E&CE 784 - STATS 902: Introduction to Stochastic Calculus, Winter Term 2011

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The stochastic calculus finds ready application in a variety of areas including electrical engineering (nonlinear filtering, stochastic optimal control, stochastic adaptive control), physics (quantum and stochastic mechanics) and mathematical economics (pricing of derivative securities).

The goal of this course is to establish the main principles of stochastic calculus within the simplest setting of stochastic integration with respect to continuous semimartingales. Our emphasis will be on the basic principles and theorems of stochastic calculus rather than on specific applications.

The main prerequisites for enrolling in this course are competence in basic measure theory (Lebesgue integral and its properties, convergence theorems, Radon-Nikodym theorem, Fubini-Tonelli theorem, extension theorem for measures) and elementary probability theory (independence, expectations, conditional expectations). These prerequisites are provided by STATS 901.

Contents:

1. Preliminaries: brief overview of prerequisites, introduction to monotone and Dynkin classes of sets, monotone and Dynkin class theorems.

2. Discrete-parameter martingales: discrete-parameter filtrations and stopping times, optional sampling theorem, supermartingale inequalities, supermartingale convergence theorem, uniform integrability and uniformly integrable supermartingales.

3. Elements of continuous-parameter stochastic processes: processes with independent increments, the Wiener process, continuous-parameter filtrations and standard filtrations, continuous-parameter stopping times, *corlol* processes, progressively measurable processes.

4. Continuous-parameter martingales: structure of (super)-martingale sample paths, continuous-parameter analogs of the main results for discrete-parameter martingales, continuous local martingales, quadratic variation process of a continuous local martingale.

5. Stochastic integration of progressively measurable integrands : sample-path integrals with respect to processes of locally-bounded variation, Kunita-Watanabe inequalities and Ito integrals with respect to continuous local martingales and continuous semimartingales, Ito's formula, exponential local martingales, Novikov theorem, martingale characterization of the Wiener process, changes of measure and the Girsanov theorem.

6. Representation of local martingales as stochastic integrals.

Text: Course notes covering the material will be available for purchase at the Mathematics Copy Centre, 5th floor MC., or can be downloaded from the URL
http://www.control.uwaterloo.ca/heunis/notes.784.902.2011.pdf

The following supplementary references are on reserve at the D.C. Library:

I. Karatzas and S. E. Shreve, *Brownian Motion and Stochastic Calculus*, 2nd. ed., Springer Verlag, 1991 (Graduate texts in Mathematics, v. 113).
R. Durrett, *Stochastic Calculus: A Practical Introduction*, CRC Press, 1996.