Second-best national saving and growth with intergenerational disagreement*

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March 2014

Abstract

We illustrate the contrast between two sources of intergenerational disagreement when generations are overlapping and governments aggregate preferences in a utilitarian manner. Social preferences tend to exhibit a present-bias because generations are imperfectly altruistic about future generations; but they tend to exhibit a future-bias because coexisting generations are imperfectly altruistic about currently older generations. When the future-bias dominates, society faces an intergenerational equity problem, in which case the present-day government tends to support institutions that enable commitments to lower growth at the expense of future generations. This is so even with perfect altruism about future generations.

JEL classification: D60; H30; O40.
Keywords: intergenerational disagreement; altruism; overlapping generations; growth; quasi-hyperbolic discounting; commitment.

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1 Introduction

Whether or not governments should promote future oriented saving policies on behalf of future generations is a perennial question at least since Ramsey (1928). For a democratic government catering solely to the interests of present generations, the relevant calculus of optimal national saving is such that the values of the present-day society are to be used to allocate consumption between present and future generations. Accordingly, a valid argument for government intervention on behalf of future generations must address the consequences of intergenerational disagreement. One problem is that governments have to strike a balance between the preferences of different agents, and presently young and old people may have different stakes at raising saving. Another one is that governments today may be unable to bind future governments and the effect of current policies on future social outcomes may be undone by future policies.

By contrast, optimal national saving is commonly viewed as the optimum of some intertemporal social welfare function within the optimal growth framework developed by Ramsey (1928), Cass (1965) and Koopmans (1965). A key assumption is that social preferences are time-consistent, which ensures that the relative valuation of utility flows at different dates remains unchanged as the planning date evolves. When applied to the intergenerational context, the familiar time-consistency requirement amounts to an assumption of perfect altruism, leaving no room for intergenerational disagreement to constrain optimal growth paths.

In this paper we explore how intergenerational disagreement constrains the calculus of second-best national saving and growth. We illustrate the contrast between two natural sources of disagreement when generations are overlapping and preferences are aggregated in a utilitarian manner. Whereas social preferences tend to exhibit a present-bias when generations are imperfectly altruistic about future generations, they also tend to exhibit a future-bias when coexisting generations are imperfectly altruistic about currently older generations. Our analysis shows that the relative strength of the two biases determines how intergenerational disagreement shapes second-best national saving and growth. It also illustrates why the ability of actual governments to improve the welfare of future generations can be greatly constrained, even in an altruistic society.

Phelps and Pollak (1968), and more recently Krusell et al. (2002), analyze equilibrium saving in the presence of imperfect altruism about future generations, where private agents suffer from a present-bias, favoring short-term consumption, under the assumption that social preferences inherit the specific form of time inconsistency that afflicts private agents. Such an assumption is natural in their non-overlapping generations setting, which is the standard setting considered in other analyses of equilibrium growth with imperfect intergenerational altruism (e.g., Kohlberg, 1976, Bernheim and Ray, 1987, Ray, 1987, Barro, 1999).

Instead, we focus on intergenerational disagreement that stems from the combination of imperfect altruism about *past* generations and the overlap of generations that takes place in actual economies, which translates into disagreement between coexisting generations about the distribution of *current* aggregate consumption. It is well known that this kind of intergenerational disagreement renders plausible social welfare functions time-inconsistent, even if individuals are perfectly altruistic towards *future* generations.[1] However, today's young generations are tomorrow's old, and so it is unclear how intergenerational disagreement at each point in time translates into intertemporal allocations. In particular, neither the specific form of time-inconsistency that one may expect to afflict social preferences nor its implications for (second-best) government intervention in this context are well understood. The goal of our analysis is to shed some new light on these issues.

Formally, we consider a tractable endogenous growth model with overlapping altruistic generations, and characterize Markov perfect equilibrium behavior of a sequence of short-lived governments. We assume that each government's objective function is a weighted sum of the utilities of generations that are currently alive. Our modeling choices reflect the facts that actual generations coexist and democratic governments are unlikely to be immune to disagreement between current generations. Our restriction to Markov perfect equilibria captures the idea that intergenerational coordination of intertemporal choices is difficult, focusing our analysis on *discretionary* government intervention.

[1] See, e.g., Burbidge (1983), Calvo and Obstfeld (1988), Bernheim (1989), and Hori (1997).

For simplicity, we develop our main arguments in the case where individuals are perfectly altruistic towards the next generation, but not altruistic about *past* generations (e.g., Barro, 1974). With overlapping generations, such a disagreement immediately translates into disagreement between current generations about the distribution of current consumption. In turn, since current generations do agree on the future distribution of consumption, there must be disagreement between current and future governments about future aggregate consumption. The simplicity of our example allows us to characterize the specific form of time-inconsistency that one may expect and its consequences for Markov perfect equilibrium plans.

We begin by showing that governments whose objective function is a weighted sum of utilities of the currently alive generations in effect have quasi-hyperbolic preferences over *aggregate consumption*, even if all individuals have standard geometric discounting. This provides a simple application where quasi-hyperbolic discounting arises naturally. Moreover, we find that the specific biases of individual and social preferences can be remarkably different. We show that social preferences are biased *against* the present when private individuals have standard time-consistent preferences. The current government's preferences exhibit a bias *against* present aggregate consumption, because the current young do not value the consumption of the current old, and social welfare puts positive weight on the current young. Since the current generations value future consumption equally, the current government favors future aggregate consumption over current aggregate consumption. We also show that the direction of the government's bias may remain unchanged even if private agents are imperfectly altruistic towards future generations, suffering from a present-bias.

The bias in social preferences underlies the manner in which intergenerational disagreement constrains equilibrium growth, even though Markov perfect equilibria are time-consistent by construction, because governments need to anticipate future equilibrium behavior. Moreover, the current government realizes that future investment will respond to future income and so it has an incentive to invest strategically in order to manipulate future investment decisions. When we analyze the properties of equilibrium growth, our main conclusion is that intergenerational disagreement can be conducive to growth, rather than inimical to it, as is commonly concluded from the analysis of non-overlapping generations models (e.g., Sen, 1967, Phelps and Pollak, 1968, Kohlberg, 1976).

Furthermore, accounting for the overlapping generations demographic structure of an economy has implications for welfare. Strotz's (1956) seminal work, and more recently Laibson's (1997), demonstrate the general relevance that the economic agents' time-inconsistency has for the design of institutions that can cope with intertemporal disagreement by facilitating commitments. With respect to this, we show that the availability of commitment mechanisms to cope with intergenerational disagreement can harm future generations, even if social preferences tend to exhibit a present-bias because generations are imperfectly altruistic about future generations. This is because social preferences also tend to exhibit a future-bias when coexisting generations are imperfectly altruistic about currently older generations.

Not surprisingly, equilibrium growth is inefficiently low when the present-bias dominates. However, when the future-bias dominates, society faces an intergenerational equity problem, in which case the present-day government tends to support institutions that enable commitments to lower growth at the expense of future generations. When accounting for imperfect altruism about future as well as past generations, the above implication is ironic, because the very commitment mechanisms that may be advocated on the grounds of intergenerational disagreement need to be introduced by current generations, and so their introduction may hurt precisely those generations that is supposed to help.

There are other sources of intergenerational disagreement that we have not considered here. Indeed, the problem of aggregation of heterogeneous preferences over consumption streams is receiving increasing attention (e.g., Weitzman, 2001, Caplin and Leahy, 2004, Blackorby et al., 2005, Gollier and Zeckhauser 2005, Jackson and Yariv, 2010, Zuber, 2010). For instance, Jackson and Yariv (2010) show that weighted averaging of utilities across individuals with geometric, but heterogeneous, time preferences tends to translate into time inconsistency that is characterized by a present-bias. Here we show that the overlap of generations is the source of a natural counteracting bias against the present, which radically changes the consequences of intergenerational disagreement for government intervention.

The next section presents the basic model. Section 3 illustrates the source of the bias in social preferences in the context of the first-best allocation for an arbitrary government. Section 4 analyzes the symmetric Markov perfect equilibrium in linear strategies. Section 5 discusses the welfare implications of our analysis. In Section 6, we extend our analysis to allow for individuals who are imperfectly altruistic about future generations, which in effect extends Phelps and Pollack's (1968) seminal work to account for overlapping generations. Section 7 concludes. Technical proofs are in the Appendix.

