Lecture 15

Performer, Variational Autoencoders

Rethinking Attention with Performers

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- “We introduce the first Transformer architectures, *Performers,* capable of **provably** accurate and practical estimations of regular (softmax) full rank attention, but of only linear space and time complexity and **not relying on any priors** such as sparsity or low-rankness. Performers use the *Fast Attention Via positive Orthogonal Random features* (FAVOR+) mechanism”

Generalized definition

- Define three different vectors corresponding to each word.
  - Input $x \in \mathbb{R}^d$ $x = [\underline{x_1} - -x_n]_{d \times n}$
  - Key
  - Query
  - Value

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Q^T: $n \times p$; K: $p \times n$

$O(n^2 m)$

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$m \times n$ $m \times n$ $n \times p$ $p \times n$

$X^T \underbrace{W_q W_k} X$

$O(n^2 m)$

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$$\phi(x) = \frac{h(x)}{\sqrt{r}} \left(f_1\left(\underline{\omega_1^T} x\right), f_1\left(\underline{\omega_2^T} x\right) \dots f_1\left(\underline{\omega_r^T} x\right) \dots f_l\left(\underline{\omega_1^T} x\right) \dots f_l\left(\underline{\omega_r^T} x\right)\right)$$

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$\phi(y)$

$K(x, y) = \phi^T(x)\phi(y)$

For example:

$h(x) = 1 \quad f_1 = \sin \quad f_2 = \cos \quad l = 2 \quad \underline{\omega} \sim N(O, I_l)$

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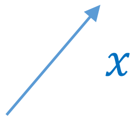

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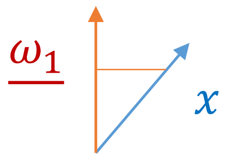

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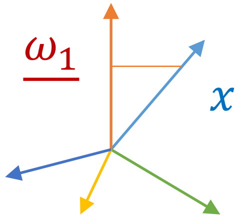

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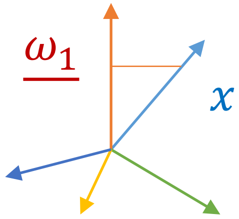

$$softmax\left(\frac{Q^T K}{\sqrt{P}}\right)$$

$$\sigma(\underline{s})_i = a_i = \frac{e^{s_i}}{\sum_j e^{s^j}}$$

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$$diag\,\underbrace{(A\underline{1})}_{D} = \begin{bmatrix} x & 0 & 0 & 0 & - \\ 0 & x & 0 & 0 & - \\ 0 & 0 & x & 0 & - \\ \cdots & \cdots & \cdots & \cdots & \end{bmatrix}$$

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$$\begin{bmatrix} \frac{1}{x} & 0 & 0 & 0 & - \\ 0 & \frac{1}{x} & 0 & 0 & - \\ 0 & 0 & \frac{1}{x} & 0 & - \\ \cdots & \cdots & \cdots & \cdots & \end{bmatrix}$$

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$$e^{-|x-y|^2} = e^{-(x-y)^T(x-y)} = e^{-[x^Tx+y^Ty-2x^Ty]} = e^{-x^Tx} \cdot e^{-y^Ty} \cdot e^{2x^Ty}$$

$$\underbrace{e^{-|x-y|^2}} = e^{\frac{-(x-y)^T(x-y)}{2}} = e^{\frac{-[x^T x+y^T y-2x^T y]}{2}} = e^{-x^T x} \cdot e^{-y^T y} \cdot \underbrace{e^{2x^T y}}$$

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$$K_{gauss} \cdot e^{\frac{x^Tx}{2}} \cdot e^{\frac{y^Ty}{2}}$$

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$$\phi(\underline{x}) = \frac{x^Tx}{2\sqrt{r}}\left(\sin\left(\underline{\omega_1^T}x\right) \sin\left(\underline{\omega_2^T}x\right) \dots \sin\left(\underline{\omega_1^T}x\right) \cos\left(\underline{\omega_1^T}x\right) \dots \cos\left(\underline{\omega_r^T}x\right)\right)$$

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$Q^T \rightarrow n \times p$

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$Q^T \rightarrow n \times p$

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$O(mn^2)$

$$\underbrace{V\ softmax\left(\frac{Q^T K}{\sqrt{P}}\right)}_{V\ (Q'^T K')}$$

$n \times p$

$p \times n$

$O(mn^2)$

$$\underbrace{m \times n \quad n \times r'}_{m \times r'}$$

$r' \times n$

$r' \times n$

$O(mr'n^2)$

$$V \, softmax\left(\frac{Q^T K}{\sqrt{P}}\right)$$

$n \times p$

$p \times n$

$V\,(Q'^T K')$

$O(mn^2)$

$\underbrace{m \times n \quad n \times r'}_{m \times r'}$

$r' \times n$

$r' \times n$

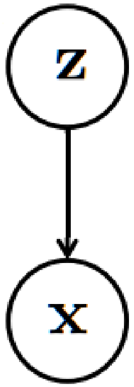
$O(mr'n^2)$

$$\begin{bmatrix} & \cdots & \\ \vdots & \ddots & \vdots \\ & \cdots & \end{bmatrix}$$

