Environmental bonds and public liability for resource extraction site cleanup

Margaret Insley[1]

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Abstract

Governments have been left with large liabilities for cleanup at natural resource extraction sites after firms have declared bankruptcy. This research studies the impact of different forms of financial assurance on a firm's optimal actions over the full life cycle of a hypothetical natural gas well, in a world of uncertain natural gas prices, when firm bankruptcy may shift cleanup costs to the government. A firm's stochastic optimal control problem is described by an HJB equation with the natural gas price modelled as a stochastic differential equation. The impact of financial assurance is examined in relation to firm investment decisions, the cleanup liability imposed on government and resource taxation revenue. A Cash Deposit and Surety Bond are contrasted with the case of no financial assurance requirement. The "fair" fee (assuming the absence of arbitrage opportunities) is determined for the Surety Bond issued by a third party. Numerical results demonstrate that in the presence of distortionary taxes, there is a trade off between indemnifying the government against cleanup costs versus maintaining government tax and royalty revenues. A numerically plausible case is presented in which the total value of the natural gas well (to the firm and the government) is not increased by the imposition of a strict form of financial surety.

Key Words: Optimal investment decisions, uncertainty, financial assurance, cleanup liability, natural gas, resource taxation revenue
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[1]Department of Economics and The Water Institute, University of Waterloo, Waterloo, Ontario, Canada

1 Introduction

Firms granted licenses to extract natural resources are, in most jurisdictions, legally obligated to protect public health and the environment during the extraction process and to cleanup the site once the project is complete, restoring the land to its previous condition. Some type of financial surety is typically required to encourage post-project site cleanup, but that surety is often inadequate resulting in the proliferation of improperly abandoned resource extraction sites in evidence across North America. For example, there are hundreds of thousands of orphaned and abandoned mine sites in the US [44] and tens of thousands in Canada [9], many posing serious health and safety hazards and leaching contaminants into groundwater systems. Former owners may have avoided cleanup costs by declaring bankruptcy, leaving the government on the hook for clean up costs of billions of dollars [44, 33, 9, 8].

The focus of this paper is on improperly abandoned oil and gas wells. The term "abandoned well" typically refers to wells with no recent production, which may or may not have been plugged.[2] A subset of abandoned wells are orphaned wells, meaning they have no responsible operator. In the U.S., unplugged abandoned oil and gas wells were estimated at 2.1 million in 2016 [43], while orphaned oil and gas wells number in the hundreds of thousands.[3] In Canada, orphaned and abandoned wells in Alberta and Saskatchewan have increased exponentially from approximately 1700 in 2007 to 17,543 in 2020 and cleanup costs are expected to reach C$1.1 billion by 2025 [35]. If improperly plugged, abandoned wells may contaminate water supplies and degrade ecosystems [27]. Unreclaimed well sites are a nuisance for landowners, preventing them from full enjoyment of their property.

A variety of financial surety instruments are used throughout the world to ensure proper closure and cleanup of resource extraction projects.[4] Surety bonds are used extensively in the U.S. for oil and gas wells drilled on federal government lands and lands under state jurisdiction. The cost to an oil and gas firm to obtain a bond depends on the credit worthiness of the firm and is typically an annual fee of three to five percent of the bond amount.[5] In the event that an oil or gas firm fails to meet its obligations to properly close a well, the guarantor would pay the regulator the bond amount; however, the guarantor is able to pursue the oil and gas company in court for the bond payout as well as court costs and fees. [19] note that in spite of bonding requirements, a substantial inventory of orphaned wells has accumulated "as complaints of neglected wells are reported, legacy wells are located, or companies become defunct" (page 3909). In Alberta, Canada, the Alberta Energy Regulator collects a levy from all well operators which goes into a fund used for the cleanup of wells held by bankrupt companies. In addition, firms may be required to provide a bond or deposit

[2] Definitions vary by jurisdiction. In Alberta, Canada, the term abandoned well refers to a well that has been plugged and properly closed.

[3] Documented orphan wells are reported as 118k in [30] and undocumented orphan wells estimated at 3k to 8k by [26].

[4] [37] surveys financial surety instruments for the mining sector including self bonding, insurance policies, letters of credit, bank guarantees, Surety Bonds, Cash Deposit and trust funds.