2 The model

Consider an economy with overlapping altruistic generations. A unit mass of individuals are born every period $t \geq 0$, each individual lives for two periods and individuals born at date t have preferences given by

$$\begin{aligned} u_t &= u\left(c_t^y\right) + u\left(c_{t+1}^o\right) + \delta u_{t+1} \\ &= u\left(c_t^y\right) + u\left(c_{t+1}^o\right) + \sum_{s=1}^{\infty} \delta^s \left(u\left(c_{t+s}^y\right) + u\left(c_{t+1+s}^o\right)\right), \end{aligned} \tag{1}$$

with $\delta \in (0,1)$, where c_t^y is the consumption of young agents at date t, and c_{t+1}^o is their consumption when old. For simplicity, we assume that individuals do not discount their second-period felicity, and also that felicity functions each period are isoelastic, with

$$u(c) = \begin{cases} \frac{c^{1-\sigma}-1}{1-\sigma} & \text{if } \sigma \neq 1, \quad \sigma > 0 \\ \ln(c) & \text{if } \sigma = 1. \end{cases} \tag{2}$$

Output is linear in the capital stock at the aggregate level, where k_t units of capital produce Ak_t units of output that become available at date $t+1$, with $A > 0$. The aggregate resources constraint in the economy is given by

$$Ak_t \geq c_t^y + c_t^o + k_{t+1} - k_t, \tag{3}$$

where we have ignored depreciation of the capital stock. We assume that $\delta(A+1) > 1$ in order to ensure positive equilibrium growth rates. We also assume that $\delta(A+1)^{\frac{1-\sigma}{\sigma}} < 1$ in order to ensure that growth is not so fast that it leads to unbounded utility.

We consider a sequence of governments. The government at date t seeks to maximize a weighted sum of the present generations' utilities:

$$v_t = u_{t-1} + a u_t, \tag{4}$$

where $a > 0$. The assumption of a utilitarian welfare objective is meant to capture in a simple manner the fact that democratic governments are unlikely to be immune to disagreement between coexisting generations. It can be interpreted as the outcome of political competition in a probabilistic voting model.[2]

Note that the utility of individuals born in period t can be written as

$$u_t = u\left(c_t^y\right) + \sum_{s=1}^{\infty} \delta^s \left(u\left(c_{t+s}^y\right) + \delta^{-1} u\left(c_{t+s}^o\right)\right).$$

The difference between individuals born at date $t-1$ and those born at date t is that the latter do not care about the former. Otherwise, their preferences are time consistent: the trade-off between dates t and $t+1$ is perceived the same way by all individuals at date $t-1$ and at date t. Accordingly, the date-t government's preferences are given by

$$v_t = u\left(c_{t-1}^y\right) + u\left(c_t^o\right) + (\delta + a) u\left(c_t^y\right) + (\delta + a) \sum_{s=1}^{\infty} \delta^s \left(u\left(c_{t+s}^y\right) + \delta^{-1} u\left(c_{t+s}^o\right)\right). \tag{5}$$

Consider the right side of equation (5). The government at date t must treat the old individuals' felicity at date $t-1$ (first term) as sunk. The felicity from consumption of old individuals at date t (second term) enters with weight 1, the weight at which the government values the old individuals' utility. However, the felicity of young individuals at date t (third term) enters not only with the direct weight on their young individuals' utility, given by a, but also indirectly because the old care about the young (with weight δ) and the government cares about the old (with weight 1). The last term reflects that both young and old individuals care about future consumption through altruism, and the young care about their own future old-age consumption.

[2] See, for example, Lindbeck and Weibull (1987).

Inspection of equation (5) indicates that the date-t government's preferences are time inconsistent as long as $a > 0$, that is, as long as the date-t government's preferences put any weight on the utility of currently young individuals. The weight of old-age consumption at t relative to old-age consumption at $t+1$ equals $1/(\delta + a)$, and this is smaller than the weight on the felicities of old-age consumption at $t+s$ relative to old-age consumption at $t+1+s$, which equals $1/\delta$. This stems from the fact that the currently old care about the future consumption of the currently young, but the young do not care about the currently old. However, both the currently young and currently old care about the consumption of all future generations. As a result, consumption of the currently old gets a relatively small weight.[3]

Thus, current and future governments evaluate future consumption streams differently, and so equilibrium behavior will be shaped by the underlying intergenerational conflict. Markov perfect equilibria provide a useful way to capture the consequences of such a conflict by focusing on behavior that depends solely on payoff-relevant state variables, reflecting the difficulties that current and future governments face to coordinate their behavior.

The welfare of each generation is influenced by the actions of different governments. Consequently, each government's optimal behavior depends on its expectation of future governments' behavior. Since every government can affect future aggregate economic conditions, equilibrium allocations depend on the interaction between current and future governments. We consider the following problem. Every period t the government chooses investment $(k_{t+1} - k_t)$, and consumption (c_t^y and c_t^o) in order to maximize (5) subject to (2) and (3), taking as given the strategies of all other governments. A Markov strategy of the date-t government consists of an investment policy $i^t(k_t)$ and consumption policies $c_y^t(k_t)$ and $c_o^t(k_t)$ that are only functions of the payoff-relevant state variable k_t. A sequence of Markov

[3] The time inconsistency of the date-t government's preferences would still arise in the presence of two-sided altruism. Suppose that $u_t = u(c_t^y) + u(c_{t+1}^o) + \delta_F u_{t+1} + \delta_B u_{t-1} = \tilde{u}_t + \delta_F u_{t+1} + \delta_B u_{t-1}$. Kimball (1987) shows that this generates, under certain conditions, the following time-consistent individual preferences: $u_t = \sum_{b=1}^{\infty} (\lambda_B)^{-b} \tilde{u}_{t-b} + \sum_{f=0}^{\infty} (\lambda_F)^f \tilde{u}_{t+f}$, where λ_B and λ_F are functions of δ_B and δ_F. Following the same procedure as above, one can verify that time inconsistency of the date-t government's preferences arises if and only if $\lambda_B \neq \lambda_F$ and that the currently old get too small a weight whenever $\lambda_B > \lambda_F$, which is the natural case. This formulation would generate the same conclusions as our simpler case of one-sided altruism.

strategies $\left\{f_t\left(i^t\left(k_t\right), c_y^t\left(k_t\right), c_o^t\left(k_t\right)\right)\right\}_{t=0}^{\infty}$ is a symmetric Markov perfect equilibrium if it is a subgame perfect equilibrium for every realization of the state variable k_t, and all governments follow the same strategy, that is, if $f_t\left(i^t\left(k_t\right), c_y^t\left(k_t\right), c_o^t\left(k_t\right)\right) = f\left(i\left(k_t\right), c_y\left(k_t\right), c_o\left(k_t\right)\right)$, for all t. We will restrict attention to symmetric Markov perfect equilibria in linear strategies.

3 Benchmark commitment solution

In order to understand the impact of the time inconsistency of social preferences on equilibrium behavior, it is useful to consider first a benchmark problem for an arbitrary government under the assumption that it can control future allocations. The goal of this section is to characterize the solution to this benchmark problem. The nature of this first-best solution is clarified by formulating the date-t government problem recursively. We will simplify notation by avoiding time subscripts and using primes to denote next-period values whenever possible.

First, consider the static intergenerational allocation of consumption every period from the viewpoint of the date-t government. At date t, only the second and third term in the social preferences given by equation (5) are relevant; therefore, the optimal intergenerational allocation of consumption solves the static problem

$$\max_{c^y, c^o} \left\{u\left(c^y\right) + (\delta + a)^{-1} u\left(c^o\right)\right\} \text{ subject to } c^y + c^o \leq c, \text{ with } c^y, c^o \geq 0, \tag{6}$$

and so the young's share of aggregate consumption is given by

$$\tau_c \equiv \frac{c^y}{c} = 1 - \frac{c^o}{c} = \frac{1}{1 + (\delta + a)^{-1/\sigma}}, \tag{7}$$

where $c = c^y + c^o$.

By contrast, from date $t+1$ onwards, the date-t government would choose intergenerational consumption allocations differently than future governments would actually do. Instead, the date-t government's optimal allocation would solve the static problem

$$\max_{c^y, c^o} \left\{u\left(c^y\right) + \delta^{-1} u\left(c^o\right)\right\} \text{ subject to } c^y + c^o \leq c, \text{ with } c^y, c^o \geq 0. \tag{8}$$

Accordingly, the young's share of aggregate consumption at every future date would be

$$\overline{\tau}_c \equiv \frac{c^y}{c} = 1 - \frac{c^o}{c} = \frac{1}{1+\delta^{-1/\sigma}}, \tag{9}$$

where $c = c^y + c^o$. It is easy to see that $\tau_c > \overline{\tau}_c$. The date-$t$ government prefers to allocate a larger share of aggregate consumption to the current young than the share he would like to allocate to the young in every future period.

