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Financial Frictions, Investment Delay and Asset Market Interventions

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Abstract

We construct a dynamic macro model to incorporate financial frictions and investment delay. Investment is undertaken by entrepreneurs who face liquidity frictions in the equity market and a collateral constraint in the debt market. After calibrating the model to the US data, we quantitatively examine how aggregate activity is affected by a shock to equity liquidity and a shock to entrepreneurs' borrowing capacity. We then analyze the effectiveness of government interventions in the asset market after such financial shocks. In particular, we compare the effects of government purchases of private equity and of private debt in the open market. In addition, we examine how these effects of government interventions depend on the option to delay investment.

Keywords: Financial frictions; Liquidity; Asset market interventions; Investment delay; Equity; Collateral.

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1. Introduction

The great recession in 2008-2009 has increased the interest in studying the importance of financial frictions. In that recession, governments in various countries intervened in asset markets on large scales. The most common type of intervention used was quantitative easing, whereby a government sells its relatively more liquid assets for less liquid, or private, assets. Such interventions were aimed at increasing the overall liquidity in the asset market and thereby reducing the negative effect of financial frictions on firms' financing ability. The effectiveness of such interventions is still being debated. One particular skepticism is that firms were observed in the recession to delay investment by hoarding liquid funds, part of which were injected by the government in quantitative easing. In this paper we construct a dynamic macro model to incorporate financial frictions and investment delay. We calibrate the model to the US data to examine the quantitative effects of financial shocks on aggregate activity and the effectiveness of government interventions in the asset market after such shocks.

Financial frictions in our model reside in the investment sector where entrepreneurs are endowed with investment projects and choose how many of these projects to implement in each period to produce new capital. The frictions appear in both the equity and the debt market. The frictions in the equity market are modeled as in Kiyotaki and Moore (2012, KM henceforth). In any period, an entrepreneur is constrained to sell no more than a fraction $\phi \in (0, 1)$ of the holdings of existing equity and to issue new equity on no more than a fraction $\theta \in (0, 1)$ of new investment. The fraction ϕ is called equity liquidity and, hence, shocks to ϕ are called liquidity shocks. In the debt market, an entrepreneur can only borrow up to an amount that depends positively on the value of equity that the entrepreneur holds at the end of the period. Such equity holdings can be interpreted as collateral. The borrowing constraint can arise from the risk of the borrower's default, as in Kiyotaki and Moore (1997). The borrowing capacity is subject to shocks, μ. These financial frictions create a wedge between the values of an entrepreneur's internal and external funds and thus, between the price of equity and the replacement cost of capital.

They also imply that the composition of equity and debt matters to an entrepreneur; i.e., the Modigliani-Miller theorem breaks down in the environment.

An entrepreneur can choose to delay investment by choosing the number of projects to be implemented. The amount of new capital produced by an entrepreneur is assumed to be increasing in both the amount of the resource allocated to investment (i.e., investment expenditure) and the stock of investment projects available to the entrepreneur. Unimplemented projects add to the future stock of projects, subject to depreciation. The option value of delay creates an additional wedge between the price of equity and the replacement cost of capital.

After calibrating this model to the US data, we first examine the dynamic effects of two negative financial shocks in the absence of government interventions in the asset market. One is a negative shock to liquidity, ϕ, and the other one to the borrowing capacity, μ. Then, we introduce government purchases of private assets, the amount of which is proportional to the size of the reduction in liquidity or the borrowing capacity. These purchases are assumed to be conducted in the open market for assets rather than targeted to specific firms. They are financed immediately by increasing the issuance of government bonds, as in a typical episode of quantitative easing, and ultimately by increasing taxes. We assess whether such interventions can significantly reduce the negative effect of financial shocks on aggregate activity. Moreover, because the composition of equity and debt is relevant to an entrepreneur's decision, we compare how equity purchases may have quantitatively different effects from debt purchases. Finally, we compare the effects of government interventions in an economy with investment delay and in an economy where delay has no benefit.[1]

On financial shocks, we find that a negative liquidity shock can have large negative effects on aggregate activity. An unanticipated reduction in liquidity by 18% can reduce

[1] This exercise does not imply that we view the large fall in asset liquidity in 2008-2009 as exogenous. On the contrary, the fall was largely caused by changes in economic fundamentals, such as the realization that mortgage related assets had a much lower quality than expected. The purpose of our analysis is to get a sense of how much the changes in asset market conditions can affect aggregate activity and how effective government interventions in the asset market can be. See section 6 for further discussions.

investment expenditure by 11.1%, employment by 2.9%, output by 1.9%, and aggregate consumption by 0.8%. It is remarkable that the liquidity shock alone can generate such positive comovement among investment, employment, output and aggregate consumption. Moreover, the negative liquidity shock induces a significant fraction of investment projects to be delayed. These negative effects are persistent if the liquidity shock is persistent. With debt financing, we calibrate the model so that debt issuance raises more funds than new equity sales in the steady state. However, unlike equity, existing debt does not raise additional funds or provide liquidity. For this reason, a negative shock to the borrowing capacity has only one twelfth of the effect of a negative liquidity shock on aggregate activity, given that the two types of shocks have the same magnitude in percentage.

On government interventions in the asset market, we find that equity purchases by the government can have sizable effects. In the case of the negative liquidity shock mentioned above, equity purchases of one trillion dollars exacerbate the initial negative effect of the liquidity shock on aggregate activity when the tax is assumed to be fixed at the time of the initial purchases. In subsequent periods, however, the purchases induce aggregate activity to recover more quickly than without interventions. In the second period after the purchases, investment recovers by 54%, output by 32%, and employment by 36% of the initial fall in period one. These large effects of equity purchases arise even though they are conducted in the open market rather than being directed to specific firms in financial distress. In contrast, government purchases of private debt have only small effects on aggregate activity. This contrast between the two types of interventions is not specific to the case of a liquidity shock. Even when the shock occurs to the borrowing capacity, equity purchases are more effective than debt purchases in helping the economy to recover.

On investment delay, we find that a negative liquidity shock induces significant delay of investment even after the government intervenes in the asset market. With equity purchases in particular, delay reduces the fraction of implemented projects by sixteen to twenty-nine percentage points more than in the hypothetical economy where delay has no benefit. Moreover, the option to delay significantly reduces the effectiveness of asset market interventions. Specifically, relative to no interventions, investment expenditure with equity purchases falls by more in the first period when the purchases take place; it recovers by more in the second period; and it recovers more slowly from the third period onward.

Our paper is related to the large literature on the role of financial frictions in macroeconomics. Some earlier references that focus on firms' borrowing constraints are Hellwig (1977), Townsend (1979), Williamson (1987), Benanke and Gertler (1989), Kiyotaki and Moore (1997), and Bernanke et al. (1999). A more recent reference is Jermann and Quadrini (2012). We model the role of equity in the collateral constraint as in Kiyotaki and Moore (1997) and the frictions in the equity market as in KM. The latter has also been used by Ajello (2010), Nezafat and Slavik (2010), Del Negro et al. (2011) and Shi (2012). Our paper follows Shi (2012) closely to employ the structure of large households in Shi (1997) to facilitate aggregation and to incorporate financial frictions in both the equity and debt market. The main additions of the current paper to Shi (2012) are the incorporation of investment delay and the evaluation of government interventions in the asset market. Even with investment delay, our analysis confirms the result in Shi (2012) that liquidity shocks alone can have large significant effects on aggregate activity and can generate positive comovement among aggregate variables. Del Negro et al. (2011) also use the large household framework to quantitatively evaluate asset market interventions. The main contrasts are as follows: (i) our model has no nominal rigidity; (ii) we incorporate the frictions in both the equity market and the debt market; (iii) we compare the quantitative responses of the equilibrium to the two types of financial shocks; (iv) we evaluate both equity and debt purchases by the government; and (v) we introduce an investment technology that allows for investment delay. We will contrast our paper with this previous work in detail at the end of section 5.2.

There is also a large literature on investment delay. Pindyck (1991) and McDonald and Siegel (1986) emphasize the option value of investment delay when there is uncertainty in the economy and when investment is partially irreversible. Boyle and Guthrie (2003) incorporate this idea in a model where firms face financing or liquidity constraints. Stokey (2012) emphasizes uncertainty about future policy as a cause of investment delay. These papers are more on the micro side of the economy. On the macro side, some examples include Bernanke (1983), Khan and Thomas (2011), and Gilchrist et al. (2012). In particular, Gilchrist et al. (2012) emphasize the importance of uncertainty shocks for business cycle fluctuations in the presence of financial market frictions. The two main ingredients of all these models are (partial) irreversibility of investment and uncertainty. In the presence of irreversibility, aggregation of individuals' decisions is tractable only under strong assumptions on preferences, which can undermine the quantitative analysis. We assume an investment technology that does not feature irreversibility. In addition, in the quantitative analyses, all shocks occur in the first period, and so there is no uncertainty in subsequent periods. This modeling is not meant to dismiss the importance of irreversibility and uncertainty for investment delay; rather, it is complementary to the existing one. The tractability of our modeling for aggregate dynamics enables us to focus on how financial frictions can induce investment delay. The quantitative evaluation of government interventions in the asset market is new relative to this literature on investment delay.

2. Environment of the Model Economy

Time is discrete and lasts forever. There is a continuum of identical households, with measure one, and we choose an arbitrary household as the representative household. A household has a large number of members, and the total measure of members is set to one.[2] At the beginning of each period, all members are identical. During a period, the members go to the market and are separated from each other until the end of the period. While in the market, a member receives a shock whose realization determines whether the member is an entrepreneur or a worker. The probability with which a member is an entrepreneur is $\pi \in (0, 1)$. These shocks are *iid* across the members and over time. An entrepreneur receives a number a of new investment projects, and can implement investment projects but has no labor endowment.[3] A worker receives one unit of labor endowment, receives no new

[2]The structure of a large household enables us to analyze aggregate dynamics tractably in the presence of heterogeneity. It is an extension of the structure used by Lucas (1991) who assumes that each household consists of three members. A similar structure has been used in other fields, e.g., monetary theory (Shi, 1997). The structure used in the current paper is a modification of that in Shi (2012).

[3]Endogenizing a requires a study of the innovation process. We abstract from it to focus on implementation.

investment project and cannot implement investment projects. The members' preferences are represented by the household's utility function:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left\{ \pi u\left(c_t^e\right) + (1-\pi)\left[U(c_t^w) - h(\ell_t)\right]\right\}, \quad \beta \in (0,1),$$

where the expectation is taken over the aggregate state of the economy that will be described later. Here, β is the discount factor, c consumption and ℓ labor supply. The superscript e indicates an entrepreneur and the superscript w a worker. The utility functions u and U are strictly increasing and strictly concave, with $u'(0) = U'(0) = \infty$ and $u'(\infty) = U'(\infty) = 0$. The disutility function of labor, h, is strictly increasing and strictly convex, with $h'(0) \geq 0$ and $h'(1) = \infty$. It is useful to interpret "workers" and "entrepreneurs" in the model broadly: Relative to their investment opportunities, entrepreneurs are individuals in an actual economy who are more financially constrained than workers.

At the beginning of each period, the household has assets, liabilities and investment projects. Assets consist of a diversified portfolio of equity claims (i.e., claims on capital), s, and government bonds, b. Liabilities consist of the household's debt, d, and (lump-sum) taxes, τ. The stock of investment projects is n. Because all members are identical at the beginning of the period and the household does not know which member will be an entrepreneur or a worker in the market, the household divides the assets and liabilities evenly among the members. After this division, the household cannot reshuffle assets and liabilities among the members during the period and, in particular, workers cannot return to the household before the end of the period. The household does not divide the stock of investment projects among all the members. Instead, the household keeps the projects until the roles of all members in the period are realized, at which time the household divides the stock of projects only among the entrepreneurs. During the period, the members undertake their activities, including consumption, separately from each other in the market. At the end of the period, the members return home and pool their assets, liabilities and unimplemented projects.

The assumptions on timing and the separation of the members during a period capture the misalignment of funds and investment projects that are important for financial frictions to affect investment and aggregate activity in reality. Although a member who becomes an entrepreneur will need more funds for investment than another member who becomes a worker, the two are given the same amount of funds by the household. The assumption that only entrepreneurs hold investment projects is not only realistic but also meant to widen the gap between investment needs and the availability of funds. If the household divided the stock of investment projects among all the members instead, the entrepreneurs as a group would have only a small fraction of these projects given any reasonable value of π. In this case, the need for investment funds at the aggregate level would be significantly smaller than under the maintained assumption. Also, entrepreneurs' decisions on how much to invest would have much smaller consequences on the future stock of projects.[4]

Two types of goods are produced in the economy, according to different technologies. Final consumption goods are produced by competitive firms according to the function $F(k^D, \ell^D)$, where k^D is the amount of capital and ℓ^D the amount of labor employed by such a firm. The function F exhibits diminishing marginal productivity in each factor and constant returns to scale. In contrast, new capital goods can only be produced by entrepreneurs. Each implemented project yields γ units of new capital, where γ is assumed to be a constant for simplicity. The number of projects implemented by an entrepreneur, m, depends on the input of final goods in the investment, x, and the stock of potential projects available to the entrepreneur, $\frac{n}{\pi} + a$. Formally, the level of investment undertaken by an entrepreneur is

$$i = \gamma \; m(x, \frac{n}{\pi} + a), \tag{2.1}$$

where $m \leq \frac{n}{\pi} + a$. The investment technology has constant returns to scale with the additional properties $m_1 > 0$, $m_2 > 0$, $m_{11} < 0$ and $m_{22} < 0$. We refer to i as an entrepreneur's investment, x as an entrepreneur's investment expenditure, and γ as the size of a project. Projects that are not implemented in the current period can be carried over to the next period, with a survival rate $\sigma_n \in (0, 1)$. Because each entrepreneur receives

[4]Note that precautionary savings do not arise in this setup. Because the shock that determines whether a member is an entrepreneur or worker is *iid* across the members and over time, a household cannot build precautionary savings conditional on whether a member will become an entrepreneur in a period.

a number a of new projects and implements m projects in the period, the net change in the total stock of projects in the household is $\pi(a-m)$. The household's stock of projects at the beginning of the next period will be

$$n_{+1} = \sigma_n \left\{ n + \pi \left[a - m(x, \frac{n}{\pi} + a) \right] \right\}. \tag{2.2}$$

The investment technology can be interpreted as follows. Suppose that an entrepreneur can choose the number of projects to experiment, $n_x \leq \frac{n}{\pi} + a$, and the resource x to spend on these projects. The probability with which an experimented project succeeds in the current period is an increasing function of the resource spent on the project, which is x/n_x. Let this probability of success be $\hat{m}(x/n_x)$, with the properties $0 < \hat{m}' < \frac{n_x}{x}\hat{m}$ and $\hat{m}'' < 0$. If a project does not succeed in the current period, it can be experimented again in the future. Then, it is optimal to choose n_x to maximize the expected number of successfully implemented projects, which is $n_x \hat{m}(x/n_x)$. With the properties of $\hat{m}$, it is easy to verify that this optimal choice of n_x is $n_x = \frac{n}{\pi} + a$; that is, the entrepreneur will experiment all available projects. The expected number of successfully implemented projects by the entrepreneur is $(\frac{n}{\pi} + a)\hat{m}(x/(\frac{n}{\pi} + a))$, which is denoted as $m(x, \frac{n}{\pi} + a)$. For simplicity, we eliminate the uncertainty in the number of successes by assuming that it is equal to the expected number of successes for each entrepreneur.[5] Clearly, the function m derived has constant returns to scale. Moreover, the assumptions on $\hat{m}'$ and $\hat{m}''$ ensure that m has strictly positive and diminishing marginal productivity of each input.