[5] As reported on the website of Higginbotham Insurance Company. `https://www.higginbotham.com/blog/oil-and-gas-bonds/`.

depending on an assessment of their financial strength.

Cleanup costs are generally well-known for conventional oil and gas wells. Hence governments could in theory impose financial assurance requirements that guarantee firms do not renege on paying these costs. However, there is a reluctance to do so as is evidenced by the large cleanup liabilities that have accumulated. The desire to hold firms accountable for environmental damages and site cleanup competes with other government objectives such as the encouragement of resource development and maintaining good relations with resource extraction firms. Governments have faced significant negative public relations as media attention focused on the large cleanup liabilities left to taxpayers by firms not meeting their cleanup obligations.[6]

In both Canada and the U.S., regulatory changes in the past five years have sought to tighten up financial assurance requirements for oil and gas companies. The Alberta government introduced in 2020 a new liability management framework intended to strengthen the assessment of firms' ability to meet their cleanup obligations and to motivate firms to more rapidly reduce their inventory of inactive wells [5, 11]. The US Bureau of Land Management recently (as of June 22, 2024) increased minimum bond amounts with the minimum lease bond rising to US$150,000 and statewide bonds to US$500,000.[7]

This purpose of this research is to examine the effect of financial surety requirements for oil and gas firms in terms of the benefits and costs for both the firm and the government in the presence of distortionary taxes. It is well known in the economics literature that in the absence of externalities, the imposition of taxes, other than a pure resource rent tax, distorts firm decisions away from the efficient choices [6, 40]. This implies that the desirability of financial insurance instruments must be examined in terms of their impact on firm behaviour and the consequences for government tax revenues.

In this paper we will examine the desirability of imposing strict financial surety requirements, in the form of a Cash Deposit or a Surety Bond, versus imposing no surety requirements at all. Both the Surety Bond and the Cash Deposit can fully indemnify the government against cleanup costs, but may push the firm's decisions significantly away from the efficient extraction path. We develop a model of the optimal decisions of a firm with a license to drill a natural gas well and produce from that well over a 50 year lease period. For simplicity it is assumed that the license to drill the gas well is the firm's only asset. Natural gas prices are modelled as a stochastic differential equation following a mean reverting process. The firm is assumed to declare bankruptcy if it is optimal to do so - i.e. if the value of the lease is negative. It is further assumed that there is no residual value for a bankrupt firm, implying that the firm avoids paying any well cleanup costs. The firm chooses when (and if) to drill a gas well, the timing of production (balancing active production with temporary shut-in periods), and when to close the well and cleanup the site or declare bankruptcy. The value of the license is described by a Hamilton-Jacobi-Bellman (HJB) equation.

[6]See for example [32] and [18].

[7]The change is described on the website of the U.S. Department of the Interior, Bureau of Land Management, https://www.blm.gov/programs/energy-and-minerals/oil-and-gas/leasing/bonding, accessed July 8, 2024.

We are interested in examining project value from the perspectives of the firm and the government, with the latter consisting of tax and royalty revenues less any cleanup costs left to government. We begin the analysis with some propositions to demonstrate that given the model's assumptions, it is not possible *a priori* to conclude whether total value (value to the firm plus value to the government) is higher or lower under the Cash Deposit compared to the case of No Financial Surety.

Further insight is obtained by implementing a numerical solution of the relevant HJB equation for a particular example. The HJB equation is solved using a standard fully implicit semi-Lagrangian approach [15] to determine the firm's optimal controls for the case of a hypothetical natural gas well in Alberta, Canada. The optimal controls are then used in a Monte Carlo simulation to determine the expected value of the project over time to the firm in terms of net profits and to the government in terms of taxes and royalties received less any cleanup costs incurred.

The numerical solution is implemented for the Cash Deposit and No Financial Surety cases. We then extend the analysis to consider financial surety in the form of a Surety Bond issued to the firm by a third party for an annual fee. In this case, the third party pays the cost of cleanup should the firm renege on its obligations. It is assumed that markets are sufficiently complete so that the third party can hedge the risk of issuing the bond. We determine the fee that would need to be charged by the third party to just break even, assuming the absence of arbitrage opportunities. We refer to this as the "fair" fee. We contrast the results for the Surety Bond, with the Cash Deposit, and No Surety cases in terms of the the firm's optimal decisions, expected value of the firm, expected value of government net cash flows, and the total expected value of the lease.