Now consider the date-t government's investment problem. Taking into account the optimal intergenerational allocations of aggregate consumption every period, by substituting the consumption shares given by equations (7) and (9) into the social preferences given by equation (5), we express the relevant preferences for the date-t government in terms of aggregate consumption levels as

$$\widetilde{v}_t = q(\tau_c, a)\, u(c_t) + q(\overline{\tau}_c, 0) \sum_{s=1}^{\infty} \delta^s u(c_{t+s}), \tag{10}$$

where

$$q(\tau, a) = \tau^{1-\sigma} + (\delta + a)^{-1} (1-\tau)^{1-\sigma}. \tag{11}$$

Note that $\widetilde{v}_t$ is simply a positive linear transformation of the preferences for the date-t government given in equation (5). The representation of social welfare in equation (10) reveals the key to understanding the governments' problem. It shows that social preferences are time inconsistent whenever $q(\tau_c, a) \neq q(\overline{\tau}_c, 0)$, which in turn is the case if and only if $a > 0$. Even though all individuals have standard time-consistent, geometric preferences, governments in effect have time-inconsistent, quasi-hyperbolic preferences (over aggregate consumption streams) of the form used by Phelps and Pollak (1968), Laibson (1997) and Krusell, Kuruşçu and Smith (2002). In our model, it is the overlapping generations demographic structure and the fact that current generations care insufficiently about previous generations that imply that governments' preferences, which aggregate the preferences of the generations currently alive, are quasi-hyperbolic. Furthermore, it can be easily seen that $q(\tau_c, a) < q(\overline{\tau}_c, 0)$, and thus, governments have an "excessive" incentive to postpone current consumption. This will be the source of our main results below.

From date $t+1$ onwards, the date-t government has time consistent preferences, and so it would solve the following problem:

$$W(k) = \max_{0 \leq k' \leq Ak} \left\{ q(\overline{\tau}_c, 0)\, u(Ak - k' + k) + \delta W(k') \right\}, \tag{12}$$

where $q(\tau, a)$ is given by equation (11) and $\overline{\tau}_c$ is given by equation (9). It is easy to verify that the corresponding first-order condition equates the marginal disutility from additional investment and the marginal value of additional capital next period:

$$-q(\overline{\tau}_c, 0) \frac{\partial u(c)}{\partial c} \frac{\partial c}{\partial k'} = \delta \frac{\partial W(k')}{\partial k'}. \tag{13}$$

Furthermore, since preferences are time consistent from date $t+1$ onwards, the solution to the above problem satisfies the familiar envelope condition

$$\frac{\partial W(k)}{\partial k} = q(\overline{\tau}_c, 0) \frac{\partial u(c)}{\partial c} \frac{\partial c}{\partial k} \tag{14}$$

every period. Combining equation (14), evaluated one period ahead, and equation (13), it is easy to see that the intertemporal allocation of aggregate consumption satisfies the familiar Euler equation

$$\frac{\partial u(c) / \partial c}{\delta \left(\partial u(c') / \partial c'\right)} = \frac{-\partial c' / \partial k'}{\partial c / \partial k'},$$

which equates the marginal rate of substitution between current and next-period consumption and the corresponding marginal rate of transformation. Taking derivatives and noting that consumption and capital grow at the common rate $\overline{g}_c$, it can be verified that the solution to the above standard dynamic programming problem implies that investment from date $t+1$ onwards is given by $k' - k = \overline{g}_c k$, where

$$1 + \overline{g}_c = \left[\delta (A + 1)\right]^{1/\sigma}. \tag{15}$$

Finally, the investment problem at date t can be formulated as:

$$W_0(k) = \max_{0 \le k' \le Ak} \{q(\tau_c, a) u(Ak - k' + k) + \delta W(k')\}, \tag{16}$$

where $q(\tau, a)$ is given by equation (11) and τ_c is given by equation (7). Once again, the corresponding first-order condition equates the marginal disutility incurred from additional investment and the marginal value of additional capital next period:

$$-q(\tau_c, a) \frac{\partial u(c)}{\partial c} \frac{\partial c}{\partial k'} = \delta \frac{\partial W(k')}{\partial k'}. \tag{17}$$

Noting that the value of future capital is derived from the appropriately weighted sum of utilities from the future consumption stream it generates, that is,

$$\delta W(k_{t+1}) = \sum_{s=1}^{\infty} \delta^s q(\overline{\tau}_c, 0) u(c_{t+s}),$$

and noting that consumption from date $t+1$ onwards grows at the constant growth rate $\overline{g}_c$, as given by equation (15), one can verify that the value of future capital is such that

$$W(k') = \text{constant} + \left(\frac{q(\overline{\tau}_c, 0)}{1 - \delta (1 + \overline{g}_c)^{1-\sigma}} \right) u(c'),$$

therefore, the marginal value of additional capital next period is given by

$$\frac{\partial W(k')}{\partial k'} = \left(\frac{q(\overline{\tau}_c, 0)}{1 - \delta (1 + \overline{g}_c)^{1-\sigma}} \right) \frac{\partial u(c')}{\partial c'} \frac{\partial c'}{\partial k'}. \tag{18}$$

Combining equations (17) and (18), it is easy to see that time inconsistency influences the date-t government's allocation of aggregate consumption at date t by introducing a wedge between the marginal rate of substitution between current and next-period consumption and the corresponding marginal rate of transformation:

$$\frac{\partial u(c) / \partial c}{\delta (\partial u(c') / \partial c')} = \left(\frac{q(\overline{\tau}_c, 0) / q(\tau_c, a)}{1 - \delta (1 + \overline{g}_c)^{1-\sigma}} \right) \frac{-\partial c' / \partial k'}{\partial c / \partial k'}. \tag{19}$$

The magnitude of the wedge takes into account that $c_t^y = \tau_c c$ and $c_{t+1}^y = \overline{\tau}_c c'$, through the term $q(\overline{\tau}_c, 0) / q(\tau_c, a) = (\overline{\tau}_c / \tau_c)^{-\sigma}$, and applies an effective discount rate equal to

$\delta\left(1+\overline{g}_c\right)^{1-\sigma}$, because the young's share of aggregate consumption in all future periods is equal to $\overline{\tau}_c$ rather than τ_c. To interpret the wedge, note that the ratio $q\left(\overline{\tau}_c,0\right)/q\left(\tau_c,a\right)$ specifies the relative weight placed on $u\left(c_{t+1}\right)$ rather than $u\left(c_t\right)$ by the social welfare function in equation (10).

Using the facts that felicity functions (given in equation (2)) are isoelastic, the aggregate resources constraint (given in equation (3)) holds with equality, and investment from date $t+1$ onwards is given by $i\left(k\right)=\overline{g}_c k$, to evaluate equation (19), one can verify that the solution to problem (16) implies that investment at date t is given by $i\left(k\right)=g_c k$, with

$$1+g_c=\frac{A+1}{1+\left(\frac{q(\tau_c,a)}{q(\overline{\tau}_c,0)}\frac{\delta^{-1}-(1+\overline{g}_c)^{1-\sigma}}{(A-\overline{g}_c)^{1-\sigma}}\right)^{1/\sigma}}, \tag{20}$$

where $q\left(\tau,a\right)$ is given by equation (11) and τ_c, $\overline{\tau}_c$ and $\overline{g}_c$ are given by equations (7), (9), and (15), respectively.

In the Appendix we show that $g_c>\overline{g}_c$, for all $\sigma>0$. If the date-t government could commit future allocations, it would choose a current growth rate that is larger than the growth rate it would dictate to future generations. This is because the current government cares more about the future old than it does about the current old generation, and so $\frac{q(\tau_c,a)}{q(\overline{\tau}_c,0)}<1$, for $a>0$. In turn, this occurs because the current government puts positive weight on the current young, but the current young does not care about the current old. Indeed, it can be verified that the right side of equation (20) is equal to $1+\overline{g}_c$ if and only if $a=0$ (see Appendix). It should be noted that our assumption that the current young do not place any weight at all on the current old is made for simplicity. The essential feature is that the current young do not place *sufficient* weight on the current old.[4]

The following proposition summarizes our discussion so far.