The presence of the stock of projects in the investment technology creates an option value of a project and, hence, the possibility of investment delay. More precisely, the assumption $m_2 > 0$, together with $\sigma_n > 0$, is necessary for unimplemented projects to yield a future benefit, by increasing investment in the next period. *So is the assumption* $\sigma_n > 0$. However, even in the special cases $m_2 = 0$ and $\sigma_n = 0$, implementing all available projects may drive down the marginal productivity of investment expenditure by too much

[5]This idiosyncratic uncertainty may be interesting for studying other issues such as consumption inequality among entrepreneurs. We abstract from this idiosyncratic uncertainty in order to focus on the aggregate behavior of asset price in the business cycles. Note that since each household has many members, the number of successfully implemented projects in a household is deterministic and given by πm.

to be optimal. In fact, we assume that the constraint, $m \leq \frac{n}{\pi} + a$, is never binding.[6] We delay the comparison of our modeling of investment with the that of the literature to the end of this section.

Investment is impeded by financial frictions in both the equity and the debt market. To specify these frictions, consider an entrepreneur, who enters the market with equity claims s, holdings of government bonds b, and net debt in the private sector, d. The entrepreneur receives capital income from equity claims, after which a fraction $1 - \sigma_k$ of capital depreciates and so equity claims are rescaled by σ_k. To finance investment, the entrepreneur can use the newly received capital income, issue new equity on the investment, sell existing equity, and issue new debt. There are two frictions in the equity market as described by KM. First, an entrepreneur can issue equity in the market on only a fraction $\theta \in (0, 1)$ of investment. The equity on the remainder of new investment, $(1 - \theta)i$, must be retained in the current period by the entrepreneur's household. The second friction is that at most a fraction $\phi \in (0, 1)$ of existing equity can be sold in the current period, and so the entrepreneur must retain the remaining amount $(1 - \phi)\sigma_k s$. These two frictions in the equity market place a lower bound on an entrepreneur's equity holdings at the end of the period, s^e_{+1}, as follows:

$$s^e_{+1} \geq (1 - \theta)i + (1 - \phi)\sigma_k s. \tag{2.3}$$

This lower bound constrains an entrepreneur's financing ability because, if the equity market frictions did not exist, the entrepreneur would rather reduce equity holdings at the end of the period to zero and use the proceeds to finance investment projects. Although it is useful to endogenize θ and ϕ by specifying asset market frictions in detail, we follow KM and Shi (2012) to treat θ and ϕ as exogenous. Also, we fix θ and focus on the effect of changes in ϕ. The shocks to ϕ are called liquidity shocks.

For (2.3) to be binding, there should also be frictions in the debt market that restrict an entrepreneur's ability to borrow. In particular, because it is difficult to enforce debt repayment, a lender demands a borrower to submit collateral to back up the borrowing.

[6] This assumption is satisfied in the computed equilibrium.

Following the argument by Kiyotaki and Moore (1997), we assume that an entrepreneur can use equity holdings at the end of the period as collateral. Precisely, the face value of debt issued by an entrepreneur, denoted d^e_{+1}, is bounded above by a multiplier z of the value of the entrepreneur's equity holdings at the end of period:

$$d^e_{+1} \leq z(\phi, \mu) q s^e_{+1}, \tag{2.4}$$

where q is the price of equity claims and $0 < z < 1$. The element μ follows a Markov process in which the innovation represents a shock to an entrepreneur's borrowing capacity that is not necessarily related to equity liquidity. We assume that the collateral multiplier z is increasing in ϕ to capture the realistic feature that a lender who can sell the collateral more easily in the market is more willing to lend a higher amount backed by the collateral.

Let us summarize the timing of events in an arbitrary period. At the beginning of the period, the shocks to ϕ and μ are realized.[7] The aggregate state in the period is $A = (K, N, \phi, \mu)$, where K is the capital stock per household and N is the stock of investment projects per household at the beginning of the period. The household evenly distributes the assets and liabilities among the members. The household also chooses consumption, investment, labor supply, and the end-of-period portfolio holdings for each member, conditional on whether the member will be an entrepreneur or worker. Then the members go the market and cannot share funds until the end of the period. The shocks are realized to determine whether an individual is an entrepreneur or a worker in the period. The household divides the stock of projects among the entrepreneurs, each of whom also receives a number a of new projects. Then, the producers of final goods rent capital and hire labor to produce consumption goods. After production, workers receive wage income, equity holders receive the rental income of capital, and a fraction $(1 - \sigma_k)$ of capital depreciates. Next, the asset market opens. Individuals repay private debt, redeem government bonds and pay taxes. An entrepreneur seeks funds to finance projects and carries out investment. Of the projects that are not implemented, a fraction σ_n survive.

[7] By assuming that the shocks are realized at the beginning of the period, we simplify the analysis by eliminating the need for precautionary savings.

After consuming goods, individuals return to the household, where they pool the assets, liabilities and unimplemented projects. Time proceeds to the next period.

It is worthwhile repeating that the separation of the members during a period is important in the model. The separation captures the realism that the entrepreneurs who need funds to finance investment have difficulty to obtain funds. If they were able to meet the workers in the household during a period, contrary to what we assume, then they would be able to circumvent the financial frictions by simply using the workers' funds. This importance of separation during a period is similar to that in the models of limited participation (e.g., Lucas, 1990).

The government in each period spends g on final goods, collects lump-sum taxes τ, issues government bonds B_{+1}, and redeems outstanding government bonds B. In addition, the government may intervene in the asset market by purchasing equity and lending to entrepreneurs. Let s^g be the amount of private equity and d^g the face value of private debt held by the government, both of which are measured in amounts per household. Let p_b denote the price of government bonds and p_d the price of private debt. The government budget constraint is:

$$g = \tau + (p_b B_{+1} - B) + [(r + \sigma_k q)s^g - q s^g_{+1}] + (d^g - p_d d^g_{+1}). \tag{2.5}$$

We assume that g is constant over time, while other terms in the above constraint can be time varying. In the baseline model, we assume that the amount of government bonds is constant over time at $B^* > 0$ and government purchases of private equity and private debt are zero. In section 5 we will introduce government purchases of equity or private debt, accompanied by changes in the amount of government bonds issued. In both cases, $(B_{+1}, s^g_{+1}, d^g_{+1})$ are only functions of (q, p_b, p_d, A).

Financial frictions put a wedge between private assets and government bonds. In particular, private debt is not a perfect substitute for government debt. While the debt issued by entrepreneurs requires collateral, government debt does not. It is possible that a household simultaneously lends to the government and borrows from the government through entrepreneurs. That is, B and d can both be positive in the equilibrium. Moreover, issuing government bonds to purchase private debt has real effects in general. Similarly, issuing government bonds to purchase equity has real effects. We will examine such government interventions in the asset market in section 5.

We end this section by comparing the modeling of the investment technology and delay in our model with three strands of the literature. First, the literature on investment delay, cited in the introduction, relies on uncertainty in the future environment of investment. Our model does allow for such uncertainty as innovations in (ϕ, μ) in the future. However, we will examine one-time shocks to (ϕ, μ) and focus, instead, on how the induced changes in the financing conditions affect investment dynamics. Second, the literature on irreversible investment assumes investment to be lumpy in the sense that there is a fixed cost to investment, and to be partially irreversible in the sense that capital is more productive for the original creator than for an outsider. The fixed cost to investment does not exist in our model, provided that $m(0, \frac{n}{\pi} + a) \geq 0$. Irreversibility does not exist in our model, either, despite the illiquidity of claims on capital. In fact, a producer of capital in our model (i.e., an entrepreneur) does not employ what he produces; instead, capital is employed by the producers of final goods and has the same productivity with all such producers. Even the illiquid claims on capital can be resold at the market price asymptotically. In lieu of the fixed cost and irreversibility, there is a smooth tradeoff between investment expenditure and the stock of projects as the two inputs in the investment technology in our model, (2.1). As explained above, this modeling captures some reasonable features of the investment process. It also simplifies the aggregation in the model significantly relative to the literature on irreversible investment. Nevertheless, our modeling is similar to this literature in the emphasis on the extensive margin of investment, i.e., the number of investment projects undertaken. With the fixed cost and irreversibility, the number of firms that undertake investment is clearly important for aggregate investment. This importance is also apparent in our model because investment is a fixed multiplier (γ) of the number of investment projects undertaken. Finally, in the special case $m(x, \frac{n}{\pi} + a) = x$, the investment technology in our model becomes the one assumed by KM and Shi (2012) who also investigate the importance of equity illiquidity for macro dynamics.

3. Optimal Decisions and the Equilibrium

3.1. A household's decisions

A household makes the decisions for each member, conditional on whether the member will be an entrepreneur or a worker in the period. For an entrepreneur, the household chooses consumption c^e, investment expenditure x, the face value of debt issuance d^e_{+1}, and the holdings of equity and government bonds at the end of the period, (s^e_{+1}, b^e_{+1}). The implied investment is $i = \gamma m$, where m is given by (2.1). Similarly, for a worker, the household chooses consumption c^w, labor supply ℓ, debt d^w_{+1}, and the holdings of equity and government bonds at the end of the period, (s^w_{+1}, b^w_{+1}). For each entrepreneur, the household faces the equity liquidity constraint (2.3) and the following budget constraint:

$$rs + (p_d d^e_{+1} - d) + (b - p_b b^e_{+1}) + q(i + \sigma_k s - s^e_{+1}) \geq c^e + x + \tau, \tag{3.1}$$

where r is the rental rate of capital and q the post-dividend price of a share of equity, measured in terms of the consumption good. The right-hand side of (3.1) represents an entrepreneur's expenditures on consumption, investment expenditure and taxes. The left-hand side represents an entrepreneur's resources. The first is the rental income on capital, rs. The second is the value of new debt minus the repayment on outstanding debt, $(p_d d^e_{+1} - d)$. The third is net income from re-balancing the holdings of government bonds, $(b - p_b b^e_{+1})$. The fourth is the net value of re-balancing equity holdings. Implemented projects create i units of new capital, the claims on which can either be sold to outsiders or retained by the household. After capital depreciates, the entrepreneur also holds $\sigma_k s$ claims on existing equity. Thus, the entrepreneur has in total $i + \sigma_k s$ of equity claims. Since the entrepreneur keeps s^e_{+1} claims at the end of the period, the rest must be sold in the asset market. The value of this sale is $q(i + \sigma_k s - s^e_{+1})$.[8]

Consider the case where the liquidity constraint binds.[9] An entrepreneur will optimally

[8] The price of new equity is the same as q, the post-dividend price of existing equity. The reason is that a share of existing equity after paying dividends and a share of new equity command the same stream of future dividends and are subject to the same frictions. Also, as in KM, we simplify the analysis by assuming that the claims on the household's own capital and other households' capital have the same liquidity, and so they have the same price.

[9] This is the case in the dynamic equilibrium computed under the parameter values calibrated later.

hold as little equity at the end of the period as possibly allowed by the constraint (3.1), set the amount of government bonds to be carried into the next period to zero, and borrow up to the bound allowed by (2.4). That is,

$$s^e_{+1} = (1-\theta)i + (1-\phi)\sigma_k s, \quad b^e_{+1} = 0, \quad d^e_{+1} = z(\phi,\mu)qs^e_{+1}. \tag{3.2}$$

Substituting these optimal choices of $(s^e_{+1}, b^e_{+1}, d^e_{+1})$ into (3.1), we obtain the following consolidated financing constraint on an entrepreneur:

$$(r + \phi_z\sigma_k q)s + b - d \geq c^e + (x - \theta_z qi) + \tau, \tag{3.3}$$

where ϕ_z and θ_z are defined as

$$\phi_z = \phi + (1-\phi)zp_d, \quad \theta_z = \theta + (1-\theta)zp_d. \tag{3.4}$$

The quantity $\phi_z q$ is the amount of funds raised from each unit of existing equity, by re-selling ϕ fraction of the equity and using the remaining fraction as collateral in borrowing. Since $z < 1$ (and $p_d < 1$), the more resaleable is equity (i.e. the higher is ϕ), the more an entrepreneur is able to finance investment by selling existing equity. Similarly, $\theta_z q$ is the amount of funds raised from equity on a unit of new investment, by issuing new equity on θ fraction of the investment and using the retained equity as collateral in borrowing. The amount of funds needed for i units of investment is x. Since the amount raised from equity is $\theta_z qi$, the remaining amount, $(x - \theta_z qi)$, is the "downpayment" on investment that must come from other sources.[10]

A worker enters the market with the same asset portfolio, (s, b), and debt, d, as an entrepreneur does. In contrast, a worker earns labor income and does not have investment projects. The worker also earns income by re-balancing his portfolio of assets, but cannot sell new equity. Hence, a worker's budget constraint is:

$$rs + w\ell + (p_d d^w_{+1} - d) + (b - p_b b^w_{+1}) + q(\sigma_k s - s^w_{+1}) \geq c^w + \tau.$$

[10]Since $u(c^e)$ is strictly concave, (3.1) holds with equality. Thus, an entrepreneur's liquidity constraint, (2.3), is binding if and only if (3.3) is binding.