To preview the results, it is found that in the presence of taxes the desirability of the Cash Deposit or Surety Bond in terms of their impact on total project value depends on how these surety instruments affect the firm's behaviour regarding whether or not a well is drilled and when a well is closed or bankruptcy is declared. The Cash Deposit and Surety Bond may reduce government receipts from taxes and royalties by more than the well cleanup costs. In the numerical example considered in the paper, if cleanup costs remain constant in real terms over time, both the government and the firm are better off when No Financial Surety is required, leaving the government to pay for the cleanup out of income taxes and royalties received. In contrast, if cleanup costs rise steeply later in the lease period, total project value is increased by imposing a imposing financial surety.

These conclusions must be tempered by the limitations of the assumed economic model. Further analysis at industry level and considering general equilibrium effects are warranted. Nevertheless, these results highlight the importance of examining the effects of financial surety using a holistic approach, considering the potential for unintended consequences.

2 Literature

There has been significant recent attention in the literature to the negative environmental consequences of natural resource extraction projects and in particular to the topic of site cleanup and remediation. This literature includes papers such as [28, 29, 46, 47], which model optimal extraction of a non-renewable resource when there is a flow of pollution damages during operations as well as a buildup of a stock externality requiring a costly cleanup at the end of the project life. These papers determine the optimal policies to internalize negative externalities, and contrast a tax on the flow versus stock of pollution. An environmental bond is shown to be desirable when there is a risk of bankruptcy. In contrast to the above mentioned papers, the current paper does not derive the socially optimal tax policy, but instead studies the desirability of applying different financial surety instruments to ensure site cleanup given the risk of firm bankruptcy. Any flow externality is included only via an exogenous pollution tax.

This paper is closely related to [3], which models optimal resource extraction for a mine with ore prices described by a stochastic differential equation. Control variables include the quantity of ore produced, waste abatement, and the timing of opening and closing the mine or going bankrupt. The current paper presents a model that is relevant for a natural gas well in which production follows a known decline curve and there is no waste abatement. In addition, the current paper focuses on two types of financial surety: a Cash Deposit and Surety Bond, and considers the consequences of imposing financial surety on top of the existing royalty and tax regime. The analysis in [3] was done using pre-tax cash flows.

Three other papers that are complementary to the current paper include [7], [31], and [16]. [31] highlights the option value of temporarily closing a resource extraction project to avoid cleanup costs and environmental liabilities. A dynamic discrete choice model of oil and gas well closure is developed to evaluate the likelihood of well reactivation after temporary closure. Using a large dataset of oil and gas wells in Alberta, the paper finds that the majority of temporarily closed wells are highly unlikely to ever reopen, and that temporary closure is used as a means to avoid the cost of site remediation. [7] considers the effect of bankruptcy protection on industry structure and environmental outcomes in the Texas oil and gas industry. Among small firms, stricter insurance requirements are found to reduce production and improve environmental outcomes. [16] presents a real options model of the decision by the operator of an oil well to decommission the well and/or reclaim the well site. They suggest that the the large inventory of unreclaimed wells in Alberta are those with high decommission/reclamation costs and that these my be the wells with the greatest environmental risk. Their research differs from the current paper in that they do not focus on bankruptcy risk and do not consider different forms of financial surety.

3 Description of the decision problem

This paper models the optimal decisions of a firm with a lease to drill a well on a specific piece of land for the purpose of extracting natural gas under different regulatory requirements for financial surety for well cleanup. The lease includes the obligation for site cleanup and reclamation once the well is permanently closed. The firm makes decisions regarding the timing of drilling, production, temporary suspension or permanent closure of a well, and whether to declare bankruptcy. A bankrupt firm avoids paying any cleanup costs. The government receives tax and royalty payments from a producing well, but may be liable for cleanup costs if the firm goes bankrupt. We highlight the implications of financial surety for the total value of the lease which includes the value to the firm and to the government. It is assumed that the maximum lease time is 50 years, after which time the well must be properly closed including site reclamation. For simplicity it is assumed that gas extraction is via a single well. The uncertain price of gas is modelled as a stochastic differential equation.