Proposition 1 *If the date-t government could precommit future allocations, optimal allocations would be given by* $i(k)=gk$, $c_y(k)=\tau\left(A-g\right)k$, *and* $c_o(k)=\left(1-\tau\right)\left(A-g\right)k$,

[4]See footnote 3 and the references in foonote 1.

with

$$(\tau, g) = \begin{cases} (\tau_c, g_c) & \textit{in the first period} \\ (\overline{\tau}_c, \overline{g}_c) & \textit{in every future period,} \end{cases}$$

where τ_c *and* $\overline{\tau}_c$ *are given by equations (7) and (9), respectively, with* $\tau_c > \overline{\tau}_c$*; while* g_c *and* $\overline{g}_c$ *are given by equations (20) and (15), respectively, with* $g_c > \overline{g}_c$*.*

Of course, the problem with the above solution is that it is time inconsistent. Accordingly, each government needs to take into account that future governments will deviate from the allocation that the current government would dictate if it could control future allocations. In the following section we consider equilibrium behavior when current governments recognize that future allocations will be chosen optimally by future governments.

4 Symmetric Markov perfect equilibrium

In this section we characterize the unique symmetric Markov perfect equilibrium in linear strategies. Now each government recognizes that every future government will choose the same optimal intergenerational allocation of consumption each period as the one chosen in the current period by the current government. This is the allocation that solves the static problem given by equation (6) and so the young's share of aggregate consumption is now given by

$$\tau^* \equiv \frac{c^y}{c} = 1 - \frac{c^o}{c} = \frac{1}{1 + (\delta + a)^{-1/\sigma}}. \tag{21}$$

every period. Of course, τ^* is equal to the young's share of aggregate consumption that the date-t government would have chosen at date t even if it could control future allocations. That is, we have $\tau^* = \tau_c$ every period.

Suppose that the current government anticipates that every future government follows the linear investment policy $i' = \widehat{g}k'$, with $\delta (1 + \widehat{g})^{1-\sigma} < 1$. Then, the current investment decision solves the following problem:

$$V_0(k) = \max_{0 \le k' \le Ak} \{q(\tau^*, a)\, u(Ak - k' + k) + \delta V(k')\}, \tag{22}$$

with

$$V(k) = q(\tau^*, 0)\, u(Ak - (1+\widehat{g})\, k + k) + \delta V((1+\widehat{g})\, k), \tag{23}$$

where $q(\tau, a)$ is given by equation (11), and τ^* is given by equation (21). An investment policy $i(k) = gk$ that is part of a symmetric Markov perfect equilibrium must have $g = \widehat{g}$.

In order to appreciate the role of commitment problems, first note that the first-order condition with respect to k' at date t is given by

$$-q(\tau^*, a) \frac{\partial u(c)}{\partial c} \frac{\partial c}{\partial k'} = \delta \frac{\partial V(k')}{\partial k'}, \tag{24}$$

which equates the marginal disutility incurred from additional investment and the marginal value of additional capital next period. Solving the recursion in equation (23) it can be verified that

$$V(k') = \text{constant} + \left(\frac{q(\tau^*, 0)}{1 - \delta (1+\widehat{g})^{1-\sigma}} \right) u(c')$$

and so we have

$$\frac{\partial V(k')}{\partial k'} = \left(\frac{q(\tau^*, 0)}{1 - \delta (1+\widehat{g})^{1-\sigma}} \right) \frac{\partial u(c')}{\partial c'} \frac{\partial c'}{\partial k'}. \tag{25}$$

The first-order condition at date t if the date-t government could control future allocations would be the same as equation (24), except that V is replaced by W. The main difference lies in the marginal effect of current investment on the value function next period. The difference can be understood as follows. Each government recognizes that a marginal increase in current investment results in extra income next period that will in turn influence investment next period. Since current and future governments disagree about future investment decisions, current governments have an incentive to manipulate future investment decisions via current investment.

By contrast, if the current government could control future allocations, time consistency of the date-t government's preferences from date $t+1$ onwards would ensure that the familiar envelope condition holds, which ensures that the above effect of current on future investment can be ignored when making current investment decisions. In turn, this guarantees that investment in every future period will be given by $\overline{g}_c k$. The difference between equations (25) and (18) lies in that, in the Markov perfect equilibrium, the current government anticipates intergenerational disagreement in every future period.

Let us return to the characterization of the Markov perfect equilibrium. Combining equations (24) and (25) we have

$$\frac{\partial u(c)/\partial c}{\delta(\partial u(c')/\partial c')} = \left(\frac{q(\tau^*,0)/q(\tau^*,a)}{1-\delta(1+\widehat{g})^{1-\sigma}}\right)\frac{-\partial c'/\partial k'}{\partial c/\partial k'}. \tag{26}$$

Although the intertemporal allocation of aggregate consumption in the Markov perfect equilibrium is time consistent by construction, there is a wedge between the marginal rate of substitution between current and next-period consumption and the corresponding marginal rate of transformation. The magnitude of the wedge takes into account that the young's share of aggregate consumption every period is equal to $\tau^* = \tau_c > \overline{\tau}_c$ rather than $\overline{\tau}_c$, and also anticipates that investment in all future periods is given by $i(k') = \widehat{g}k'$.

Recognizing that

$$\frac{\partial c'}{\partial k'} = A - \frac{\partial i(k')}{\partial k'} = A - \widehat{g},$$

since $c' = Ak' - i(k')$ and $i(k') = \widehat{g}k'$, and using the facts that felicity functions (given in equation (2)) are isoelastic and the aggregate resources constraint (given in equation (3)) holds with equality, it is now straightforward to write the above Euler equation as

$$\left(\frac{k'}{Ak-k'+k}\right)^{\sigma} = \frac{q(\tau^*,0)(A-\widehat{g})^{1-\sigma}}{q(\tau^*,a)\left(\delta^{-1}-(1+\widehat{g})^{1-\sigma}\right)},$$

which describes the best response k' to the anticipation of $\widehat{g}$, for given k. Clearly, the best response to any given $\widehat{g}$ is linear in k. Consequently, we obtain the best-response mapping

$$1+g = \frac{A+1}{1+\left(\frac{q(\tau^*,a)}{q(\tau^*,0)}\frac{\delta^{-1}-(1+\widehat{g})^{1-\sigma}}{(A-\widehat{g})^{1-\sigma}}\right)^{1/\sigma}} \equiv 1 + B(\tau^*,\tau^*,\widehat{g}). \tag{27}$$

The best response function $g = B(\tau,\tau',\widehat{g})$ characterizes the best investment response by a government that allocates a share τ of current consumption to the current young and anticipates that future governments will allocate a share τ' of consumption to the young and invest according to $i(k') = \widehat{g}k'$. Note that the structure of the best response function in equation (27) is identical to that in equation (20). The above commitment solution has $g_c = B(\tau_c, \overline{\tau}_c, \overline{g}_c)$, whereas a symmetric Markov perfect equilibrium has $g^* = B(\tau^*, \tau^*, g^*)$.

Proposition 2 *(i) There exists a unique symmetric, interior, Markov perfect equilibrium in linear strategies. The equilibrium is characterized by $i(k) = g^*k$, $c_y(k) = \tau^*(A - g^*)k$, and $c_o(k) = (1 - \tau^*)(A - g^*)k$, where τ^* is given by equation (21); $g^* = B(\tau^*, \tau^*, g^*) \in (\overline{g}_c, A)$, where B is given by equation (27) and $\overline{g}_c$ is given by equation (15). (ii) For all $\widehat{g} \in (\overline{g}_c, A)$, $\partial B(\tau^*, \tau^*, \widehat{g})/\partial\widehat{g} \geq 0$ if and only if $\sigma \geq 1$, with equality if and only if $\sigma = 1$.*

Part (i) characterizes the unique symmetric, interior, Markov perfect equilibrium in linear strategies. Part (ii) provides additional insight into the role of commitment problems. Note that the disagreement between governments about investment decisions takes the particular form that the date-$(t+1)$ government invests too much from the viewpoint of the date-t government. The best response mapping (27) indicates how each government will attempt to manipulate investment next period. Part (ii) of the proposition implies that locally around the equilibrium current and next-period investments are "strategic" complements if $\sigma > 1$ and "strategic" substitutes if $\sigma < 1$. The panels in Figure 1 plot the different types of best responses.