Here, w is the real wage rate. Because a worker does not have investment to finance, a worker is a buyer of new and existing equity and a lender in the equilibrium. As a result, a worker at the end of the period will hold more equity than the lower bound imposed by equity market frictions (i.e., $s^w_{+1} > (1-\phi)\sigma_k s$), lend to the government (i.e., $b^w_{+1} > 0$), and lend to entrepreneurs (i.e., $d^w_{+1} < 0$). This result supports the earlier statement that the workers in the model should be broadly interpreted as financially unconstrained individuals in an actual economy.

Denote average consumption per member in the household as c and the average holdings of the portfolio and debt per member, including projects, at the end of the period as (n, s, b, d). Then

$$c = \pi c^e + (1-\pi)c^w,$$

and similar equations hold for (n, s, b, d). The household's budget constraint can be obtained by weighting the entrepreneur's and the worker's budget constraints by π and $1-\pi$, respectively, and adding up:

$$(1-\pi)w\ell + (p_d d_{+1} - d) + (b - p_b b_{+1}) + (r + \sigma_k q)s - qs_{+1} \geq c + \tau + \pi(x - qi). \tag{3.5}$$

Recall that the aggregate state is $A = (K, N, \phi, \mu)$.[11] Equilibrium prices are functions of A, which include equity price $q(A)$, the price of government bonds $p_b(A)$, the price of private bonds $p_d(A)$, the rental rate of capital $r(A)$ and the wage rate $w(A)$. All prices are expressed in terms of the consumption good, which is the numeraire. The household's value function is $v(n, s, b, d; A)$, where (n, s, b, d) are the individual household's state variables. The household's choices in a period are $(x, c^e, s^e_{+1}, b^e_{+1}, d^e_{+1})$ for each entrepreneur, ℓ for each worker, and $(c, n_{+1}, s_{+1}, b_{+1}, d_{+1})$ for the average quantities per member, which together imply the choices for each worker. As explained earlier, when the financing constraint (3.3) binds, the optimal choices of $(s^e_{+1}, b^e_{+1}, d^e_{+1})$ are given by (3.2). The other choices,

[11] Strictly speaking, the aggregate state should also include the supply of government bonds, B, and government purchases of private assets, (s^g, d^g). We omit them from the list because they are assumed to be functions of $(q, p_b, p_d, K, \phi, \mu)$.

$(x, c^e, \ell, c, n_{+1}, s_{+1}, b_{+1}, d_{+1})$, solve:

$$v(n, s, b, d; A) = \max \quad \{\pi u(c^e) + (1-\pi)[U(c^w) - h(\ell)] + \beta \mathbb{E} v(n_{+1}, s_{+1}, b_{+1}, d_{+1}; A_{+1})\}$$

subject to (3.3), (3.5), and the following constraints:[12]

$$\begin{aligned} & c^w = (c - \pi c^e)/(1-\pi), \quad i = \gamma\, m(x, \tfrac{n}{\pi} + a), \\ & n_{+1} = \sigma_n \left\{n + \pi \left[a - m(x, \tfrac{n}{\pi} + a)\right]\right\}, \\ & x \geq 0,\ c^e \geq 0,\ c^w \geq 0, \quad n_{+1} \geq 0,\ s^w_{+1} \geq 0,\ b^w_{+1} \geq 0. \end{aligned}$$

The expectation in the objective function is taken over A_{+1}.

Denote λ as the Lagrangian multiplier of the household's budget constraint, (3.5). The optimal choice of c yields $\lambda = U'(c^w)$. Let $\lambda^e \pi U'(c^w)$ be the Lagrangian multiplier on the financing constraint, (3.3), so that λ^e is the shadow price of the financing constraint measured in a worker's consumption units. The liquidity constraint (2.3) binds if and only if λ^e is positive. The optimal choices of (ℓ, c^e) yield:

$$h'(\ell)/U'(c^w) = w, \tag{3.6}$$

$$u'(c^e) = U'(c^w)(1 + \lambda^e). \tag{3.7}$$

Condition (3.6) is the familiar condition of optimal labor supply. Condition (3.7) shows that a marginal unit of the consumption good yields the additional value $\lambda^e U'(c^w)$ to an entrepreneur relative to a worker by relaxing the financing constraint (3.3). Thus, λ^e is indeed the marginal value of liquid funds to an entrepreneur in terms of a worker's consumption. We will exhibit and explain the condition of optimal investment and the value of an investment project in the next subsection.

In addition, the optimality conditions of asset holdings and debt, (s_{+1}, b_{+1}, d_{+1}), and the envelope conditions of (s, b, d) yield the following asset-pricing equations:

$$q = \beta \mathbb{E} \left\{ \frac{U'(c^w_{+1})}{U'(c^w)} \left[r_{+1} + \sigma_k q_{+1} + \pi \lambda^e_{+1} (r_{+1} + \phi_{z,+1} \sigma_k q_{+1}) \right] \right\} \tag{3.8}$$

[12] The constraints $c^e \geq 0, c^w \geq 0, s^w_{+1} \geq 0, b^w_{+1} \geq 0$ and $n_{+1} \geq 0$ do not bind. In particular, $n_{+1} > 0$ under the assumption that $m_2(x, 0)$ is sufficiently large.

$$p_b = \beta \mathbb{E}\left[\frac{U'(c^w_{+1})}{U'(c^w)}\left(1+\pi\lambda^e_{+1}\right)\right] \tag{3.9}$$

$$p_d = p_b. \tag{3.10}$$

These asset pricing equations require the effective rate of return to an asset, evaluated with the marginal utility of consumption, to be equal to $1/\beta$. The effective return to an asset includes the direct return and liquidity services provided by the asset. For example, in (3.9), an additional unit of government bond in the hand of an entrepreneur enables the entrepreneur to reduce the extent to which the financing constraint (3.3) binds, which generates liquidity service in the amount $\pi\lambda^e_{+1}$. Moreover, the price of private debt is equal to the price of government bonds because a worker, who is a lender, is indifferent between lending to the government without the collateral requirement and lending to entrepreneurs with the collateral requirement (2.4). This equality between the two prices does not contradict the earlier statement that the debt issued by an entrepreneur is not a perfect substitute for government debt. The debt issued by an entrepreneur requires collateral according to the constraint (2.4), but government debt does not. This borrowing constraint (2.4) imposes an additional cost to an entrepreneur on issuing debt. An entrepreneur strictly prefers issuing government bonds to issuing private debt, but doing the former is not possible.

3.2. Optimal investment and delay

To characterize optimal investment, let us define the implicit price of an investment project in terms of a worker's consumption as

$$p_n = \frac{1}{U'(c^w)}\frac{\partial v}{\partial n}. \tag{3.11}$$

The optimal choice of x and the envelope condition of n in the household's optimization problem yield:

$$q - \frac{1}{\gamma m_1} \le \lambda^e\left[\frac{1}{\gamma m_1} - \theta_z q\right] + \frac{\sigma_n}{\gamma}\beta\mathbb{E}\left[\frac{U'(c^w_{+1})}{U'(c^w)}p_{n,+1}\right], \tag{3.12}$$

$$p_n = \gamma m_2(1+\theta_z\lambda^e)q + (1-m_2)\sigma_n\beta\mathbb{E}\left[\frac{U'(c^w_{+1})}{U'(c^w)}p_{n,+1}\right]. \tag{3.13}$$

The inequality in (3.12) and the inequality $x \geq 0$ hold with complementary slackness.

Let us explain (3.12) first. Given the stock of investment projects available to an entrepreneur, the direct cost of producing one unit of new capital at the margin is $1/(\gamma m_1)$. Let us refer to this cost as the replacement cost of capital. Because each unit of capital has value q, then $(q - \frac{1}{\gamma m_1})$ is the marginal benefit of one unit of new capital in excess of the replacement cost. For investment to be optimal, this excess benefit must be equal to the opportunity cost of investment, which consists of the two terms on the right-hand side of (3.12). One is the cost of downpayment on investment. Since the amount of funds that can be raised through equity on each unit of investment is $\theta_z q$, the downpayment on a marginal unit of investment is $(\frac{1}{\gamma m_1} - \theta_z q)$, and so the cost of such a downpayment is $\lambda^e(\frac{1}{\gamma m_1} - \theta_z q)$. The other implicit cost of investment is the option value of delaying investment. Delaying one unit of investment saves $1/(\gamma m_1)$ units of the resource and, hence, increases the number of unimplemented projects by $m_1/(\gamma m_1) = 1/\gamma$. As a result, the household's stock of projects in the next period will increase by σ_n/γ. Because each project in the next period will have a value $p_{n,+1}$ in terms of a worker's consumption in the next period, its expected value in terms of a worker's current consumption is $\beta E \frac{U'(c^w_{+1})}{U'(c^w)} p_{n,+1}$. Thus, the last term in (3.12) is the option value of one unit of delayed investment. In this economy, the cost of downpayment on investment and the option value of an investment project both drive equity price above the replacement cost of capital.

The price of an investment project obeys the intertemporal equation, (3.13). The two terms on the right-hand side are the values that a higher stock of investment projects today can generate in the current and the next period. A higher stock of investment projects increases current investment by γm_2. The equity associated with a unit of new capital is q. In addition, a unit of investment can be used to raise the amount $\theta_z q$ units of funds to relax an entrepreneur's financing constraint, the implicit value of which is $\theta_z q \lambda^e$. Thus, the increased investment brought about by a higher stock of investment projects has the marginal value $\gamma m_2(1 + \theta_z \lambda^e)q$ in the current period. Moreover, a higher stock of investment projects increases the stock in the next period by $(1 - m_2)\sigma_n$. The future value of this higher stock of projects is given by the last term in (3.13) according to the above explanation for the option value of delayed investment.[13]

As mentioned before, it is never optimal to implement all available projects because of diminishing marginal productivity of investment expenditure. This is true even when the financing constraint is not binding and unimplemented projects have zero option value, i.e., when $\lambda^e = 0$ and $\sigma_n = 0$. Thus, investment delay is not indicated by the mere existence of a positive gap, $(\frac{n}{\pi} + a - m)$, but rather by how much of this gap is caused by financial frictions and how much by the option value of unimplemented projects.

Of particular interest is the extent of investment delay caused by financial frictions. To measure this, let us characterize optimal investment in the hypothetical economy where the financing constraint is not binding and an unimplemented project has the same option value as in the model economy. Precisely, in this hypothetical economy, $(n, q, p_d, p_{n,+1}, c^w, c^w_{+1})$ are the same as in the model economy but $\lambda^e = 0$. Then, an entrepreneur's optimal investment expenditure, denoted as x^f, satisfies (3.12) with $\lambda^e = 0$. That is,

$$q - \frac{1}{\gamma m_1^f} = \frac{\beta\sigma_n}{\gamma}\mathbb{E}\left[\frac{U'(c^w_{+1})}{U'(c^w)}p_{n,+1}\right], \tag{3.14}$$

where $m^f = m(x^f, \frac{n}{\pi} + a)$ and (m_1^f, m_2^f) denote the partial derivative of $m(x^f, \frac{n}{\pi} + a)$. By construction, investment delay in this hypothetical economy is caused entirely by the option value of an investment project and not by the financing constraint. The additional delay caused by financial frictions can be measured by $1 - fimp$, where $fimp$ is defined as

$$fimp = m/m^f. \tag{3.15}$$

Subtracting (3.14) from the equality form of (3.12) yields:

$$\frac{1}{\gamma m_1^f} - \frac{1}{\gamma m_1} = \lambda^e\left(\frac{1}{\gamma m_1} - \theta_z q\right). \tag{3.16}$$

The left-hand side of (3.16) is increasing in x^f and decreasing in x. Thus, intuitively, financial frictions are likely to cause a larger delay in investment if the current period has a tighter financing constraint (i.e., higher λ^e), a lower ability to use equity to raise funds (i.e., a lower $\theta_z q$), or a lower productivity of investment expenditure (i.e., a lower γm_1).

[13]Note that this term is not divided by γ because all terms in (3.13) are measured in a worker's current consumption, which contrasts with (3.12) where all terms are measured in current investment.

3.3. Definition of a Recursive Equilibrium

Let $\mathcal{K} \subset \mathbb{R}_+$ and $\mathcal{N} \subset \mathbb{R}_+$ be compact sets that have as their elements all possible values of K and N, respectively. Let $\Phi \subset [0,1]$ be a compact set that contains all possible values of ϕ and let μ lie in the set $[0,1]$. Denote $\mathcal{A} = \mathcal{K} \times \mathcal{N} \times \Phi \times [0,1]$. Let C_1 be the set of all continuous functions that map $\mathcal{A}$ into $\mathbb{R}_+$, C_2 the set of all continuous functions that map $\mathcal{N} \times \mathcal{K} \times [0,B] \times \mathbb{R} \times \mathcal{A}$ into $\mathbb{R}_+$ and C_3 the set of all continuous functions that map $\mathcal{N} \times \mathcal{K} \times [0,B] \times \mathbb{R} \times \mathcal{A}$ into $\mathbb{R}$. Asset and factor prices, (q, p_b, p_d, r, w), are functions of the aggregate state A and, hence, lie in C_1. The value function v is a function of the household's own state variables (n, s, b, d) and the aggregate state A. So are the household's policy functions for optimal choices, $(x, c^e, s^e_{+1}, b^e_{+1}, d^e_{+1}, \ell, c, s_{+1}, b_{+1}, d_{+1})$. Given government policies (B, s^g, d^g), a *recursive competitive equilibrium* is a list of asset and factor price functions $(q, p_b, p_d, r, w) \in C_1$, a household's policy functions $(x, c^e, s^e_{+1}, b^e_{+1}, d^e_{+1}, \ell, c, s_{+1}, b_{+1}, d_{+1}) \in C_2$, the value function $v \in C_3$, the factor demand functions, (k^D, ℓ^D), and the laws of motion of the aggregate capital and project stocks that meet the following requirements:

(i) Given price functions and the aggregate state, a household's policy and value functions solve a household's maximization problem;

(ii) Given price functions and the aggregate state, (k^D, ℓ^D) maximize producers' profit, i.e., $r = F_1(k^D, \ell^D)$ and $w = F_2(k^D, \ell^D)$;

(iii) Given the law of motion of the aggregate state, prices clear the markets:

$$\text{goods: } F(k^D, \ell^D) = c + g + \pi x \tag{3.17}$$

$$\text{capital: } k^D = K = s + s^g \tag{3.18}$$

$$\begin{aligned}
&\text{labor: } \ell^D = (1-\pi)\ell \\
&\text{government bonds: } b_{+1} = B_{+1} \\
&\text{equity: } s_{+1} + s^g_{+1} = \sigma_k(s + s^g) + \pi i
\end{aligned}$$

$$\text{private debt: } d_{+1} = d^g_{+1}. \tag{3.19}$$

(iv) Symmetry and aggregate consistency: $(s, n, b) = (K, N, B)$, and the laws of motion of the aggregate capital and project stocks are consistent with the aggregation of individual households' choices:

$$K_{+1} = \sigma_k K + \pi i, \quad N_{+1} = \sigma_n [N + \pi(a - m)]. \tag{3.20}$$

In the above conditions, we have suppressed the arguments of the policy functions. Condition (3.18) says that all capital is claimed by the private sector and the government. To explain (3.19), recall that d_{+1} is the household's average debt per member. Because all households are symmetric and each worker in a household lends to other households' entrepreneurs, d_{+1} is a household's debt position after netting out the liabilities with other households. This private debt, if positive, must be held by the government, which is what (3.19) requires. The consistency conditions in (iv) are required in order for households to compute expectations in their optimization problem. Note that since $K = s + s^g$, the law of motion for the aggregate capital stock is identical to the equity market clearing condition.