We study the impact of different regulatory requirements for financial surety via three different scenarios.

- **Cash Deposit.** The government requires financial surety from the firm in the form of a Cash Deposit of 100 percent of estimated cleanup costs if the well were to be closed immediately. If cleanup costs rise over time, the firm must increase the Cash Deposit accordingly. This is a strong form of financial surety which ensures the government faces no cleanup liability. The deposit is refunded once cleanup of the well site has been done in accordance with regulations. If the firm goes bankrupt, the deposit is used by the government to pay for the cleanup costs. It is assumed that the government pays the firm interest on the Cash Deposit at the firm's opportunity cost of capital. This assumption is used to isolate the impact of the deposit requirement, as opposed to the return offered on the deposit.

  The assumption that the government pays interest at the firm's opportunity cost of capital was also used in [3]. It was shown in that paper that given this assumption the firm's optimal controls under a 100% cash deposit surety are the same as for a firm with no option to declare bankruptcy.

- **No financial surety.** In this scenario, no financial assurance is required. The firm has the option to declare bankruptcy and thereby avoid paying any cleanup costs. We calculate the expected cleanup liability left to government.

- **Surety Bond.** The government requires the firm to guarantee well cleanup via a financial guarantor. The firm agrees to make a stream of payments to a financial guarantor and, in return, the guarantor assumes the firm's cleanup liability in the event of firm bankruptcy. It is assumed that markets are sufficiently complete that the guarantor can hedge the risk of this bond. In this case, we calculate the required fee that just equals the expected value of monetary losses under the risk neutral probability measure (the Q-measure). This is referred to as the "fair" fee as it allow the financial guarantor to just break provided the risk from natural gas price volatility is being dynamically hedged, assuming no arbitrage opportunities exist in the economy.

3.1 State and control variables

The firm's optimal decisions depend on three state variables: the price of the commodity, $P(t)$, the stock of the resource, $S(t)$, and the stage of operation, $\delta(t)$. There are three possible states of operation, $\delta(t) = \delta_i,\ i = 1, 2, 3$. Stage 1 ($i = 1$) is pre-drilling, Stage 2 ($i = 2$) is active production, and Stage 3 ($i = 3$) is inactive or suspended. The suspended or inactive state retains the option to reopen. From a mathematical point of view, permanent closure of operations is a stopping time and hence is not specified as an operating stage. To avoid notation clutter, we will not show the explicit dependence of state variables on t, when there is no ambiguity.

The firm's controls are defined to be impulse controls, meaning that controls are exercised only at fixed times, rather than continuously. All are feedback controls, dependent on the state variables. The fixed decision times for impulse controls are specified as follows:

$$\mathcal{T}_d \equiv \{t = 0 < ... < t_m < ...t_M < T\}, \quad m = 0, ..., M, \tag{1}$$

where T represents the lease end date when the well must permanently closed. t_M denotes the last time that a decision can be made to switch operating stages. The impulse controls for $t_m \in \mathcal{T}_d$ are (i) the choice of operation stage, $\delta^+(P, S, \delta, t_m)$ and (ii) the time to declare bankruptcy, $T_b(P, S, \delta, t_m)$ (when the firm walks away from the well and incurs no cleanup costs), and (iii) the time of permanent closure, $T_c(P, S, \delta, t_m)$, when the firm pays cleanup costs and permanently closes the well.

The project will be terminated at the lease end date or at the time when it is optimal to permanently close the well or declare bankruptcy, whichever is sooner. Denote the project termination time as $\hat{T}$ which is defined as:

$$\hat{T} = \min(T_c, T_b, T). \tag{2}$$

At the lease end date, T, if the well is still in the operating or suspended states, it is assumed that the well will be closed and the firm will pay for cleanup and reclamation.

Let Z_{δ^+} denote the admissible values for the control, δ^+, where:

$$Z_{\delta^+} = \{\delta_1, \delta_2, \delta_3\}. \tag{3}$$

In other words, in applying the control the decision maker can choose any of the δ_i stages, which may involve changing to a different stage or staying in the current stage. The firm incurs costs to switch between stages, as will be described in Section 3.4.1. In the numerical example presented in Section 6, the firm is not permitted to switch from stage 2 (operations) to stage 1 (pre-drilling). This is accomplished through the imposition of a large cost to switch from stage 2 back to stage 1.