[FIGURE 1]

Panel (1) shows that $B(\tau^*, \tau^*, \widehat{g})$ decreases at first, reaching a minimum at $\overline{g}_c$, and then increases, when $\sigma > 1$. Panel (2) shows that the best response is flat when $\sigma = 1$. In this case, $g^* = B(\tau^*, \tau^*, g^*)$ has a closed-form solution and the equilibrium growth rate is given by

$$1 + g^* = \frac{A + 1}{1 + \left(\frac{\delta + a + 1}{\delta + a}\right)\left(\frac{1 - \delta}{1 + \delta}\right)}. \tag{28}$$

Panel (3) in the above figure illustrates that $B(\tau^*, \tau^*, \widehat{g})$ increases at first, peaking at $\overline{g}_c$, and then decreases, when $\sigma < 1$.

The role of the elasticity of intertemporal substitution, given by $1/\sigma$, is worth noting. With respect to a generation's lifetime, higher values of σ indicate greater aversion to differences in consumption over the life cycle. However, since individuals are altruistic, higher values of σ also indicate greater aversion to unequal consumption across generations. With balanced growth, the higher the value of σ, the less individuals are willing to tolerate larger positive, or smaller negative, growth rates. Indeed, it is not difficult to show that the equilibrium growth rate decreases as individuals are less willing to substitute consumption intertemporally. This is as expected. The following proposition illustrates the role of σ when comparing growth rates with and without commitment.

Proposition 3 *$g^* > g_c$ if and only if $\sigma > 1$, with $g^* > \overline{g}_c$ for all $\sigma > 0$.*

This proposition compares long-run growth in the Markov perfect equilibrium and the commitment solution. Proposition 3 is a striking result for two reasons. First, $\overline{g}_c$ is the first-best growth rate from the viewpoint of the old, and it is also the growth rate that every young generation, and every government, would dictate on every future generation, if they could do so. In this sense, commitment problems lead to equilibrium growth that is too high, relative to the preferences of all generations. Second, the private return to investment is lower than the social return to investment. The latter is given by the constant marginal product of capital A, whereas the former is given by $A - \partial(g^*k)/\partial k = A - g^*$.

In order to understand the source of the result stated in Proposition 3, it is useful to consider the relationship between the current investment decisions of a government with and without commitment. Note that the investment problem of the date-t government at date t, given by equation (22), can be written as

$$V_0(k) = \max_{0 \leq k' \leq Ak} \left\{ \widetilde{W}_0(k, k') - \delta\left(W(k') - V(k')\right) \right\} \tag{29}$$

where W and V are given by equation (12) and equation (23), respectively, and where $\widetilde{W}_0(k, k')$ is precisely the objective to be maximized at date t under the assumption that the date-t government can commit future allocations (see equation (16)), that is,

$$W_0(k) = \max_{0 \le k' \le Ak} \widetilde{W}_0(k, k'). \tag{30}$$

Clearly, it must be that $V_0(k) < W_0(k)$, since the commitment solution from date $t+1$ onwards is the date-t government's first-best solution (i.e., since $W(k') - V(k') > 0$). Thus, whenever the current government anticipates $(\tau^*, \widehat{g})$ to deviate from $(\overline{\tau}_c, \overline{g}_c)$ in the future, it anticipates a welfare loss. Accordingly it has an incentive to invest strategically to compensate for this loss. Formally, the marginal effect of additional current investment on the welfare loss associated with the difference between $\widehat{g}$ and $\overline{g}_c$ in the future is given by $\partial W(k')/\partial k' - \partial V(k')/\partial k'$, where $\partial W(k')/\partial k'$ is given by equation (18) and $\partial V(k')/\partial k'$ is given by equation (25). It is easy to verify that $\partial W(k')/\partial k' - \partial V(k')/\partial k' \le 0$ if and only if $B(\tau^*, \tau^*, \widehat{g}) \ge B(\tau_c, \overline{\tau}_c, \overline{g}_c)$, where recall that the best-response mapping $B(\tau^*, \tau^*, \widehat{g})$ is given by equation (27), with $B(\tau^*, \tau^*, g^*) = g^*$ and $B(\tau_c, \overline{\tau}_c, \overline{g}_c) = g_c$.

To understand the above "strategic-compensation effect", note that the anticipated discrepancy between $(\tau^*, \widehat{g})$ and $(\overline{\tau}_c, \overline{g}_c)$ gives rise to two opposing effects. The problem arises because future governments weigh future consumption too little relative to the current government, that is, $q(\tau^*, a) < q(\tau^*, 0)$. On the one hand, for given next-period consumption, next-period utility is anticipated to be lower because next-period's government will misallocate consumption over the two generations. The current government can compensate for this loss by strategically raising investment in order to increase next-period aggregate consumption. On the other hand, transferring wealth to the future has a lower return, because the increase in production is misallocated over the two coexisting generations: by strategically lowering investment the current government can substitute intertemporally away from misallocated future investment.

With log utility, the current government is in effect unable to use current investment strategically to its advantage, as the welfare loss from misallocation of future investment exactly offsets the welfare gain from additional future consumption. Consequently, the current government's best response to any future growth rate $\widehat{g} \ge \overline{g}_c$ is given by $i(k) = g_c k$ when $\sigma = 1$. Since every future government faces the same problem, each government will choose the growth rate that it would be chosen in the first period if future allocations could be controlled. Thus, the resulting equilibrium growth rate g^* must be equal to g_c. This outcome reflects the pro-growth bias inherent to the social preferences that aggregate the preferences of coexisting generations that disagree about current investment: the young would like faster current growth than the old. This is the "imperfect altruism effect" that underlaid the relatively high *short-run* growth rate associated with the benchmark commitment solution. Now, however, this effect translates into higher *long-run* equilibrium growth, relative to the commitment solution.

If inequality aversion is large enough (i.e., if $\sigma > 1$), the welfare loss from misallocation of future investment cannot offset the welfare gain from additional future consumption and thus, each government has a strategic incentive to overinvest, relative to $i(k) = g_c k$. This explains the strategic complementarity between current and next-period investments (see Proposition 2), which leads to a long-run equilibrium growth rate g^* that is not only higher than $\overline{g}_c$, but also higher than g_c.

Finally, if $\sigma < 1$, the welfare loss from misallocation of future investment more than offsets the welfare gain from additional future consumption. This explains the strategic substitutability between current and next-period investments (see Proposition 2), which leads to a long-run equilibrium growth rate g^* that is lower than g_c while still higher than $\overline{g}_c$.

It is also worth noting the following limiting results.

Proposition 4 *Suppose that $\sigma > 1$. Then,*

$$\lim_{\delta \to 1} g^* > \overline{g}_c, \text{ for } a > 0\text{; and } \lim_{a \to 0} g^* = \overline{g}_c < A = \lim_{a \to \infty} g^*, \text{ for } 0 < \delta \leq 1.$$

Thus, even in the limit as the discount rate on future generations approaches zero, the equilibrium growth rate is strictly higher than $\overline{g}_c$. This is because as long as governments puts weight on the current young the time inconsistency of their preferences creates a nontrivial problem, which does not disappear as δ approaches 1. Furthermore, as the weight governments put on the current old becomes negligible, the equilibrium growth rate becomes arbitrarily close to A (for $\sigma > 1$), and so the savings rate approaches 1. In this sense, the equilibrium growth rate can be arbitrarily higher than the growth rate that is preferred by all generations.

5 Welfare implications

Accounting for the overlapping generations demographic structure of an economy not only gives different results, but it also has implications for welfare. Strotz's (1956) seminal work, and more recently Laibson's (1997) demonstrate the general relevance of economic agents' time-inconsistency for the design of institutions that can cope with intertemporal disagreement by facilitating commitments. With respect to this, a remarkable corollary of our previous analysis is that the availability of commitment mechanisms to cope with intergenerational disagreement can harm future generations.

Proposition 5 *In the Markov perfect equilibrium, the date-t government would like to commit all future allocations optimally, even though this implies that there is a time $T \geq t$ such that all generations born after date T will be made worse off, from the perspective of both young and old age.*

The proposition follows from comparing the two allocations characterized above. Clearly, starting at the Markov perfect equilibrium, the date-t government prefers the commitment solution, which is the one it would optimally choose if it could control future allocations. Mechanically, the fact that an infinite number of generations must be made worse off by the move (at date t) from the equilibrium allocation to the commitment solution follows from the fact that the wealth differential between the two allocations grows without bound and the original equilibrium allocation has higher growth at all times. Moreover, this is so even in the limit as individuals do not discount the future, which is well defined for all $\sigma > 1$.