Finding an equilibrium amounts to finding the asset price functions, $q(A)$ and $p_b(A)$, that solve (3.8) and (3.9), which are part of the requirements for optimality in (i) above. Once the asset price functions are solved, factor price functions, and the value and policy functions can be recovered from other equilibrium conditions. Appendix B describes the procedure for computing the equilibrium.

4. Quantitative Analysis on the Effect of Financial Shocks

4.1. Calibration

To calibrate the model, we assume the following functional forms:

production function:	$F(K, (1-\pi)\ell) = K^\alpha((1-\pi)\ell)^{1-\alpha}$
worker's utility function:	$U(c^w) = \frac{(c^w)^{1-\rho} - 1}{1-\rho}$
entrepreneur's utility function:	$u(c^e) = u_0 U(c^e)$
disutility of labor:	$h(\ell) = h_0 \ell^\eta$
investment technology:	$m(x, \frac{n}{\pi} + a) = \left\{\frac{1}{2}(\delta x)^\xi + \frac{1}{2}(\frac{n}{\pi} + a)^\xi\right\}^{\frac{1}{\xi}}$
collateral multiplier:	$z(\phi, \mu) = \phi\ \mu.$

In the investment technology, δ converts investment expenditure into the same unit as the available stock of projects, and $1/(1-\xi)$ is the elasticity of substitution between the two inputs. The functional form of the collateral multiplier has the maintained property that it is increasing in equity liquidity. The multiplicative form conveniently implies that a shock to ϕ and a shock to μ of the same percentage points affect the collateral multiplier in the same percentage points. Denote $\tilde{\phi} = \frac{1}{\phi} - 1$ and $\tilde{\mu} = \frac{1}{\mu} - 1$. We assume the following processes for $\tilde{\phi}$ and $\tilde{\mu}$:

$$
\begin{aligned}
\log \tilde{\phi}_{t+1} &= (1 - \sigma_\phi) \log \tilde{\phi}^* + \sigma_\phi \log \tilde{\phi}_t - \varepsilon_{\phi,t+1} \\
\log \tilde{\mu}_{t+1} &= (1 - \sigma_\mu) \log \tilde{\mu}^* + \sigma_\mu \log \tilde{\mu}_t - \varepsilon_{\mu,t+1},
\end{aligned}
\tag{4.1}
$$

where $\tilde{\phi}^*$ is the steady state level of $\tilde{\phi}$, $\tilde{\mu}^*$ the steady state level of $\tilde{\mu}$, and $\sigma_\phi, \sigma_\mu \in (0, 1)$.

Table 1. Parameters and calibration

Parameter	Value	Target
β: discount factor	0.9879	$\beta^{-4} = 1.05$
ρ: relative risk aversion	2	exogenously chosen
η: curvature in labor disutility	2	labor supply elasticity $\frac{1}{\eta-1} = 1$
u_0: constant in entrep. utility	75.757	capital stock/annual output = 3.32
h_0: constant in labor disutility	18.840	hours of work = 0.25
α: capital share	0.36	capital income share = 0.36
σ_k: survival rate of capital	0.981	annual replacement of capital = 7.6%
g: government spending	0.1928	government spending/GDP = 0.18
π: fraction of entrepreneurs	0.06	annual fraction of investing firms = 0.24
B^*: steady state liquid assets	1.831	fraction of liquid assets = 0.12

Table 1. Parameters and calibration (continued)

Parameter	Value	Target
θ: fraction of new equity sold	0.0629	$\frac{\text{funds raised in markets}}{\text{investment expenditure}} = 0.284$
μ^*: steady state borrowing capacity	0.0891	$\frac{\text{issuance of debt}}{\text{sale of stock}} = 1.287$
ϕ^*: steady state equity liquidity	0.2913	annual equity premium in the deterministic steady state = 0.02
a: endowment of new projects	0.3361	normalization $\frac{n^*}{\pi} + a = 1$
γ: converting m into i	15.480	$c^*/F^* = 0.70$
δ: efficiency unit of x in m	0.0795	annual return to liquid assets=0.02
σ_n: survival rate of projects	0.9363	exogenously chosen
ξ: $\frac{1}{1-\xi}$ = elasticity of sub. in m	-1.0	$fimp^* = 0.78$
σ_ϕ: survival rate of $\tilde{\phi}$	0.9	exogenously chosen
σ_μ: survival rate of $\tilde{\mu}$	0.9	exogenously chosen

The deterministic steady state is described in Appendix A.1 for any given government policy (B^*, s^{g*}, d^{g*}). In the calibration, we set government purchases of equity and private debt to zero, i.e., $s^{g*} = d^{g*} = 0$. The length of a period is one quarter. Table 1 lists the parameters, their values, and the targets. Appendix A.2 describes how to use the targets to solve the parameters. With these parameter values, the financing constraint (3.3) is binding.

The targets that determine $(\beta, \rho, \eta, u_0, h_0, \alpha, \sigma_k, g)$ are standard in macro analyses.[14] One exception might be the elasticity of labor supply, which is deliberately set to be relatively low in order to ensure that the response of employment to financial shocks does not come from very elastic labor supply. Note that u_0 affects investment undertaken by an entrepreneur and, hence, setting the ratio of the steady state capital stock to output helps in identifying u_0. The parameter, π, is the fraction of firms that opt to invest in a period. Several papers estimate this fraction at an annual frequency, and the estimates vary from 0.20 (Doms and Dunne, 1998) to 0.40 (Cooper et al., 1999). The value 0.24 lies within the range of these estimates, and so we set the quarterly value to $\pi = 0.06$. Del Negro et al. (2011), using the U.S. Flow of Funds between 1952 and 2008, calculate the share of liquid assets in total asset holdings to be 0.12, which is used here to calculate B^*.

The three parameters, (θ, μ^*, ϕ^*), describe financial frictions. The target for θ comes from the evidence in Nezafat and Slavik (2010). Using the U.S. Flow of Funds, these authors construct a time series for the ratio of funds raised in the market to investment expenditure by nonfarm nonfinancial corporate firms. They find that the mean of this ratio is 0.284. In their definition, the amount of funds raised in the market is equal to new equity issuance plus credit market instruments. In our model, this amount is the sum of the value of new equity issuance, $q\theta i$, and the amount borrowed in the market, $p_d zqs^e_{+1}$. We set the ratio of this sum to x in the steady state to 0.284. The target for μ^* comes from the evidence in Covas and den Haan (2011). Using the data COMPUSTAT, they report the ratio of debt issuance to assets and the ratio of stock sales to assets in each period for US nonfarm nonfinancial corporate firms. Dividing these two ratios yields the ratio of debt issuance to stock sales, denoted as DE. This ratio exhibits large variation across firms and is increasing in firm size. We target the value for the bottom 50% of firms, 1.287, in

[14]A target may involve more than one parameter. To identify the parameters, we use several targets jointly to solve a number of parameters. However, to link the parameters to the targets intuitively, we describe the identification as if each target could identify a parameter separately.

order for the collateral constraint to be binding. Setting the equity premium as a target helps identifying ϕ^* because the lower liquidity of equity relative to government bonds is the cause of equity premium in our model. Although the target, 0.02, is smaller than the equity premium in the US data, it may be justified by noting that it is the equity premium in a deterministic steady state. Even with this relatively low target on the equity premium, ϕ^* is far below one. If the equity premium were set at a higher target, say 0.03, ϕ^* would be very close to zero.

It is clear that the size of a project, γ, cannot be identified separately from the number of projects received in each period, a. We pin down a by normalizing the number of projects in the steady state, $\frac{n^*}{\pi} + a$, to one, which amounts to choosing the unit of a project. The target on c^*/F^* enables us to determine steady state investment expenditure, x^*, through the goods market clearing condition. Because i^* was identified earlier, the relationship $i^* = \gamma m(x^*, \frac{n^*}{\pi} + a)$ helps us identifying γ. With the chosen unit of a project, the value of γ suggests that each project is relatively large, and so the extensive margin of investment (i.e., the number of implemented projects) is important. The target on the annual rate of return to liquid assets comes from Del Negro et al. (2011). They report that the net rate of return to U.S. government liabilities is 1.72% for one-year maturities and 2.57% for 10-year maturities. The value we choose, 0.02, lies in this range. This target enables us to solve the price of government bonds and, hence, the Lagrangian multiplier of the financing constraint in the steady state, λ^{e*} (see (3.9)). Because the marginal cost of the downpayment on investment depends on λ^{e*}, knowing λ^{e*} helps us in identifying the marginal replacement cost of capital that is needed for the investment decision to be optimal (see (3.12)). In turn, this helps identifying δ since the replacement cost of capital depends on δ.

The target on the fraction of implemented projects is the ratio of the number of projects implemented in the steady state relative to the hypothetical case where the financing constraint is not binding. This target comes from Graham and Harvey (2013). In this quarterly survey of CFOs around the world, the CFO's are asked whether they postponed attractive investment projects specifically because external financing was limited.[15] With the responses in the US to this question, we calculate the average number of implemented projects with financial constraints as a fraction 0.57 of the number that would be implemented in the absence of financial constraints. In the model, this fraction is $fimp$ defined by (3.15). However, because the survey was conducted in the great recession, this fraction is lower in the survey than in the long run. We set the target on this fraction in the steady state at $fimp^* = 0.78$ so that the fraction indeed drops to 0.57 after a negative shock to equity liquidity of a reasonable size (see section 4.2). This target gives a restriction on (ξ, σ_n) (see Appendix A.2). For investment delay to be important, unimplemented projects must be able to survive to the next period with high probability. We set σ_n to a high level and use the restriction just described to pin down ξ. In subsection 5.3, we will contrast the results with those obtained in the case $\sigma_n = 0$ where delay yields no benefit to an entrepreneur.

Finally, for financial shocks to have persistent effects on aggregate activity, the shocks cannot be temporary. Thus, we set the persistence parameters of the shocks as $\sigma_\phi = \sigma_\mu = 0.9$.[16]

4.2. Financial shocks and aggregate activities

In this model, financial shocks are shocks to equity liquidity and entrepreneurs' borrowing capacity, modeled by ε_ϕ and ε_μ,respectively. Suppose that the economy has been in the deterministic steady state before period $t = 1$. At the beginning of period 1, there is an unanticipated realization of either $\varepsilon_{\phi,1} < 0$ or $\varepsilon_{\mu,1} < 0$. The size of the shock is chosen such that it reduces $z = \phi\mu$ by 18% in period 1 from the steady state level. After the initial shock, the shock variable returns to the steady state asymptotically according to

[15] The question has been asked seven times since December 2008, and the exact question varies slightly each time it is asked. For example, in December 2008, the question was: "When external capital is limited, are your corporate investments postponed or canceled?" In September 2009, the question was: "Over the past 18 months, did you company pass up attractive investment project specifically because of the cost or availability of credit?"

[16] We have also computed the responses of the model and evaluated government interventions in the asset market with $\sigma_\phi = \sigma_\mu = 0.7$. Predictably, the effects of the shocks are less persistent. However, the comparisions between different types of interventions and the importance of delay are robust.

the process in (4.1). We examine how aggregate activities respond to the shock. The computation of equilibrium dynamics is described in Appendix B.

Consider first a negative shock to equity liquidity ϕ. Figure 1a depicts the dynamics of equity liquidity, investment expenditure x, and the marginal replacement cost of capital $1/(\gamma m_1)$. The negative liquidity shock has a large effect on investment. On impact, the shock reduces investment expenditure by about 11.1%, which is comparable to the magnitude of the fall in investment from the trend to the trough in 2009. Given the persistence of the shock, the effect of the liquidity shock on investment is also persistent. Two years after the shock, investment expenditure is still about 2.5% below the steady state. The fall in investment increases the marginal productivity of the resource in investment and, hence, reduces the replacement cost. As investment recovers, so does the replacement cost of capital. This is the case despite that the stock of projects rises after the shock due to delay, which tends to increase the marginal productivity of the resource in investment and depress the marginal replacement cost of capital.

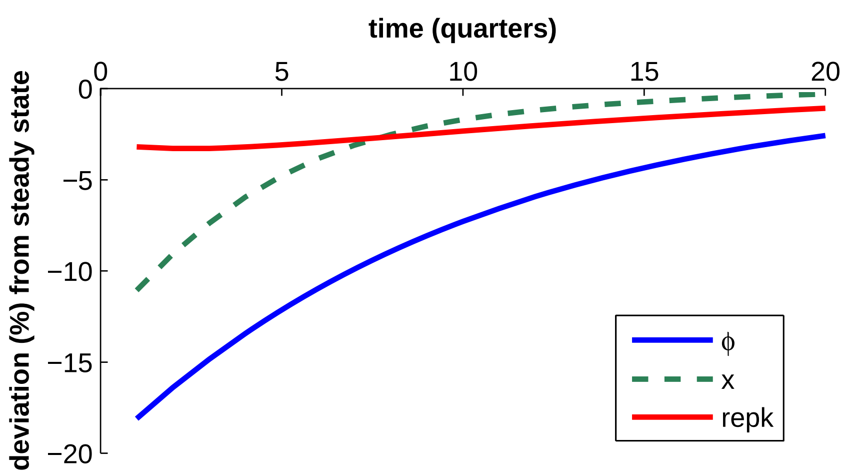

Figure 1a. Liquidity ϕ, investment expenditure x and the replacement cost of capital $repk$ after a negative shock to ϕ

Figure 1b depicts the dynamics of aggregate output, employment and consumption after the negative liquidity shock. On impact of the shock, output falls by about 1.9%, employment by 2.9% and aggregate consumption by 0.8%. All three variables take a significant amount of time to return to the steady state. Thus, liquidity shocks alone can generate significant aggregate fluctuations and induce positive comovement among aggregate quantities. This result extends the finding in Shi (2012) to an environment with investment delay. The result is remarkable, since there is no shock to total factor productivity or the existence of nominal rigidity. It is expected that the negative liquidity shock reduces investment and output. To explain the response of employment, note that workers do not face the liquidity constraint because they do not have the opportunity to invest. The negative liquidity shock shifts wealth from the liquidity constrained individuals (i.e., entrepreneurs) to the unconstrained individuals (i.e., workers). This increase in workers' consumption induces them to reduce labor supply. Despite the increase in workers' consumption, aggregate consumption falls because entrepreneurs' consumption falls by a much larger magnitude than the increase in workers' consumption.