Remark: *The choice of whether to stay in the current stage or switch stages depends on the current stage, $\delta(t)$, which is a state variable. Hence we make the distinction between the control, denoted as δ^+, and the stage at time t, $\delta(t)$.*

For convenience, we specify the control set K as follows:

$$K = \{\delta^{+}, T_c, T_b\},\ t_m \in \mathcal{T}_d, \tag{4}$$

where t_m refers to times when impulse controls may be applied (see Equation (1)).

Once the decision is made to drill the well, gas production is determined by a known initial extraction rate and declining extraction rates thereafter as determined by a hyperbolic decline curve.[8] The hyperbolic decline curve is considered a good model of oil and gas well production, particularly for horizontal wells [20, 45]. A horizontal well's output has a high decline rate at the beginning of its life, levelling off during the later stages of the life cycle. Let $t_p(K)$ denote the time in production (the number of periods the well has been actively producing in stage 2), which is a function of the control variables, K. The rate of production, $q(t_p(K))$ is given as:

$$\begin{aligned} q(t_p(K)) &= \min\left[\frac{q_s}{(1 + b \times d_s \times t_p(K))^{(1/b)}}, S(t_p)\right], \quad b \neq 0, q(t_p) \leq \bar{q},\ \delta = \delta_2, t_p \geq 0. \\ &= 0, \quad \delta = \delta_1, \delta_3. \end{aligned} \tag{5}$$

where q_s is the initial well production level and t_p is time in production (in stage 2). $\bar{q}$ (the production capacity constraint), d_s (the initial hyperbolic decline rate) and b are parameters. $q(t_p)$ and d_s are expressed as annual rates. $S(t_p)$ is the stock of reserves after t_p periods in production. Production cannot exceed the remaining stock of reserves.

Cumulative production up to t_p, denoted as $Q(t_p(K))$, is given as the integral of the production rate over time.

$$\begin{aligned} Q(t_p(K)) &= \int_{t^s}^{t} q(t_p)dt_p, \quad q(t^s) = q_s. \\ &= \frac{q_s}{d_s(1-b)}[(1 + d_s b t^s)^{(b-1)/b} - (1 + d_s b t)^{(b-1)/b}], \\ &= \frac{q_s^b}{d_s(1-b)}[q_s^{(1-b)} - q_{t_p}^{(1-b)}]. \end{aligned} \tag{6}$$

The third line in Equation (6), shows cumulative production as calculated from initial and current production levels.

Prior to drilling, the reservoir is assumed to have a known fixed quantity of reserves denoted as S_0. Remaining reserves at any time t are then given by:

$$S(t) = S_0 - Q(t_p(K)), \quad S(t) \geq 0. \tag{7}$$

[8]The initial rate of extraction could be modelled as an additional control variable, with a larger initial extraction rate implying different extraction or development costs. If development or extraction costs are a nonlinear function of the initial extraction rate, then this feature may yield interesting results. If development or extraction costs are linear functions of the initial extraction rate then the maximum initial extraction rate would be chosen. For simplicity, this feature is not included in the current model.

The commodity price, $P(t)$, is assumed to be described by a simple one-factor Ito process, which is mean reverting in the drift term. Mean reversion in the drift term is a common in the modelling of commodity prices [39, 17]. The specific model used in this paper, under the $\mathbb{P}$-measure is:

$$dP(t) = \eta(\bar{P} - P(t))\; dt + \sigma P(t) dz;\; P(0) = p_0 \; given \tag{8}$$

where $\eta, \bar{P}, \sigma$ are parameters to be estimated from the data, t denotes time where $t \in [0, T]$, and dz is the increment of a Wiener process. This model is used in other papers such as [22, 23, 24]. Equation (8) is transformed to the $\mathbb{Q}$-measure by deducting a risk premium, $\lambda\sigma$, from the drift term where λ refers to the market price of risk.[9]