Variational Auto encoder (VEA)

Variational Inference

- Problem Definition
  - Observable Data: $x = \{x_1, x_2, ..., x_n\}$
  - Hidden Variable: $z = \{z_1, z_2, ..., z_n\}$



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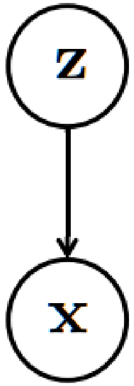


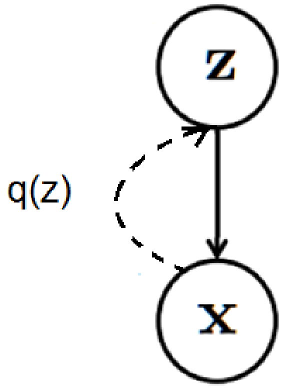
$$p(z|x) = \frac{p(z,x)}{p(x)} = \frac{p(x|z)p(z)}{\int p(x|z)p(z)dz}$$

Variational Inference

- Solutions
    - Monte Carlo Sampling
        - Metropolis Hasting
        - Gibbs Sampling
    - Variational Inference

Variational Inference

- Approximate $p(z|x)$ by $q(z)$



- Minimize the KL Divergence:

$$D_{KL}\left[q(z)||p(z|x)\right] = -\int q(z) log \frac{p(z|x)}{q(z)} dz$$

Variational Lower Bound

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Variational Lower Bound

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Variational Lower Bound

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$$= -\int q(z)log\frac{p(z,x)}{q(z)}dz + \int q(z)log\big(p(x)\big)dz$$

Variational Lower Bound

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$$= -\int q(z)log\frac{p(z,x)}{q(z)}dz + \int q(z)log\left(p(x)\right)dz$$

$$= -\int q(z)\Big(log\left(p(z,x)\right) - log\left(q(z)\right)\Big)dz + log\left(p(x)\right)$$

Variational Lower Bound

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$$= -\int q(z) log \frac{p(z,x)}{q(z)} dz + \int q(z) log\big(p(x)\big) dz$$

$$= -\int q(z)\Big(log\big(p(z,x)\big) - log\big(q(z)\big)\Big) dz + log\big(p(x)\big)$$

$$= -\underbrace{\left(E_{q(z)}\left[log\big(p(z,x)\big)\right] - E_{q(z)}\left[log\big(q(z)\big)\right]\right)}_{\text{Evidence Lower Bound (ELBO)}} + log\big(p(x)\big)$$

Variational Lower Bound

$$D_{KL}\left[q(z)||p(z|x)\right] = -\underbrace{\left(E_{q(z)}\left[log\left(p(z,x)\right)\right] - E_{q(z)}\left[log\left(q(z)\right)\right]\right)}_{\text{Evidence Lower Bound (ELBO)}} + log\left(p(x)\right)$$

Variational Lower Bound

$$D_{KL}\Big[q(z)||p(z|x)\Big] = -\underbrace{\left(E_{q(z)}\Big[log\big(p(z,x)\big)\Big] - E_{q(z)}\Big[log\big(q(z)\big)\Big]\right)}_{\text{Evidence Lower Bound (ELBO)}} + log\big(p(x)\big)$$

$$D_{KL}\Big[q(z)||p(z|x)\Big] = -L\Big[q(z)\Big] + log\Big(p(x)\Big)$$

Variational Lower Bound

$$D_{KL}\Big[q(z)||p(z|x)\Big] = -\underbrace{\left(E_{q(z)}\Big[log\big(p(z,x)\big)\Big] - E_{q(z)}\Big[log\big(q(z)\big)\Big]\right)}_{\text{Evidence Lower Bound (ELBO)}} + log\big(p(x)\big)$$

$$D_{KL}\Big[q(z)||p(z|x)\Big] = -L\Big[q(z)\Big] + log\Big(p(x)\Big)$$

$$log\Big(p(x)\Big) = D_{KL}\Big[q(z)||p(z|x)\Big] + L\Big[q(z)\Big]$$

Variational Lower Bound

$$D_{KL}\Big[q(z)||p(z|x)\Big] = -\underbrace{\left(E_{q(z)}\Big[log\big(p(z,x)\big)\Big] - E_{q(z)}\Big[log\big(q(z)\big)\Big]\right)}_{\text{Evidence Lower Bound (ELBO)}} + log\big(p(x)\big)$$

$$D_{KL}\Big[q(z)||p(z|x)\Big] = -L\Big[q(z)\Big] + log\Big(p(x)\Big)$$

$$log\Big(p(x)\Big) = D_{KL}\Big[q(z)||p(z|x)\Big] + L\Big[q(z)\Big]$$

Minimizing $D_{KL}\Big[q(z)||p(z|x)\Big]$

is equal to Maximizing $L\Big[q(z)\Big]$