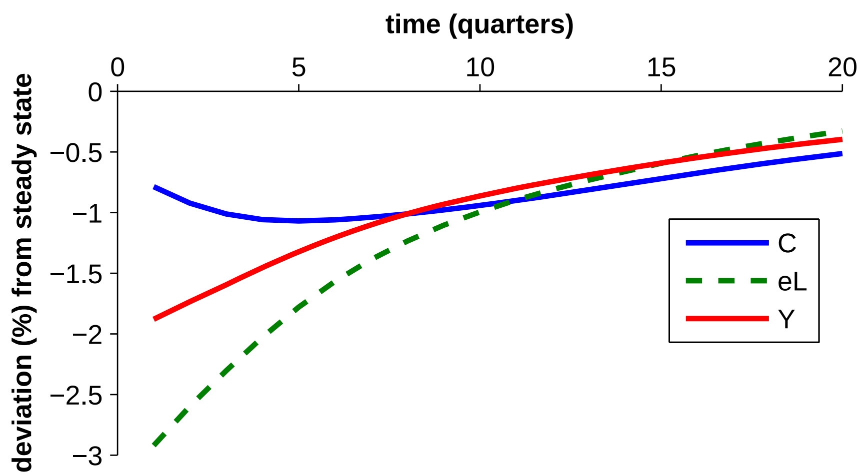

Figure 1b. Aggregate consumption c, employment eL and output Y after a negative shock to ϕ

Figure 1c shows the ratio of the number of implemented projects to that in the hypothetical case where the financial constraint is not binding. Note that it is the level of $fimp$ instead of its percentage change that is depicted in Figure 1c. On impact of the negative liquidity shock, the ratio $fimp$ drops from 0.78 to 0.57. The magnitude of the reduction is what we intended to achieve by setting the target $fimp^* = 0.78$ in the calibration.

Figure 1d shows the responses of the stock of projects available to an entrepreneur and the implicit price of a project to the negative liquidity shock. The stock of projects does not change in period 1 because it is predetermined. Starting in period 2, the stock of projects increases until reaching the peak in the eighth period, which is about 7.3% higher than the steady state level. Such a long delay in reaching the peak and the magnitude of the peak reflect the significant amount and duration of investment delay. After the eighth period, the stock of projects starts to fall toward the steady state, as the reduction in investment has abated and the stock of projects has been accumulated sufficiently. The implicit price of a project follows a non-monotonic path after the shock. The immediate fall in this price reflects the feature that the stock of projects and investment expenditure are complementary with each other in the investment technology. As investment expenditure falls, the marginal productivity of the stock of projects falls, which reduces the price of this stock. The price of the stock of projects continues to fall until the third period after the shock. This response is caused by delay. As investment projects are delayed, the stock of projects increases, which reduces the marginal productivity of this stock. After the fourth period, the price of the stock of projects starts to rise to return the steady state gradually.

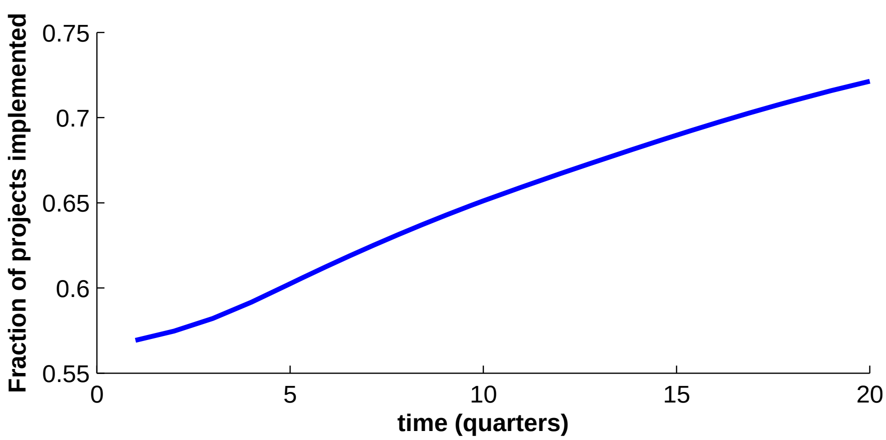

Figure 1c. The number of implemented projects relative to the unconstrained case, after a negative shock to ϕ

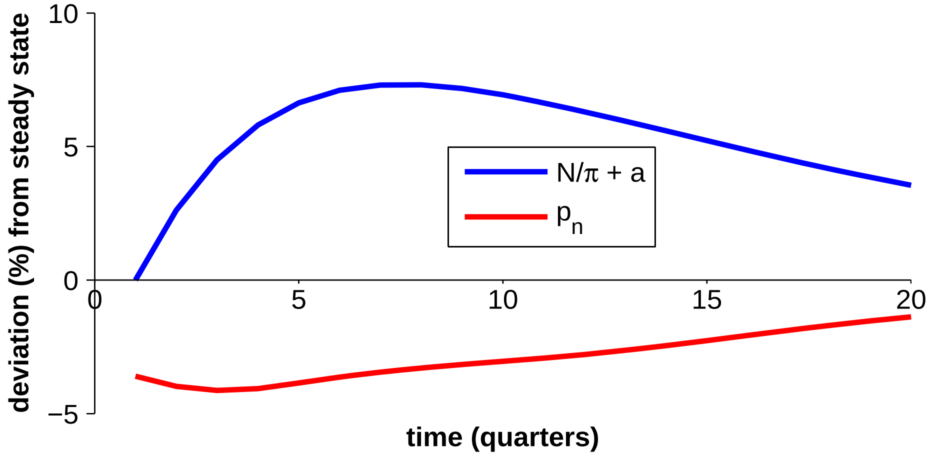

Figure 1d. The stock of investment projects, $N/\pi + a$, and the implicit price of this stock p_n after a negative shock to ϕ

Consider next a negative shock to μ of the same size as the shock to ϕ. For brevity, we only present the responses of investment expenditure, output and employment to the μ shock, as depicted in Figure 2. The responses of these variables are qualitatively similar to the responses to the negative liquidity shock depicted in Figures 1a and 1b, but the magnitude is twelve times smaller. The qualitative similarity between the responses to the two shocks reflects the fact that both shocks reduce entrepreneurs' ability to finance investment. To understand why a shock to μ has much smaller effects than a shock to ϕ, note that the calibration sets the funds raised from debt to be 1.287 times that from new equity in the steady state. Thus, the difference in the magnitude between the effects of the two shocks arises not because debt is insignificant relative to new equity in financing investment. Rather, the difference arises from the fact that a liquidity shock affects an entrepreneur's financing ability in more ways than a μ shock does. A negative μ shock reduces the financing ability solely by reducing an entrepreneur's borrowing capacity. A negative liquidity shock has this effect as well, because a lower liquidity of equity reduces the effectiveness of equity used as collateral in borrowing. In addition, a negative liquidity shock directly reduces the amount of existing equity that can be re-sold to raise funds for investment. This additional effect is much larger than the effect through the borrowing capacity.

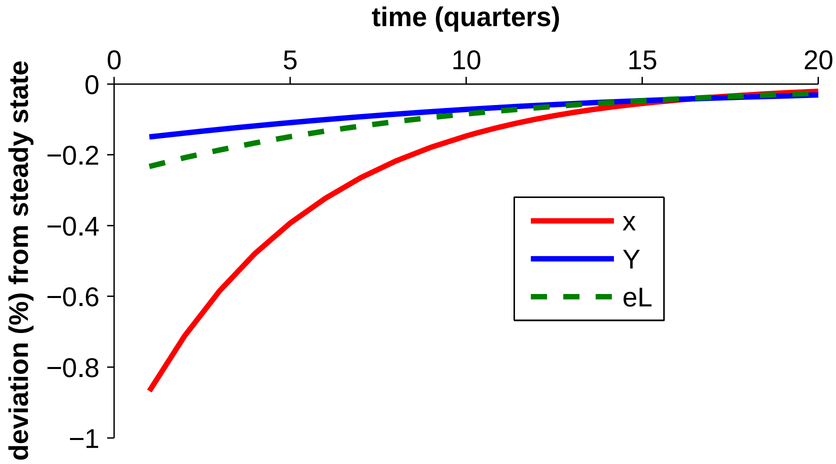

Figure 2. Investment expenditure x, employment eL and output Y after a negative shock to μ

5. Effects of Government Interventions in the Asset Market

In this section, we examine how government interventions in the asset market affect the response of the economy to financial shocks and how investment delay affects the impact of these interventions. The interventions are government purchases of either private equity or debt issued by entrepreneurs.

5.1. Government purchases of private assets in the open market

Let J denote the type of private assets that the government holds and p_J the price of this asset. If the assets are private equity, then $J = s$ and $p_J = q$; if the assets are private debt, then $J = d$ and $p_J = p_d$. Assume that the government did not hold private assets before period $t = 1$; i.e., $J_1^g = 0$. After the realization of the shock in period 1 to either equity liquidity or the borrowing capacity, the government announces the following path of holdings of private assets:

$$J_{t+1}^g = \psi_J \; K^* \left(1 - \frac{z_t}{z^*}\right) \text{ for all } t \geq 1, \tag{5.1}$$

where $\psi_J > 0$ is a constant. Given the size of the shock, the coefficient ψ determines the magnitude of the intervention. We set $\psi_s = 0.115$. To gauge the magnitude of these interventions, recall that the capital stock is 3.32 times as large as annual output in the steady state and the equity liquidity shock reduces z by 18% in period 1. The parameter value $\psi_s = 0.115$ implies that the initial purchase of private assets in period 1 is 6.87% of annual output, which amounts to about one trillion dollars in 2008. Given the persistence of the shock, (5.1) requires a sufficiently long time to unload this initial purchase. For comparison between purchases of equity and debt, we set ψ_d to be such that the two types of purchases have the same value in period 1 when they are evaluated with steady state asset prices. That is, $q^* s_2^g = p_d^* d_2^g$.[17] This amounts to

$$\psi_d = \psi_s q^* / p_d^*. \tag{5.2}$$

The path of asset purchases above can be financed by infinitely many combinations of government bond issuing and taxes. We consider two specific combinations that are indexed by $\tau 1_d = 0$ or 1. Taxes satisfy the government budget constraint, (2.5), while the issuance of government bonds satisfies:

$$p_{b,t} B_{t+1} = \tilde{p}_b B^* + p_{J,t} J_{t+1}^g, \quad t \geq 1, \tag{5.3}$$

where $\tilde{p}_b = p_b^*$ if $\tau 1_d = 0$ and $\tilde{p}_b = p_b$ if $\tau 1_d = 1$. In both cases of $\tau 1_d$, the tax τ satisfies the government budget constraint (2.5). It is easy to verify that in the case $\tau 1_d = 0$, the tax in period 1 stays at the steady state level, which explains the meaning of $\tau 1_d = 0$. In the case $\tau 1_d = 1$, the tax in every period $t \geq 1$ can change. We focus on the case $\tau 1_d = 0$ because it resembles quantitative easing: In period 1, government purchases of private assets are financed entirely by the change in the value of public debt issued and not by the change in the tax, although taxes in future periods may be adjusted.

Note the following features of the interventions specified above. First, the government is assumed to purchase private assets in the open market rather than directly from entrepreneurs. The latter resembles bailouts or lending through discount windows, rather than

[17] We use steady-state asset prices in this requirement in order to simplify the calculation of ψ_d. If we use actual prices of assets in period 1, this requirement becomes $q_1 s_2^g = p_{d,1} d_2^g$. This alternative requirement is much more difficult to use, because actual prices of assets in period 1 are equilibrium objects and, hence, are affected by the interventions. Finding the value of ψ_d that satisfies this alternative requirement involves solving a fixed-point problem.

quantitative easing, and its effects are larger than those of the interventions described above. Second, government purchases of assets eventually return to zero as the effect of the shock dissipates, because z_t approaches the steady state asymptotically. Also, because $z = \phi\mu$, the size of the intervention is the same under the shocks to ϕ and μ. Finally, government policies $(B_{+1}, s^g_{+1}, d^g_{+1})$ are indeed functions of only (q, p_b, p_d, A), as we have assumed.

5.2. Effects of government interventions

Consider the same negative shock to equity liquidity as examined in subsection 4.2. The left panel in Figure 3a depicts the response of investment expenditure with equity purchases, debt purchases and no intervention, and the right panel depicts the response of output in the three cases. The intervention by purchasing equity has large effects on investment and output. One period after the purchase (in period 2), investment recovers 54% and output recovers 32% of the initial loss. With equity purchases, the paths of investment expenditure and output from the second period onward are significantly above the ones with no intervention. The intervention also induces employment in period 2 to recover 36% of the initial loss (not depicted). These large effects of equity purchases are remarkable for two reasons. First, they cannot be explained by merely referring to the large size of the intervention. Even with the same size, debt purchases have only a small effect, as the responses with debt purchases are close to the ones with no intervention. Second, as mentioned before, the interventions are conducted in the open market and, hence, they are not specifically aimed at only taking illiquid assets out of entrepreneurs' hands. Instead, among the equity purchased by the government, the fraction that will become illiquid is the same as that held by households.