Not only the date-t government would support institutions to enable commitments to lower the growth rate of the economy permanently, but they may also be unanimously supported by all private agents at date t. With log utility, for example, the move from the Markov perfect equilibrium to the commitment solution does not affect the first period consumption allocation, and it leads to the first best allocation for both generations after the current period. Hence, both current generations must be better off.

The above implications follow immediately from our previous analysis, even though it may seem at first counterintuitive that individuals who are perfectly altruistic towards future generations would support institutions that will necessarily harm future generations. The main insight here is that they do so because future equilibrium growth is too high from the perspective of current generations, not because equilibrium investment is dynamically inefficient.

In principle, allocations can be dynamically inefficient on the production side and/or the consumption side of the economy. As usual, we say that an investment allocation is dynamically efficient if there is no alternative allocation that provides more aggregate consumption in one period and at least the same consumption in every other period. We say that consumption allocations are dynamically efficient if there is no alternative allocation of aggregate consumption across generations that provides higher utility for one generation and at least the same utility for any other generation.

It is not difficult to verify that equilibrium investment is dynamically efficient. To see why, note that this is the case if the growth rate is lower than the *social* return to investment, that is, if $g^* < A$.[5] That this condition must hold follows immediately from the aggregate resources constraint: $Ak_t \geq c_t + k_{t+1} - k_t$.

By contrast, equilibrium consumption is dynamically inefficient if and only if the growth rate is higher than the *private* return to investment. For a fixed sequence of aggregate consumption levels, $\{C_t^*\}_{t \geq 0}$, the private return to investment at date t is given by

$$\frac{\partial u\left(\tau^* C_t^*\right)/\partial c_t^y}{\partial u\left(\left(1-\tau^*\right)\left(1+g^*\right)C_t^*\right)/\partial c_{t+1}^o} - 1 = \left(\frac{\tau^*}{\left(1-\tau^*\right)\left(1+g^*\right)}\right)^{-\sigma} - 1. \tag{31}$$

[5] The following proof replicates the argument in Saint Paul (1992). Consider an allocation $\left\{\widetilde{k}_t\right\}$ with $\widetilde{k}_s < k_s$, for some s, with $\widetilde{c}_t \geq c_t$ for $t \geq s$. Since $\widetilde{k}_{t+1} = (A+1)\widetilde{k}_t - \widetilde{c}_t$ and $k_{t+1} = (A+1)k_t - c_t$, for $t \geq s$, it must be that $k_{t+1} - \widetilde{k}_{t+1} \geq (A+1)\left(k_t - \widetilde{k}_t\right)$, for $t \geq s$. In turn this implies that $k_{s+T} - \widetilde{k}_{s+T} \geq (A+1)^T\left(k_s - \widetilde{k}_s\right)$, and thus $\widetilde{k}_{s+T} \leq \left(1+g^*\right)^T k_s - (A+1)^T\left(k_s - \widetilde{k}_s\right)$, for any $T \geq 1$. Clearly, if $g^* < A$, the right side of the inequality becomes negative for T sufficiently large, contradicting the hypothesis that there is a feasible deviation $\widetilde{k}_s < k_s$, for some s, with $\widetilde{c}_t \geq c_t$ for $t \geq s$. This concludes the proof.

Since τ^* maximizes $u(\tau C_t^*) + (\delta + a)^{-1} u((1-\tau) C_t^*)$, we know that $(\tau^* / (1-\tau^*))^{-\sigma} = (\delta + a)^{-1}$. Thus, consumption is dynamically inefficient if and only if $\delta + a > (1 + g^*)^{\sigma - 1}$.

It should be noted that the Markov perfect equilibrium in our context is Pareto inefficient, because the private and the social return to investment are different. A Pareto improvement would result from investing optimally from the viewpoint of the currently young generation at the socially optimal rate of return, without changing the allocation for any other generation. This is in contrast with the common perception that perfect altruism about the following generation must lead to Pareto efficiency (Streufert, 1993). This is the case in the non-overlapping generations models studied in the literature, because perfect altruism then amounts to time-consistent preferences. However, with time-inconsistent social preferences, as is the case here, the private return to investment is necessarily lower than the social return, because the incentive to manipulate future investment does not disappear.

Finally, consider the corresponding Ramsey-Cass-Koopmans problem, which amounts to the case where the social welfare function is given by the utility of the currently old agents. Interestingly, the solution to the Ramsey-Cass-Koopmans problem is Pareto inefficient. A marginal transfer of consumption from the current old to the current young leaves the old indifferent but is strictly preferred by the young. This presents a problem for the argument that a "solution" is to have the government "choose" a time-consistent social welfare function at the outset (e.g., Strotz, 1956, Calvo and Obstfeld, 1988). The problem is that this requires a non-trivial commitment technology.

6 Imperfect altruism about future generations

The most natural departure from the perfect altruism assumption made in the Ramsey-Cass-Koopmans optimal growth model is simply that individuals are imperfectly altruistic *about future generations*. Indeed this is the standard assumption in the literature on equilibrium growth with imperfect intergenerational altruism that originated with Phelps and Pollak's (1968) seminal work, although imperfect altruism has been formalized in several different ways.[6] Arguably, this is precisely what those who justify government intervention to target social investments on the grounds of intergenerational disagreement usually have in mind.

One can easily extend our previous analysis to allow for the fact that individuals themselves may be imperfectly altruistic about future as well as past generations. Specifically, the following analysis allows for individuals having quasi-hyperbolic discounting, with a present-bias, as in Phelps and Pollak (1968). Conversely, our analysis extends Phelps and Pollak's (1968) work to account for overlapping generations.

Consider the above model, but assume that individuals born at date t have preferences

$$u_t(\beta) = u(c_t^y) + \beta \left[u\left(c_{t+1}^o\right) + \sum_{s=1}^{\infty} \delta^s \left(u\left(c_{t+s}^y\right) + u\left(c_{t+1+s}^o\right)\right)\right], \tag{32}$$

whose discount structure implies that consumption streams are discounted according to the sequence of discount factors $1, \beta\delta, \beta\delta^2, \beta\delta^3, \ldots$. In this section, we assume that $\beta \leq 1$. When $\beta = 1$, discounting is geometric, and so individuals have standard, time-consistent preferences. This is the case we analyze above. When $\beta < 1$, individuals have quasi-hyperbolic discounting, with a present-bias, in the sense that the *rate* at which utility flows at date $t+2$ are discounted falls between date t and date $t+1$.

It should be noted that the specification of individual preferences in equation (32) assumes that the quasi-hyperbolic discounting structure applies equally to all future utility flows, rather than applying only to the future generations' utility flows. This assumption implies that individuals' relative valuation of utility flows for young and old agents at a given date does not evolve with the passage of time. If it did, there would be an additional source of disagreement, referring to the distribution of valuations between generations at a point in time. Accordingly, equation (32) can be written as

$$u_t(\beta) = u(c_t^y) + \beta \sum_{s=1}^{\infty} \delta^s \left(u\left(c_{t+s}^y\right) + \delta^{-1} u\left(c_{t+s}^o\right)\right).$$

Quasi-hyperbolic discounting implies preference reversals, and so the relevant preferences for the old in the social welfare function should be their current preferences. Accordingly, we

[6] Compare, for example, Sen (1967), Phelps and Pollak (1968), and Kohlberg (1976).

assume that the objective function of the date-t government is

$$v_t(\beta) = u_{t-1}^t(\beta) + a u_t(\beta), \tag{33}$$

with $a > 0$, where $u_{t-1}^t(\beta)$ is the utility of the currently old from the viewpoint of date t:

$$u_{t-1}^t(\beta) = u(c_t^o) + \delta\left(u(c_t^y) + \beta\sum_{s=1}^{\infty}\delta^s\left(u(c_{t+s}^y) + \delta^{-1}u(c_{t+s}^o)\right)\right). \tag{34}$$

Hence, we have

$$v_t(\beta) = u(c_t^o) + (\delta + a)u(c_t^y) + (\delta + a)\beta\sum_{s=1}^{\infty}\delta^s\left(u(c_{t+s}^y) + \delta^{-1}u(c_{t+s}^o)\right). \tag{35}$$

Equivalently, one can think of the government as maximizing

$$\widetilde{v}_t(\beta) = u(c_t^y) + (\delta + a)^{-1}u(c_t^o) + \beta\sum_{s=1}^{\infty}\delta^s\left(u(c_{t+s}^y) + \delta^{-1}u(c_{t+s}^o)\right). \tag{36}$$

One can verify that our previous analysis goes through essentially unchanged. Intuitively, the two pro-growth forces we identified above, as reflected by an imperfect altruism effect and a strategic compensation effect, are now complemented by an anti-growth bias arising from the present-bias inherent in the preferences of individuals.

