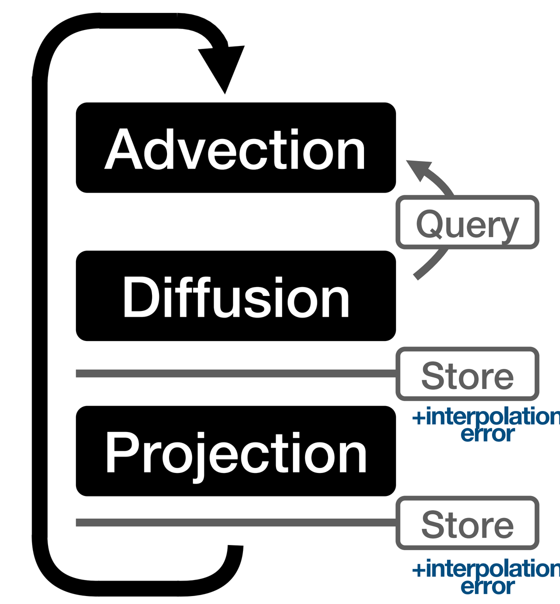
with solid boundaries — Time-dependent diffusion
Walk-on-Boundary method [Sabelfeld and Simonov 1994].

Advantages of Monte Carlo

The solver for each substep supports the *pointwise* evaluation: it estimates the velocity at *any* spatial point we are interested in, assuming we can likewise query the velocity estimates from the previous steps at any spatial point, in contrast to traditional solvers that require globally-coupled solves.

The advantages of our Monte Carlo method thus include

- Handling complex boundaries without cut-cell or conforming mesh.
- Flexible choice of the underlying discretization of the velocity field.
- Removal of excessive interpolation errors.

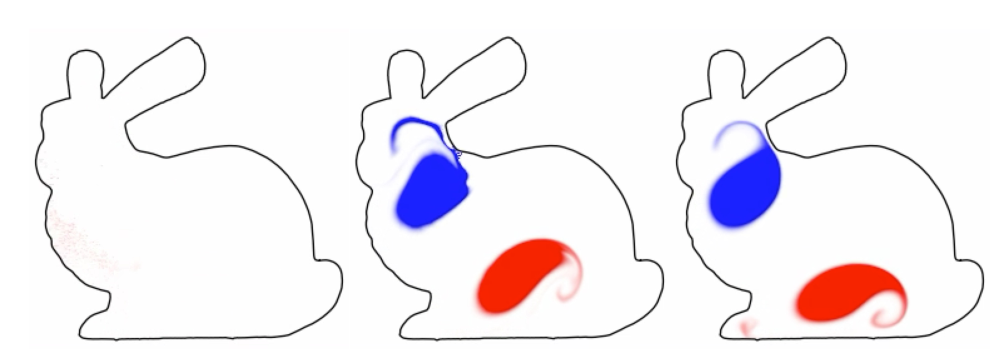


Results

We can integrate various velocity-based techniques into our solver.

Open Problems

- Investigation for when the removal of excessive interpolation errors may have advantages.
- Improvement of efficiency by utilizing spatio-temporal coherency in the velocity field.



0.01x samples (collapse) | 0.1x samples (jitter) | 1x samples (correct)

Reference

Damien Rioux-Lavoie*, Ryusuke Sugimoto*, Tümay Özdemir, Naoharu H. Shimada, Christopher Batty, Derek Nowrouzezahrai, and Toshiya Hachisuka. 2022. A Monte Carlo Method for Fluid Simulation. ACM Trans. Graph.
Karl K. Sabelfeld and Nikolai A. Simonov. 1994. Random Walks on Boundary for Solving PDEs. De Gruyter, Berlin.
Jos Stam. 1999. Stable Fluids. In Proceedings of the 26th Annual Conference on Computer Graphics and Interactive Techniques (SIGGRAPH '99).
Ryusuke Sugimoto, Terry Chen, Yiti Jiang, Christopher Batty, and Toshiya Hachisuka. 2023. A Practical Walk-on-Boundary Method for Boundary Value Problems. ACM Trans. Graph.