Equity purchases by the government generate the large effect on aggregate activity by creating liquid funds based on temporarily illiquid assets. To see this, let us contrast the values of one unit of equity held by an entrepreneur and by the government after the depreciation of capital. When a unit of equity is held by an entrepreneur, a fraction ϕ can be resold immediately, and the remaining fraction can be used as collateral to borrow

$(1-\phi)zp_d$ units of funds. The total amount of liquid funds raised from one unit of equity is ϕ_z, as defined in (3.4). If a unit of equity is held by the government, again, only a fraction ϕ can be resold immediately. However, the government can make use of all of the remaining fraction of equity. In particular, because equity can be resold eventually and the government does not face the borrowing constraint as entrepreneurs do, the government can borrow against the currently illiquid fraction $(1-\phi)$ of equity and reduce current taxes. This reduction in taxes increases an entrepreneur's liquid funds by the amount $(1-\phi_z)q\sigma_k$ per equity. Since ϕ_z is significantly below one as a result of the calibration, this increase in liquid funds is significant and leads to a large recovery of investment from the initial fall after the shock.

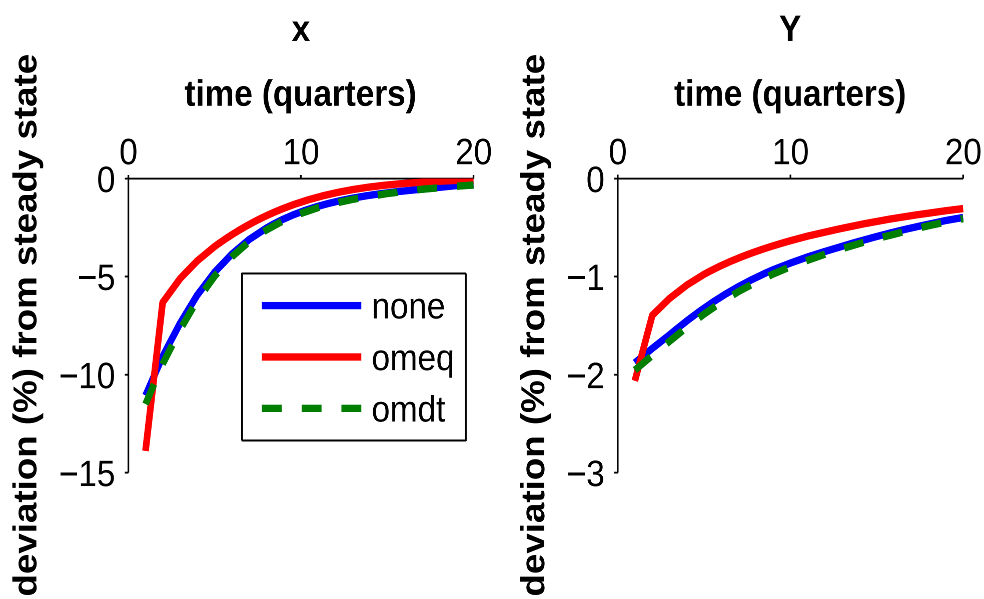

Figure 3a. Investment x and output Y under no intervention, equity purchases (*omeq*) and debt purchases (*omdt*): liquidity shock

From the above explanation, it is also easy to see why purchases of private debt by the government do not affect aggregate activity by as much as equity purchases. Debt purchases increase the demand for private debt, but the benefit to an entrepreneur of borrowing from the government is limited for two reasons. First, any borrowing from the government by an entrepreneur is subject to the same collateral constraint (2.4) as borrowing from the private sector. Second, the household must repay the debt to the government next period, which reduces the scope of creating liquid funds based on temporarily illiquid assets.[18]

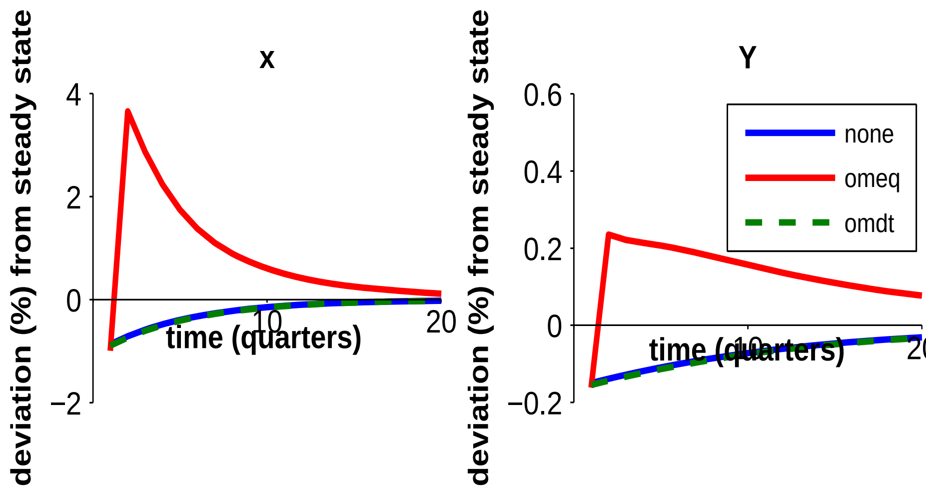

Figure 3b. Investment x and output Y under no intervention, equity purchases ($omeq$) and debt purchases ($omdt$): μ shock

Figure 3a also shows that the interventions exacerbate the impact of the shock in period 1. In particular, investment expenditure falls by 13.9% in period 1 with equity purchases, in comparison with 11.1% with no intervention. Most of this difference is caused by the assumption that the tax in period 1 is fixed under the interventions. To see this, note that the negative liquidity shock increases the price of government bonds, because such bonds perform a greater role of providing liquidity when the financing constraint is tighter. The increase in the price of government bonds increases government revenue raised by issuing new bonds, which leads to a reduction in the tax in period 1 when there is no intervention. This lower tax mitigates the shortfall in an entrepreneur's liquid funds and, hence, in investment. When the tax in period 1 is assumed to be fixed under interventions, the mitigating effect is absent. However, this difference between the cases with and without interventions is temporary. From the second period onward, the tax revenue is allowed to change in all cases.

[18] It is then reasonable to conjecture that the effect of debt purchases can be increased if the maturity of private debt is increased.

Figure 3b depicts the effects of government interventions on investment expenditure and output in the case of a negative shock to μ. These effects are smaller than in the case of the liquidity shock. However, relative to the case of no interventions, the effects of equity purchases are more dramatic. With equity purchases by the government, investment and output do not merely recover from the initial fall – they overshoot the steady state in period 2 and return to the steady state from above. This overshooting indicates that the amount of liquid funds created by equity purchase, $(1 - \phi_z)q\sigma_k K$, is large relative to the negative effect of the μ shock on the amount borrowed by an entrepreneur. As in the case of the liquidity shock, debt purchases have only small effects on investment and output.

Let us contrast our analysis with Del Negro et al. (2011) who evaluate the extent to which asset market interventions reduced the recession in 2008-2009. They integrate the KM frictions in the equity market into a new Keynesian model that features nominal rigidities and a monetary policy rule prominently. Our paper and Del Negro et al. share the emphasis on equity liquidity. Moreover, both papers use the large household framework of Shi (1997) and both find that asset market interventions can have quantitatively large effects on aggregate activity. However, there are important differences between the two papers. On the modeling side, we incorporate frictions in both the equity and the debt market, while Del Negro et al. (2011) assume that entrepreneurs cannot borrow. Moreover, we introduce investment delay but abstract from nominal rigidities and monetary policy rules. On the quantitative importance of financial shocks, our model with no nominal rigidity shows that liquidity shocks can have large effects on aggregate activity and can induce significant positive comovement among aggregate variables. In contrast, Del Negro et al. find that such shocks have only small effects when there is no nominal rigidity. In addition, we find that the shock to the borrowing capacity has only small effects on aggregate variables. This quantitative comparison of debt shocks with liquidity shocks seems new relative to the literature on financial frictions in general. On the policy analysis, we evaluate both equity purchases and debt purchases by the government, while Del Negro et al. (2011) examine only equity purchases and only after equity liquidity shocks. Moreover, we assume that government purchases of private assets are initially financed entirely by selling liquid government assets, as in a typical episode of quantitative easing, while Del Negro et al. (2011) assume that the tax adjusts immediately to the interventions. As a result, our analysis reveals that government interventions in the asset market can exacerbate the negative effect of a liquidity shock initially, although they help aggregate activity to recover in subsequent periods.

5.3. The role of investment delay

We now investigate the quantitative importance of investment delay for the responses of the equilibrium to financial shock and to government interventions in the asset market. To this end, suppose counter-factually that investment delay does not yield any benefit to an entrepreneur, in the sense that an unimplemented project cannot survive to the next period. We refer to this economy with $\sigma_n = 0$ as an economy without delay. To recalibrate the model with the new value $\sigma_n = 0$, we discard the target on the fraction of implemented projects in the steady state, because the target was obtained from a survey in a real economy where delay has a positive option value. Since this target was used to pin down ξ in the baseline calibration, we set ξ in the economy without delay to the same value as in the baseline economy. Also, we discard the normalization $\frac{n^*}{\pi} + a = 1$ and, instead, set a to the same value as in the baseline calibration. The reason for doing so is to maintain comparability between the two economies. Because $n^* = 0$ with $\sigma_n = 0$, the normalization would yield a much higher endowment of new projects ($a = 1$) in the economy without delay than in the economy with delay. Appendix A.2 describes the recalibration, which yields the following new parameter values: $\sigma_n = 0$, $\gamma = 15.283$, and $\delta = 0.1225$. All other parameters are the same as in Table 1.

Financial frictions affect the number of implemented projects even when $\sigma_n = 0$, as explained in section 3.2. Thus, as in the baseline economy, we can calculate investment expenditure in the economy with $\sigma_n = 0$ in the hypothetical situation where the financing constraint is not binding. This is done by setting $\sigma_n = 0$ in (3.14). Then, we can calculate $fimp$ for the economy with $\sigma_n = 0$, which is the ratio of the number of implemented projects to that in the case with $\lambda^e = 0$. The difference in this ratio between the baseline economy and the economy with $\sigma_n = 0$ indicates the effect of investment delay caused by the option value of investment. It is worth noting that $fimp = 0.94$ in the steady state with $\sigma_n = 0$.

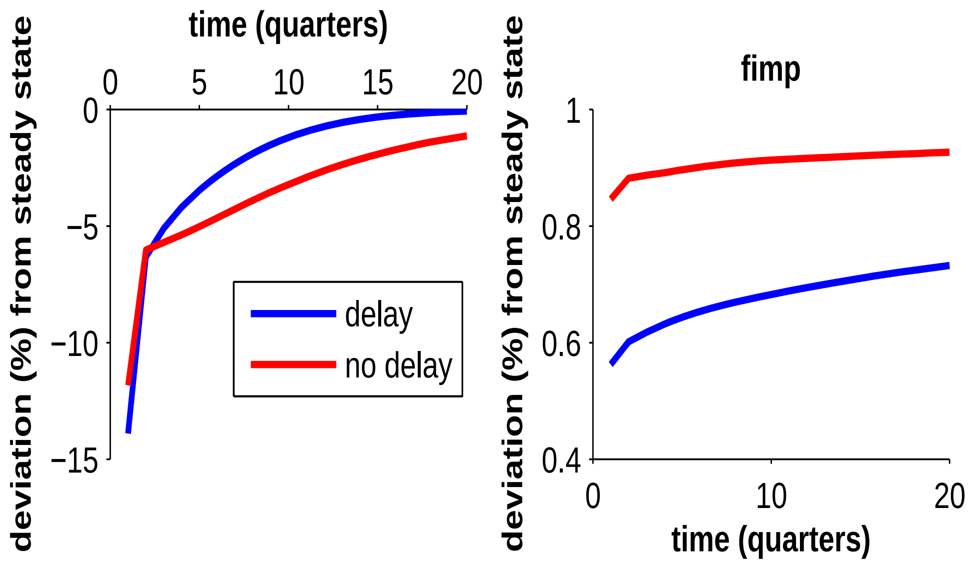

Figure 4a. Delay versus no delay: investment x and the ratio $fimp$ with equity purchases after a liquidity shock

We compare equilibrium responses to financial shocks in the baseline economy and the economy with $\sigma_n = 0$. To economize on space, we present the results for only the liquidity shock and, when interventions are introduced, only equity purchases. The responses of the equilibrium are much smaller with debt purchases or the shock to μ. The left panel in Figure 4a depicts investment expenditure in the two economies after a negative liquidity shock, where the government purchases equity, and the right panel depicts the ratio $fimp$. Even with the large equity purchase, there is still significant delay in investment, because the ratio $fimp$ in the economy with delay is sixteen to twenty-nine percentage points below that in the economy without delay. This large delay indicates that the financing constraint is binding severely even with equity purchases. Moreover, delay plays a quantitatively important role in the response of the equilibrium to the liquidity shock. Relative to the economy without delay, investment expenditure in the economy with delay falls by more in period 1 and recovers more quickly in subsequent periods. After period 3, investment expenditure in the economy with delay is significantly above that in the economy without delay.

These effects of delay on investment dynamics are intuitive. Delay increases the initial reduction in investment because entrepreneurs postpone part of the investment until the financing constraint becomes less tight. Delay induces investment to recover more quickly in subsequent periods because investment expenditure is complementary with the stock of projects in the investment technology. By increasing the stock of projects in the next period, delay increases the marginal productivity of investment expenditure in the future, thus encouraging future investment to rise more quickly toward the steady state. The right panel in Figure 4a confirms this intuition. The ratio $fimp$ is smaller in the economy with delay than in the economy without delay. Moreover, the ratio $fimp$ falls by a larger amount in period 1 and increases more quickly in subsequent periods with delay than without delay.