If the date-t government could precommit future allocations, optimal allocations would be given by $i(k) = gk$, $c_y(k) = \tau(A - g)k$, and $c_o(k) = (1 - \tau)(A - g)k$, with

$$(\tau, g) = \begin{cases} (\tau_c, g_c(\beta)) & \text{in the first period} \\ (\overline{\tau}_c, \overline{g}_c) & \text{in every future period,} \end{cases}$$

where τ_c and $\overline{\tau}_c$ are given by equations (7) and (9), respectively, with $\tau_c > \overline{\tau}_c$, while $\overline{g}_c$ is given by equation (15), and $g_c(\beta)$ solves

$$1 + g_c(\beta) = \frac{A + 1}{1 + \left(\frac{q(\tau_c, a)}{\beta q(\overline{\tau}_c, 0)}\frac{\delta^{-1} - (1 + \overline{g}_c)^{1-\sigma}}{(A - \overline{g}_c)^{1-\sigma}}\right)^{1/\sigma}}. \tag{37}$$

Moreover, $g_c(\beta) > \overline{g}_c$ if and only if $\beta > q(\tau_c, a)/q(\overline{\tau}_c, 0)$, where $q(\tau_c, a)/q(\overline{\tau}_c, 0) < 1$.

Similarly, there is a unique symmetric, interior, Markov perfect equilibrium in linear strategies. The equilibrium is characterized by $i(k) = g^* (\beta) k$, $c_y(k) = \tau^* (A - g^* (\beta)) k$, and $c_o(k) = (1 - \tau^*) (A - g^* (\beta)) k$, where τ^* is given by equation (21), $\overline{g}_c$ is given by equation (15), and $g^* (\beta) = B_\beta (\tau^*, \tau^*, g^* (\beta))$, where B_β is given by

$$1 + g = \frac{A + 1}{1 + \left(\frac{q(\tau^*,a)}{\beta q(\tau^*,0)} \frac{\delta^{-1} - (1+\widehat{g})^{1-\sigma}}{(A-\widehat{g})^{1-\sigma}} \right)^{1/\sigma}} \equiv 1 + B_\beta (\tau^*, \tau^*, \widehat{g}) . \tag{38}$$

Finally, it is easy to verify that $g^* (\beta) > \overline{g}_c$ if and only if $\beta > q(\tau^*, a) / q(\tau^*, 0)$, where, using the definitions of $q(\tau, a)$ and τ^*,

$$\frac{q(\tau^*, a)}{q(\tau^*, 0)} = \frac{1 + (\delta + a)^{-1} (\delta + a)^{1-1/\sigma}}{1 + \delta^{-1} (\delta + a)^{1-1/\sigma}} < 1, \tag{39}$$

An immediate implication is that the effect of commitment depends on whether or not the individuals' present-bias dominates the future-bias that results from aggregation of preferences in the social welfare function.

Proposition 6 *In the Markov perfect equilibrium, the date-t government would like to commit all future allocations optimally, in which case: (i) If* $\beta < q(\tau^*, a) / q(\tau^*, 0)$*, there is a time* $T \geq t$ *such that all generations born after date* T *will be made better off, from the perspective of young age as well as old age. (ii) If* $\beta > q(\tau^*, a) / q(\tau^*, 0)$*, there is a time* $\widehat{T} \geq t$ *such that all generations born after date* $\widehat{T}$ *will be made worse off, from the perspective of young age as well as old age. The ratio* $q(\tau^*, a) / q(\tau^*, 0) < 1$ *is given by equation (39).*

Clearly, starting at the Markov perfect equilibrium, the date-t government prefers the commitment solution, which is the one it would optimally choose if it could control future allocations. In this sense, an incentive to commit to lower growth and larger transfers from the young to the old in the future can be explained by altruism. Once again, note that current generations must be made better off by the institutional change whenever $\sigma = 1$, because the date-t allocation does not change and the new allocation thereafter is their first-best allocation.

Parts (i) and (ii) of Proposition 6 follow from the fact that the wealth differential between the equilibrium allocation and the commitment solution implemented at date t grows without bound. Hence, whether future generations are made eventually worse off or better off by the move (at date t) to the commitment solution depends only on whether the new growth path lies below or above the original path, which is determined by whether β is greater or less than $q(\tau^*, a)/q(\tau^*, 0)$. It should also be noted that, for the case where $\beta \leq q(\tau^*, a)/q(\tau^*, 0)$, whether or not some generations are made worse off depends in general on the configuration of parameter values for β, δ, a and σ.

Part (ii) of Proposition 6 implies that an infinite number of future generations will necessarily be made worse off by the switch to the commitment solution whenever $\beta > q(\tau^*, a)/q(\tau^*, 0)$. It is easy to verify that this condition holds for a wide range of circumstances. For example, assuming that $a = \sigma = 1$, equation (39) implies that $q(\tau^*, a)/q(\tau^*, 0)$ goes from 0 to 3/4 as δ goes from 0 to 1. If $\delta = 0.2$, future generations will be made worse off by the switch to the commitment solution whenever $\beta > 0.31$.

7 Conclusion

In this paper we have explored how intergenerational disagreement constrains the calculus of second-best national saving and growth. We have illustrated the contrast between two natural sources of disagreement when generations are overlapping and preferences are aggregated in a utilitarian manner. Social preferences tend to exhibit a present-bias because generations are imperfectly altruistic about future generations, but they tend to exhibit a future-bias because coexisting generations are imperfectly altruistic about currently older generations. Not surprisingly, equilibrium growth is inefficiently low when the former bias dominates. Otherwise, society faces an intergenerational equity problem, in which case the present-day government has an incentive to support institutions that enable commitments to lower future growth at the expense of future generations. This is so even with perfect altruism about future generations and even without discounting.

More generally, we have argued that intergenerational disagreement can imply that the incentive of individuals, and governments, are such that institutions that enable commitments to cope with intergenerational disagreement will tend to favor the introducing generations at the expense of future generations. Thus, our analysis warns that the ability of actual governments to improve the welfare of future generations can be greatly constrained, even in an altruistic society.

Appendix

Proof of Proposition 1

All statements in the proposition are proven in the main text, except for the inequality $g_c > \bar{g}_c$. To prove this, let

$$1 + \tilde{B}(\hat{g}, Q) \equiv \frac{A+1}{1 + Q\left(\frac{\delta^{-1} - (1+\widehat{g})^{1-\sigma}}{(A-\widehat{g})^{1-\sigma}}\right)^{1/\sigma}},$$

and inspect equation (20) to note that $\tilde{B}(\bar{g}_c, Q_c) = g_c$, where

$$Q_c \equiv \left(\frac{q(\tau_c, a)}{q(\overline{\tau}_c, 0)}\right)^{1/\sigma}.$$

Using equations (7) and (9), one can verify that $Q_c = \overline{\tau}_c/\tau_c < 1$. Moreover, one can easily verify that $\tilde{B}(\bar{g}_c, 1) = \bar{g}_c$. Since $\partial\tilde{B}(\hat{g}, Q)/\partial Q < 0$, it follows that $\tilde{B}(\bar{g}_c, Q_c) = g_c > \tilde{B}(\bar{g}_c, 1) = \bar{g}_c$, as required. **QED**

Proof of Proposition 2

Consider Part (ii) first. From equation (27), $B(\tau^*, \tau^*, \hat{g})$ can be rewritten as $\tilde{B}(\hat{g}, Q^*)$, with

$$Q^* \equiv \left(\frac{q(\tau^*, a)}{q(\tau^*, 0)}\right)^{1/\sigma},$$

where $\tilde{B}$ is defined in the proof of Proposition 1. Using equation (21), one can verify that

$$Q^* = \left(\frac{1}{1 + (1-\tau^*)a/\delta}\right)^{1/\sigma} < 1.$$

Next, note that the sign of $\partial\tilde{B}(\hat{g}, Q)/\partial\hat{g}$ is given by the sign of $(\sigma - 1)\left[(1+\widehat{g})^{\sigma} - \delta(A+1)\right]$. It is easy to verify that, for given Q^*, $\tilde{B}(\hat{g}, Q^*)$ has a global minimum at $\widehat{g} = \bar{g}_c$ if $\sigma > 1$; it has a global maximum at $\widehat{g} = \bar{g}_c$ if $\sigma < 1$; and it is flat at $\widehat{g} = g_c$ if $\sigma = 1$. This proves Part (ii) of the proposition.