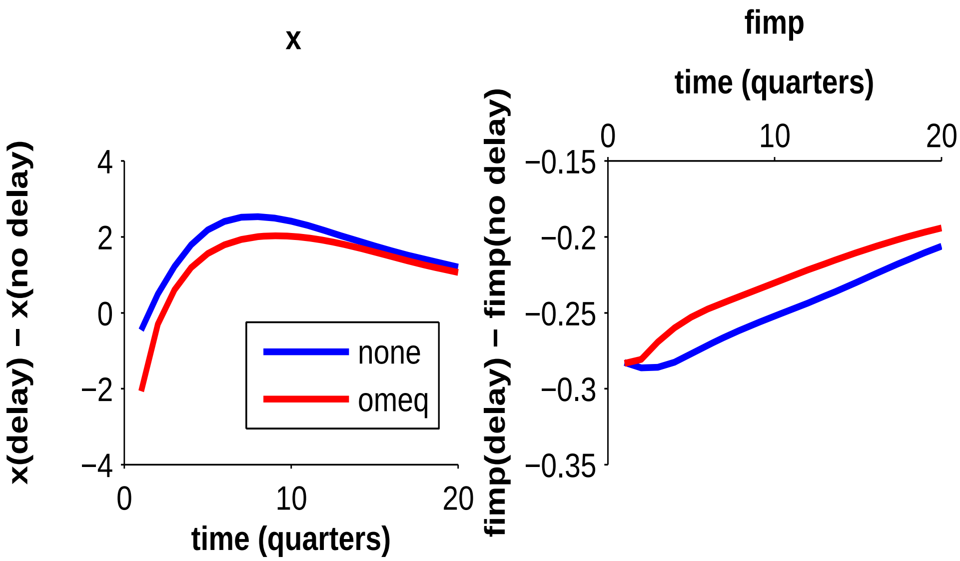

Figure 4b. Net effects of delay with equity purchases and no interventions: investment expenditure x and the ratio $fimp$ after a liquidity shock

Investment delay affects equilibrium responses to the liquidity shock both with and without the interventions in the asset market. To see the net effect of delay with interventions, we subtract equilibrium responses without delay from the ones with delay. This difference is computed separately with equity purchases and with no interventions, and then the differences in the two cases are put in the same figure. The left panel in Figure 4b depicts this difference in investment expenditure and the right panel in the ratio $fimp$. In both cases, the difference in investment expenditure between delay and no delay is negative in period 1 and positive from period 3 onward. This means that investment expenditure falls by more in period 1 and recovers more quickly in subsequent periods with delay than without delay, regardless of whether there are equity purchases. In comparison with the case of no interventions, delay generates the following differences in the quantitative responses: (i) Investment expenditure falls by more in period 1 and recovers by a larger amount in period 2; (ii) Investment expenditure recovers less quickly from period 3 onward; (iii) There is less delay from period 2 onward, as depicted by the right panel in Figure 4b. Taken together, these differences suggest that the option to delay reduces the effectiveness of equity purchases in counteracting the negative liquidity shock.

We explain the above effects of delay as follows. First, delay reduces investment expenditure in period 1 by more with equity purchase than with no interventions because the tax in period 1 is assumed to be fixed with equity purchases. As explained before, this difference in the tax in period 1 implies that the negative liquidity constraint tightens the financing constraint to a greater extent in the first period when the government purchases equity purchases. As a result, entrepreneurs curtail investment by a larger amount given the option to delay. Most of this difference in the tax between equity purchases and no interventions disappears from period 2 onward. This explains why investment expenditure recovers by a larger amount in period 2 in the case of equity purchases. Second, from period 2 onward, the main difference between the case of equity purchases and the case of no interventions lies in how quickly the financing constraint is relaxed over time as equity liquidity increases back to the steady state. In the case of equity purchases, the government gradually unloads private equity purchased in period 1, by selling equity for government bonds. Because this operation extracts part of liquid assets from the market, the financing constraint is relaxed relatively slowly over time. The benefit to delay is smaller in this case, since the option value of delay is to undertake investment at a future time when the financing constraint is less tight. This explains why there is less delay from period 2 onward in the case of equity purchases than in the case of no interventions. Third, less delay means that the stock of projects next period will be lower in the case of equity purchases. Because the stock of projects is complementary with investment expenditure, a lower stock of projects next period will reduce the marginal productivity of investment expenditure next period, thus inducing future investment expenditure to rise less quickly.

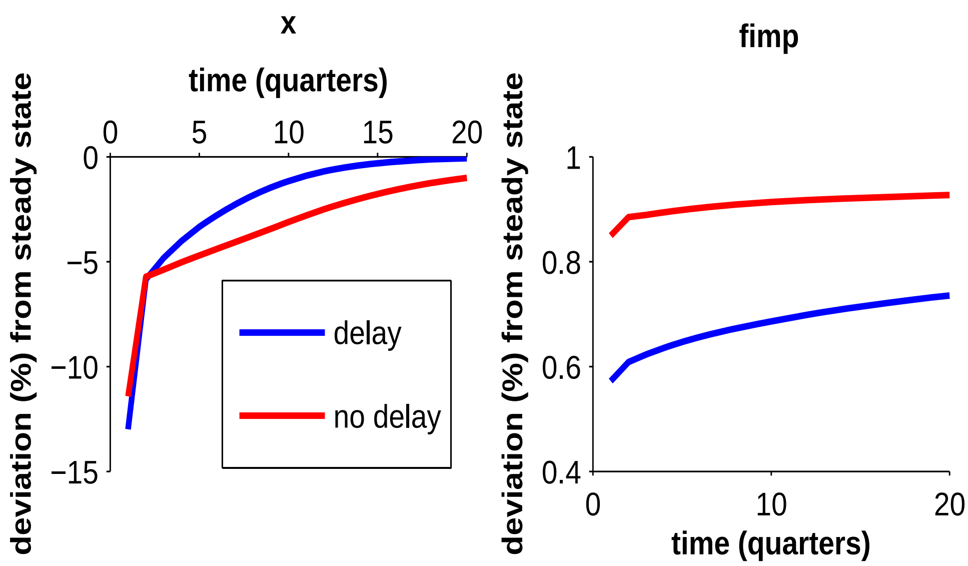

Figure 5. Delay versus no delay with $\tau 1_d = 1$: investment x and the ratio $fimp$ with equity purchases after a liquidity shock

So far we have assumed $\tau 1_d = 0$ so that the tax in period 1 is fixed at the steady state level when the government purchases private assets. To see whether the main quantitative results above depend on this particular assumption, let us change the assumption to $\tau 1_d = 1$. Under this alternative assumption, the tax in period 1 can change with the shock under government interventions as under no intervention. Figure 5 depicts the responses of investment expenditure and the ratio $fimp$ to the negative liquidity shock in the economy with delay and in the economy without delay. The two panels in Figure 5 are similar to their counterparts in Figure 4a. The only discernible difference is that the initial fall in investment expenditure with equity purchases is smaller in Figure 5 than in Figure 4a. Thus, the assumption $\tau 1_d = 0$ is not critical for the main results on the effect of government interventions and the role of investment delay.

6. Conclusion

We construct a dynamic macro model to incorporate financial frictions and investment delay. Investment is undertaken by entrepreneurs who face liquidity frictions in the equity market and a collateral constraint in the debt market. After calibrating the model to the US data, we find that shocks to equity liquidity can affect aggregate activity significantly and, in particular, can induce positive comovement among investment, employment, output and aggregate consumption. In contrast, a shock to entrepreneurs' borrowing capacity has only small effects on aggregate activity. After such shocks, if the government intervenes in the asset market by using its liquid assets to buy private assets, the effectiveness of the intervention depends on the type of private assets that the government buys. Equity purchases can speed up the recovery of the economy significantly, although they exacerbate the negative effect of the shock in the first period of the intervention. In contrast, debt purchases have only very small effects. Moreover, the option to delay investment reduces the effectiveness of government intervention through equity purchases.

The findings in this paper provide guidance to future research on the macro importance of financial frictions. First, it is important to build a microfoundation for the frictions in equity liquidity and the cause of their fluctuations. In this paper, we followed KM to parameterize equity liquidity by ϕ and assumed that ϕ follows an AR(1) process. Given the finding that equity liquidity can cause large fluctuations in aggregate activity, a microfoundation for such liquidity frictions will enhance the understanding of where these frictions come from and why they vary over time. It may also help resolving the puzzling response of equity price to changes in asset liquidity in this class of models (see Shi, 2012). Second, it is useful to conduct welfare analysis of asset market interventions. In this paper, we focused on the effects of asset market interventions on aggregate activity. The interventions also redistribute wealth from unconstrained individuals in the model (workers) to constrained individuals (entrepreneurs). The structure of a representative household is a convenient device for a welfare evaluation of the interventions. However, we refrained from evaluating the welfare consequence of the interventions because such an evaluation would be more credible if the primary frictions in the model were microfounded rather than being exogenously imposed. With a strong microfoundation of these frictions, the welfare analysis of the interventions will provide theoretical guidance to the policy debate on how often and how large the interventions should be carried out.

Appendix

A. The Non-stochastic Steady State and Calibration

A.1. Determining the non-stochastic steady state

In the deterministic state, (3.20) implies $i^* = (1-\sigma_k)K^*/\pi$ and $n^* = \frac{\pi\,\sigma_n}{1-\sigma_n}(a - m^*)$, where $i^* = \gamma m(x^*, \frac{n^*}{\pi} + a)$. These results solve for unique (n^*, x^*, i^*, m^*) for any given $K^* > 0$. Denote these solutions as $n^* = n(K^*)$, $x^* = x(K^*)$, $i^* = i(K^*)$ and $m^* = i(K^*)/\gamma$. Next, we express $(r^*, \lambda^{e*}, q^*, p_b^*)$ as functions of (K^*, ℓ^*). The optimization by the producers of consumption goods yields $r^* = r(K^*, \ell^*)$, where $r(K, \ell) = F_1(K, (1-\pi)\ell)$. In the steady state, (3.13) yields:

$$p_n^* = \frac{\gamma m_2^*(1+\theta_z^*\lambda^{e*})q^*}{1-\beta\sigma_n(1-m_2^*)}, \tag{A.1}$$

where $\theta_z^* = \theta + (1-\theta)z^*p_d^*$ and $z^* = \phi^*\mu^*$. Substituting the solution into (3.12) yields:

$$q^* = \frac{1+\lambda^{e*}}{1+\theta_z^*\lambda^{e*}}\left(1 + \frac{\beta\sigma_n m_2^*}{1-\beta\sigma_n}\right)\frac{1}{\gamma m_1^*} \tag{A.2}$$

The asset pricing equations, (3.8)-(3.10), imply:

$$q^* = \frac{\beta(1+\pi\lambda^{e*})}{1-\beta\sigma_k(1+\pi\phi_z^*\lambda^{e*})}r^* \tag{A.3}$$

$$p_b^* = p_d^* = \beta(1+\pi\lambda^{e*}), \tag{A.4}$$

where $\phi_z^* = \phi + (1-\phi)z^*p_d^*$. With (A.4), we express $\phi_z^* = \phi_z(\lambda^{e*})$ and $\theta_z^* = \theta_z(\lambda^{e*})$. Substituting these functions and (n^*, x^*, i^*, m^*) as functions of K^*, (A.2) and (A.3) become equations involving only the variables $(q^*, \lambda^{e*}, K^*, \ell^*)$. Thus, we can use these equations to solve (q^*, λ^{e*}) as $q^* = q(K^*, \ell^*)$ and $\lambda^{e*} = \lambda^e(K^*, \ell^*)$.[19] With these solutions, we express $(p_n^*, p_d^*, \phi_z^*, \theta_z^*)$ as functions of (K^*, ℓ^*).

[19] For arbitrarily given (K^*, ℓ^*), these equations can have one or two pairs of solutions for (q^*, λ^{e*}). However, if there are two pairs, only the one with the larger value of q^* leads to admissible solutions for (K^*, ℓ^*) below.

To solve (K^*, ℓ^*), we use the steady-state version of (2.5) to solve τ^*. Substituting this solution, we write the steady-state version of (2.5) and (3.3), and (3.7) as

$$\begin{aligned} c^{e*} = & \quad [r^* + \phi_z^* \sigma_k q^*] K^* - (x^* - \theta_z^* q^* i^*) - g \\ & \quad + p_b^* B^* - p_d^* d^{g*} - [1 - \sigma_k(1 - \phi_z^*)] q^* s^{g*} \\ u'(c^{e*}) = & \quad (1 + \lambda^{e*}) U'(c^{w*}) \end{aligned}$$

where $r^* = F_1(K^*, (1-\pi)\ell^*)$. To obtain the first equation above, we have used the fact that $s^* = K^* - s^{g*}$, $b^* = B^*$ and $d^* = d^{g*}$. Substituting τ^* and the solutions for $(i^*, q^*, \lambda^{e*}, p_b^*, p_d^*, \phi_z^*, \theta_z^*)$, we use the two equations above to solve (c^{e*}, c^{w*}) as functions of (K^*, ℓ^*). Then, $c^* = \pi c^{e*} + (1-\pi) c^{w*}$ can be expressed as a function of (K^*, ℓ^*). Finally, in the steady state, (3.6) and (3.17) become:

$$F_2(K^*, (1-\pi)\ell^*) = \frac{h'(\ell^*)}{U'(c^{w*})}$$

$$F(K^*, (1-\pi)\ell^*) = c^* + g + \pi x^*.$$

These equations determine (K^*, ℓ^*).

A.2. Calibration procedure

We use the targets listed in Table 1 and the steady state characterized above to identify the parameters. The values β, ρ, σ_ϕ, σ_μ, α and π are determined directly either as their exogenously chosen values or their targets as explained in the main text. The other parameters are identified as follows:

Part 1. The parameters $(\eta, \sigma_k, g^*, B^*)$. Because the elasticity of labor supply is $\frac{1}{\eta-1}$, the target on this elasticity solves η. In the steady state, the replacement of capital in a period is $\pi i^* = (1-\sigma_k)K^*$. The ratio of annual replacement to capital is $\frac{4\pi i^*}{K^*} = 4(1-\sigma_k)$. Equating this to the target, 0.076, yields σ_k. The target on total hours of work in the steady state requires $(1-\pi)\ell^* = 0.25$, and the target on the ratio of capital to annual output requires $\frac{K^*}{4F^*} = 3.32$. These two requirements solve (ℓ^*, K^*). Then, we can solve $\gamma m^* = i^* = (1-\sigma_k)K^*/\pi$, $F^* = F(K^*, (1-\pi)\ell^*)$, $r^* = F_1$, and $w^* = F_2$. Subsequently, the target on the ratio of government spending to GDP solves $g^* = 0.18F^*$, the target on the ratio of private consumption to GDP solves $c^* = 0.7F^*$, and the goods market clearing condition solves $x^* = (F^* - c^* - g^*)/\pi$. The target on the annualized net rate of return to liquid assets requires $\frac{1}{(p_b^*)^4} - 1 = 0.02$, which solves for p_b^* and, hence, p_d^*. Substitution into (A.4) solves for λ^{e^*}. The target on the annual equity premium requires $(\frac{r^*}{q^*} + \sigma_k)^4 - (\frac{1}{p_b})^4 = 0.02$. Because r^* and σ_k are already solved, this requirement solves for q^*. Using the target on the share of liquid assets gives $\frac{p_b^* B^*}{p_b^* B^* + q^* K^*} = 0.12$, which solves for B^* and, hence, $\tau^* = g^* - (p_b^* - 1)B^*$. Notice that the solutions of $(\ell^*, K^*, c^*, p_b^*, q^*)$ used five targets but have not led to the solution for any parameter yet. We will utilize the solutions of these five variables to identify five parameters in the parts below.

Part 2. The parameters $(\phi^*, \theta^*, \mu^*, u_0, h_0)$. We use three targets to solve for (ϕ^*, θ^*, z^*) and, hence, μ^*. The first target is the annual equity premium, which is equal to $(\frac{r^*}{q^*} + \sigma_k)^4 - (p_b^*)^{-4}$ in the steady state of the model. We already used this target to solve for q^*. Using this solution for q^* in (A.3) solves

$$\phi_z^* = \frac{1}{\pi \lambda^{e*}} \left(\frac{1}{\beta \sigma_k} - 1 \right) - \frac{r^*}{\sigma_k q^*} \left(\frac{1}{\pi \lambda^{e*}} + 1 \right). \tag{A.5}$$

The second target is the ratio of funds raised in the market to fixed investment expenditure, which is 0.284 as estimated by Nezafat and Slavik (2010). In our model, the value of new equity issuance is $q\theta i$ and the amount borrowed in the market is $p_d z q s_{+1}^e$. Thus, the target on the ratio of funds raised in the market to fixed investment expenditure in the steady state requires

$$\frac{q^*}{x^*} \left[\theta i^* + p_d^* z^* s^{e*} \right] = 0.284. \tag{A.6}$$

The third target for identifying (ϕ^*, θ^*, z^*) is the ratio of debt issuance to stock sales, which requires $\frac{p_d^* z^* s^{e*}}{\theta i^*} = 1.287$. Using this requirement to express $p_d^* z^* s^{e*} = 1.287 \theta i^*$ and substituting into (A.6), we solve $\theta = \frac{0.284 x^*}{2.287 q^* i^*}$. Using $s^{e*} = (1-\theta) i^* + (1-\phi^*) \sigma_k K^*$, we have

$$z^* = \frac{1.287 \theta i^*}{p_d^* \left[(1-\theta) i^* + (1-\phi^*) \sigma_k K^* \right]}. \tag{A.7}$$

This is a relationship between z^* and ϕ^*. Substituting the definition of ϕ_z^* into (A.5) yields another relationship between z^* and ϕ^*. The two relationships together solve for (ϕ^*, z^*).

Then, the definition $z^* = \phi^* \mu^*$ leads to $\mu^* = z^*/\phi^*$, and (ϕ_z^*, θ_z^*) can be calculated from their definitions.

Substituting the solved parameters and steady state variables into the steady state version of (3.3), we solve for c^{e^*}. The definition of c^* yields $c^{w^*} = (c^* - \pi c^{e^*})/(1-\pi)$. Substitution into the steady state versions of (3.7) and (3.6) solves for u_0 and h_0.

Notice that the second target and the third target above are new ones in this part. The equity premium is not a new target because it was already used in Part 1. With the two new targets, we identified five new parameters. This was achieved by using the variables $(\ell^*, K^*, c^*, p_b^*, q^*)$ that were calculated in Part 1 with additional targets. We still have two restrictions from this list to be utilized below.

Part 3. The parameters $(\sigma_n, \xi, \delta, \gamma, a)$. We utilize the target on $fimp^*$, the exogenously set value σ_n, and the normalization of $\frac{n^*}{\pi} + a$. The target $fimp^* = 0.78$ is the ratio of investment in the steady state to that in the hypothetical economy where the financing constraint does not bind. Rewrite (3.16) as

$$\frac{m_1}{m_1^f} = 1 + \lambda^e (1 - \theta_z q \gamma m_1). \tag{A.8}$$

Using the functional form of m to derive γm_1 and substituting $\gamma = i/m$, we get $m^\xi = \frac{i\,\delta^\xi}{2\gamma m_1} x^{\xi-1}$. Inverting the m function, we get:

$$(\delta x)^\xi = 2m^\xi - (\frac{n}{\pi} + a)^\xi, \quad (\delta x^f)^\xi = 2(fimp^{-\xi}) m^\xi - (\frac{n}{\pi} + a)^\xi.$$

Subtracting these two equations, substituting m^ξ just derived, and re-arranging, we get: $(x^f)^\xi - x^\xi = \left(fimp^{-\xi} - 1\right) \frac{i}{\gamma m_1} x^{\xi-1}$. That is,

$$\left(\frac{x^f}{x}\right)^\xi = 1 + \left(fimp^{-\xi} - 1\right) \frac{i/x}{\gamma m_1}.$$

Dividing the formula of m_1 by m_1^f, and substituting x^f/x from above, we get:

$$\frac{m_1}{m_1^f} = \left[\left(1 - fimp^\xi\right) \frac{i/x}{\gamma m_1} + fimp^\xi\right]^{(1-\xi)/\xi}. \tag{A.9}$$

Equating the two expressions for m_1/m_1^f in (A.8) and (A.9) yields:

$$\gamma m_1 = \frac{1}{\lambda^e q\theta_z}\left\{1+\lambda^e - \left[\left(1-fimp^\xi\right)\frac{i/x}{\gamma m_1} + fimp^\xi\right]^{(1-\xi)/\xi}\right\}. \tag{A.10}$$

The target on the fraction of implemented projects is $fimp^* = 0.75$. With this target, the steady state version of (A.10) involves only two unknowns, $(\gamma m_1^*, \xi)$. In addition, the steady state equation, (A.2), can be rewritten as

$$\sigma_n = \frac{(sxc)\gamma m_1^* - 1}{\beta[(sxc)\gamma m_1^* - 1 + m_2^*]}, \text{ where } sxc \equiv q^*\frac{1+\theta_z^*\lambda^{e*}}{1+\lambda^{e*}}. \tag{A.11}$$

This equation involves only three unknowns, $(\sigma_n, \gamma m_1^*, m_2^*)$. Moreover, m_2^* can be expressed as a function of $(\gamma m_1^*, \xi)$, as shown below.

For any exogenously set value of $\sigma_n > 0$, we can solve $(\gamma m_1^*, \xi)$ from (A.11) and the steady-state version of (A.10). It is convenient to reverse this process. We choose a value of ξ, set the target $fimp^* = 0.78$, and use the steady-state version of (A.10) to solve γm_1^*. Using this solution and the target $\frac{n^*}{\pi} + a = 1$, we solve $(\gamma m_2^*, m^*)$ as

$$\gamma m_2^* = \frac{i^* - x^*\gamma m_1^*}{\frac{n^*}{\pi} + a}, \quad m^* = \left[\frac{i^*}{2\gamma m_2^*}\left(\frac{n^*}{\pi} + a\right)^{\xi-1}\right]^{1/\xi}.$$

The first equation comes from the feature that m has constant returns to scale, and the second equation from differentiating m with respect to $\frac{n}{\pi} + a$ and substituting $\gamma = i^*/m^*$. Once $(\gamma m_1^*, \gamma m_2^*, m)$ are solved, we can further solve:

$$\gamma = \frac{i^*}{m^*},\ m_1^* = \frac{\gamma m_1^*}{\gamma},\ m_2^* = \frac{\gamma m_2^*}{\gamma},\ \delta = \frac{1}{x^*}\left[2(m^*)^\xi - \left(\frac{n^*}{\pi} + a\right)^\xi\right]^{1/\xi}.$$

The last equation comes from inverting the m function. Substituting the solutions for $(\gamma m_1^*, m_2^*)$ into (A.11) yields a value of σ_n. If this value of σ_n differs from the intended one, adjust the value of ξ and repeat the process until the intended value of σ_n is achieved.

With the solution of m^* and the normalized value of $\frac{n^*}{\pi} + a$, we obtain:

$$n^* = \sigma_n\pi\left(\frac{n^*}{\pi} + a - m^*\right), \quad a = \left(\frac{n^*}{\pi} + a\right) - \frac{n^*}{\pi}.$$

Notice that this part used three new targets, $(fimp^*, \sigma_n, \frac{n^*}{\pi} + a)$, but identified five parameters, $(\sigma_n, \xi, \delta, \gamma, a)$. The two additional restrictions used in this part are the two that remained in the list $(\ell^*, K^*, c^*, p_b^*, q^*)$ after Part 2.

In the special case $\sigma_n = 0$, we do not use the target $fimp^* = 0.78$, its associated equation, (A.10), or the normalization $\frac{n^*}{\pi} + a = 1$. Instead, we set ξ and a at their values identified in the case $\sigma_n > 0$ (see section 5.3 for the explanation). Then, (A.11) solves $\gamma m_1^* = 1/(sxc)$. The other variables and parameters, especially $(\gamma m_2^*, m^*, \gamma, m_1^*, m_2^*, \delta)$, are solved in the same way as above.

B. Computing the Dynamic Equilibrium and Responses

We have assumed that government policies, $(B_{+1}, s_{+1}^g, d_{+1}^g)$, are only functions of (q, p_b, p_d, A), where $A = (K, N, \phi, \mu)$ is the aggregate state. After substituting these policies, the value function v, the policy functions and price functions can be expressed as functions of A in the equilibrium. We compute these functions and simulate the responses of the equilibrium to the shocks. Because the shocks are sizable, we do not linearize the equilibrium around the steady state.

Part 1. Computing equilibrium functions.

The computation iterates on four functions, $(q, p_b, p_n, x)(A)$, where p_n and x are added to the list for convenience. To do so, we substitute all other variables. Note that $p_d = p_b$, and so (ϕ_z, θ_z) defined in (3.4) are functions of only (p_b, ϕ). We use (3.12) to solve:

$$\lambda^e = \frac{q - p_n/\gamma - (1 - m_2)/(\gamma m_1)}{(1 - m_2)/(\gamma m_1) - \theta_z q}. \tag{B.1}$$

This is a function of $(N, \phi, \mu, q, p_n, x)$. With this, we use (3.7), (3.6) and $w = F_2(K, (1-\pi)\ell)$ to solve (c^e, ℓ, w) as functions of (c^w, A, q, p_n, x). Then, (r, Y, c) can be written as functions of (c^w, A, q, p_n, x) by using the following equilibrium relationships:

$$r = F_1(K, (1 - \pi)\ell),\ Y = F(K, (1 - \pi)\ell),\ c = \pi c^e + (1 - \pi)c^w.$$

Substituting (Y, c, x) into the clearing condition of the goods market, (3.17), we solve c^w as a function of (A, q, p_n, x) and recover $(\lambda^e, c^e, \ell, w, r, Y, c)$ as functions of (A, q, p_n, x).

Next, we construct equilibrium mappings on the functions, $(q, p_b, p_n, x)(A)$. Substituting τ from (2.5) into (3.3) and using $s = K - s^g$, $b = B$ and $d = d^g$, we obtain:

$$\begin{aligned} x = \;& (r + \phi_z \sigma_k q)K + [\sigma_k(1-\phi_z)s^g - s^g_{+1}]q \\ & -c^e + \theta_z q i - g + p_b B_{+1} - p_d d^g_{+1}. \end{aligned} \tag{B.2}$$

Given government interventions specified in section 5.1, especially (5.3), we can write this expression for x further as

$$x = (r + \phi_z \sigma_k q)K - c^e + \theta_z q i - g + \tilde{p}_b B^* + \sigma_k (1-\phi_z) q s^g. \tag{B.3}$$

This depends on s^g. Rather than adding s^g as a state variable, we express s^g as a function of the current state A. This is trivial if government interventions are purchases on private debt, in which case $s^g = 0$. When government purchases are on private equity, $s^g_t = \psi_s K^*(1 - \frac{z_{t-1}}{z^*})$ for all $t \geq 2$ and $s^g_1 = 0$. Because there are no new shocks in any period $t \geq 2$ (i.e., $\varepsilon_{\phi,t} = \varepsilon_{\mu,t} = 0$ for all $t \geq 2$), we can use (4.1) to get the following relationships for all $t \geq 2$:

$$\begin{aligned} &\log \tilde{\phi}_{t-1} = \log \tilde{\phi}_t - (\tfrac{1}{\sigma_\phi} - 1) \log \tilde{\phi}^* \\ &\log \tilde{\mu}_{t-1} = \log \mu_t - (\tfrac{1}{\sigma_\mu} - 1) \log \tilde{\mu}^*. \end{aligned}$$

Using these relationships, we can express ϕ_{t-1} as a function of ϕ_t and μ_{t-1} as a function of μ_t for all $t \geq 2$. Then, we can express z_{t-1} as a function of (ϕ_t, μ_t) for all $t \geq 2$. Denote this function as $z_{t-1} = \zeta(\phi_t, \mu_t)$ for some function ζ. For $t = 1$, we have $s^g_1 = 0$. Note that $K_1 = K^*$. Let $\chi_{(K=K^*)} = 1$ if $K = K^*$, and $\chi_{(K=K^*)} = 0$ otherwise. We put the case $t = 1$ and the case $t \geq 2$ together as

$$s^g_t = \psi_s K^* \left[1 - \frac{\zeta(\phi_t, \mu_t)}{z^*}\right] \chi_{(K_t = K^*)} \text{ for all } t \geq 1. \tag{B.4}$$

This expresses s^g as a function of (K, ϕ, μ).

Now, the right-hand sides of (3.8), (3.9), (3.13) and (B.3) form the mapping on the functions $(q, p_b, p_n, x)(A)$. Denote this mapping as $T : C_1 \to C_1$, where C_1 is the set of all continuous functions that map $\mathcal{A}$ into $\mathbb{R}_+$. We iterate on the mapping T on a discretized state space of A until convergence. The result is a fixed point of T for the functions (q, p_b, p_n, x) on the discretized state space. We then use Chebyshev projection to approximate the functions (q, p_b, p_n, x) on the entire state space. Other equilibrium functions can be recovered accordingly.

Part 2. Simulate dynamic responses to shocks

Once a shock is realized at the beginning of period 1, ϕ_1 and μ_1 are known. The dynamics of A are described by (3.20) and (4.1). We simulate the equilibrium forward as follows. For any t, starting with $t = 1$, compute $A_t = (K_t, N_t, \phi_t, \mu_t)$ and evaluate the functions (q, p_b, p_n, x) to obtain $(q_t, p_{b,t}, p_{n,t}, x_t)$. Obtain other equilibrium functions accordingly. Then, use (3.20) and (4.1) to compute A_{t+1}. Repeat this process for as many periods as desirable.

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