Now consider Part (i). To prove existence of a unique fixed point $g^* \in (\overline{g}_c, A)$, evaluate $g = \tilde{B}(\hat{g}, Q)$ at $\hat{g} = g$ and rewrite it as

$$\delta^{-1}Q^{\sigma}(1+g)^{\sigma} + (1-Q^{\sigma})(1+g) - (A+1) = 0. \tag{40}$$

As long as $Q \leq 1$, the left side of the equation is increasing in g. Moreover, it is negative when $g = -1$ and positive when $g = A$. Hence, there is exactly one fixed point, $\tilde{g}(Q) < A$, where $g^* = \tilde{g}(Q^*)$. Differentiating equation (40), one can verify that $\partial\tilde{g}(Q)/\partial Q < 0$ if and only if $1 > \delta\left(1+\tilde{g}(Q)\right)^{1-\sigma}$. For $\sigma \leq 1$, the latter inequality holds since $g \leq A$ and we have assumed $1 < (A+1)^{1-\sigma}\delta$. For $\sigma > 1$, first note that $\tilde{g}(1) = \bar{g}_c$ and $1 > \delta\left(1+\tilde{g}(1)\right)^{1-\sigma}$, so $\partial\tilde{g}(1)/\partial Q < 0$; hence for all $Q \leq 1$ the inequality $1 > \delta\left(1+\tilde{g}(Q)\right)^{1-\sigma}$ holds. Since $Q^* < 1$, we have $\tilde{g}(Q^*) = g^* > \tilde{g}(1) = \bar{g}_c$. Therefore, $A > g^* > \bar{g}_c$. All other statements in the proposition are proven in the main text. **QED**

Proof of Proposition 3

That $g^* > \bar{g}_c$, for all $\sigma > 0$, is proven in Proposition 2. To prove that $g^* > g_c$ if and only if $\sigma > 1$, first note that

$$\frac{Q_c}{Q^*} = \left(\frac{q\left(\tau_c, 0\right)}{q\left(\bar{\tau}_c, 0\right)}\right)^{1/\sigma} = \left(\frac{\tau_c^{1-\sigma} + \delta^{-1}(1-\tau_c)^{1-\sigma}}{\bar{\tau}_c^{1-\sigma} + \delta^{-1}(1-\bar{\tau}_c)^{1-\sigma}}\right)^{1/\sigma},$$

where the first equality follows from the definitions of Q_c and Q^* given in the previous two proofs, and the fact that $\tau^* = \tau_c$, and the second equality follows from the definition of $q\left(\tau, a\right)$ given in equation (11). Next, note that the right side of the second equality above is increasing in τ_c for $\sigma > 1$ and is decreasing in τ_c for $\sigma < 1$. Since $\tau_c > (1+\delta^{-1/\sigma})^{-1} = \bar{\tau}_c$, it follows that $Q_c > Q^*$ if $\sigma > 1$ and $Q_c < Q^*$ if $\sigma < 1$. Hence, we have the following: (1) If $\sigma > 1$, then $\tilde{B}(g^*, Q^*) = g^* > \tilde{B}(g^*, Q_c) > \tilde{B}(\bar{g}_c, Q_c) = g_c$, where the first inequality follows from the fact that $\partial\tilde{B}(\hat{g}, Q)/\partial Q < 0$, and the second one from the fact that $g^* > \bar{g}_c$ and $\partial\tilde{B}(g, Q)/\partial g > 0$ for $g > \bar{g}_c$. (2) If $\sigma < 1$, then $\tilde{B}(g^*, Q^*) = g^* < \tilde{B}(g^*, Q_c) < \tilde{B}(\bar{g}_c, Q_c) = g_c$, where the first inequality follows from the fact that $\partial\tilde{B}(\hat{g}, Q)/\partial Q < 0$, and the second one from the fact that $g^* > \bar{g}_c$ and $\partial\tilde{B}(g, Q)/\partial g < 0$ for $g > \bar{g}_c$. It follows that $g^* > g_c$ if and only if $\sigma > 1$, as required. **QED**

Proof of Proposition 4

Suppose that $\sigma > 1$. First, note that the limit of g^* as δ approaches 1 is well defined for all $\sigma > 1$. That $g^* > \overline{g}_c$, for $a > 0$, is proven in Proposition 2 for $\delta \leq 1$. Next, note that $\lim_{a\to 0} Q^* = 1$, so $\lim_{a\to 0} g^* = \lim_{a\to 0} \tilde{g}(Q^*) = \tilde{g}(1) = \bar{g}_c$, as required. Finally, one can apply l'Hopital's rule to find $\lim_{a\to\infty} Q^*$. If $\sigma > 1$, $\lim_{a\to\infty} Q^* = 0$, and $\lim_{a\to\infty} g^* = \tilde{g}(0) = A$, where the last equality follows from substituting $Q = 0$ into equation (40). This concludes the proof. **QED**

Proof of Proposition 5 and Proposition 6

Consider Proposition 6 first. The date-t government prefers the allocation under commitment, since it is the unique optimal allocation from its viewpoint. The arguments leading to Proposition 6 in the main text show that $g^*(\beta) > \overline{g}_c$ if and only if $\beta > q(\tau^*, a)/q(\tau^*, 0)$, where $q(\tau^*, a)/q(\tau^*, 0) < 1$ is given by equation (39). Proposition 6 follows immediately.

To prove Proposition 5, note it is just the special case of Proposition 6 where $\beta = 1$; hence it follows from the fact that $q(\tau^*, a)/q(\tau^*, 0) < 1$. **QED**

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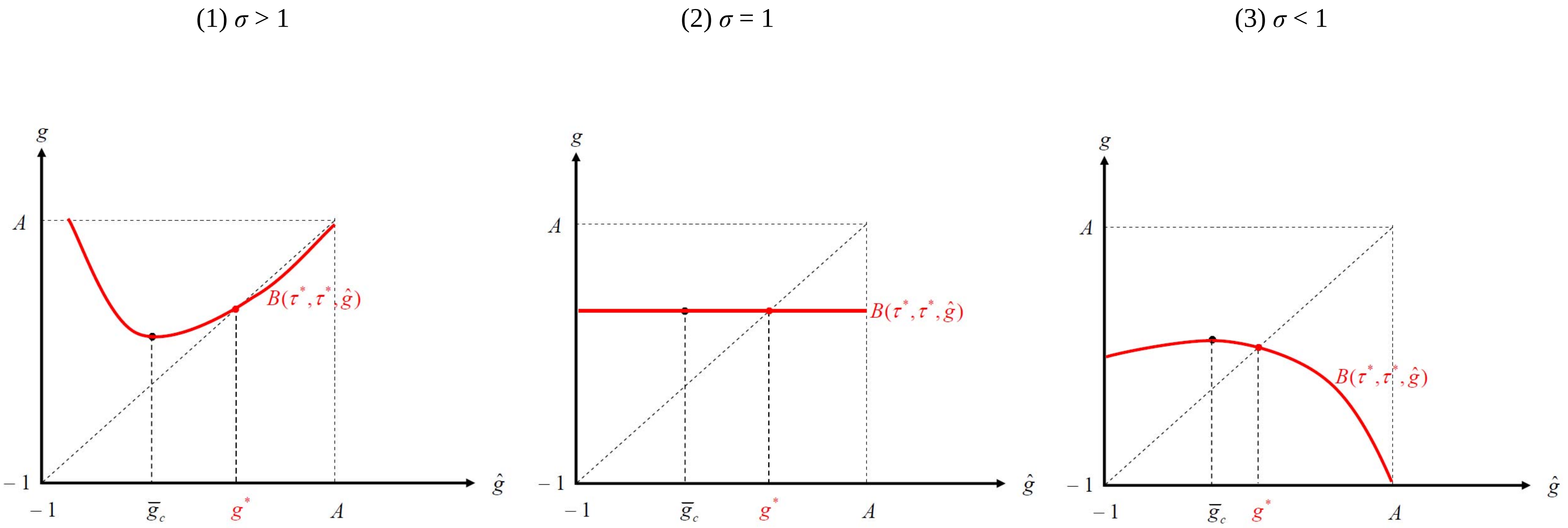

Figure 1