3.2 Closure costs and financial surety

[19] and [36] study the factors that determine well cleanup costs. There is evidence that decommissioning older wells is more costly than newer wells. To allow for this feature, we specify a model with a rising cleanup cost beginning at some time, t^o, in the life of the license. $C_c(\delta, t)$ denotes all costs of closure and reclamation if the well is permanently closed at time t. It is dependent on δ because cleanup cost is zero if the well remains in stage 1, $\delta = \delta_1$, implying the well is never drilled.

$$C_c(\delta, t) = \zeta_c, \;\; \text{for } t^s \leq t \leq t^o \tag{9a}$$

$$= \zeta_c e^{\gamma t}, \;\; \text{for } t^o < t \leq \hat{T}; \tag{9b}$$

ζ_c is a constant, t^s denotes the time when drilling occurs, and t^o an alternate time where $t^o > t^s$, and $\gamma \geq 0$.

At the time the well is drilled, the firm may be required to provide financial surety to indemnify the government for the cost of well cleanup should the firm be unable to meet its obligations. As noted, we consider two forms of surety - a Cash Deposit covering up to one hundred percent of cleanup costs and a Surety Bond .

- **Cash Deposit, $\Omega(\delta, t)$.** The firm must make an cash initial deposit to the government when the well is drilled at $t = t^s$ of ζ_c, that is $\Omega(\delta, t_s) = \zeta_c$. If $\gamma > 0$ in Equation (9b), the firm must make subsequent Cash Deposits so that the value of the deposit will always be equal to cleanup costs if the well were to be closed and the site cleaned up immediately. The cash accrued in the Cash Deposit at time t can be expressed as:

$$\Omega(\delta, t) = \left[\zeta_c + \int_{t'=t^{s+}}^{t} \frac{dC_c(t')}{dt} dt' \middle| \; \delta(t) \neq \delta_1 \right], \; \text{for } t \geq t^s, \tag{10}$$
$$= 0, \; \text{for } t < t^s$$

  where drilling is undertaken at t^s, and t^{s+} refers to the instant after t^s. The Cash Deposit is defined for the operating (δ_2) and mothballed (δ_3) stages,and is liquidated when the project is terminated.

[9]See [24, 13] for a discussion of the $\mathbb{P}$ and $\mathbb{Q}$-measures.

The required cash payment by the firm is $d\Omega/dt$ which is given as:

$$\frac{d\Omega}{dt} \quad = \quad \mathcal{D}(t-t^s)\zeta_c + \frac{dC_c(t)}{dt}\mathbb{1}_{\delta\neq\delta_1} \qquad \text{for } t \geq t^s, \tag{11}$$

where $\mathcal{D}(t-t^s)$ is the Dirac function, also know as the Dirac delta function.[10] The indicator function, $\mathbb{1}_{\delta\neq\delta_1}$, is included for clarity where $\mathbb{1}_{\delta\neq\delta_1}$ equals 1 if the condition $\delta \neq \delta_1$ is true and equals zero otherwise. For simplicity, it will be assumed that the firm self finances the required Cash Deposit and the government pays interest at rate, ρ, which reflects the opportunity cost of capital to the firm, with ρ greater than or equal to the risk free rate r.[11] The annual cash flow associated with the Cash Deposit from the firm's point of view is given as:

$$\Pi^{\Omega} = -\frac{d\Omega}{dt} + \rho\Omega(\delta,t)\mathbb{1}_{\delta\neq\delta_1}. \tag{12}$$

where interest at rate ρ is received only after the well is drilled, i.e. when $\delta = \delta_2$ or δ_3.

- **Surety bond.** According to the Surety Bond contract, the firm agrees to make a stream of payments to the guarantor and, in return, the guarantor agrees to pay a specified amount to government in the event of firm bankruptcy. We denote the firm's annual payment to the guarantor as $X(t)$. Logically, the guarantor will only agree to this contract if *in the risk neutral or Q-measure*, the expected payments from the firm cover the guarantor's expected payment to government, as is stated in Equation (13).

$$\mathbb{E}^Q\left[\int_{t^s}^{T} e^{-rt}X(t)dt\right] = \mathbb{E}^Q[C_c(\delta,t)]. \tag{13}$$

According to Equation (9), cleanup costs are know with certainty if permanent closure happens before t_o. After t_o cleanup costs are time dependent.

3.3 Cash flows associated with operations and the financial surety

Cash flows associated with operations and the financial surety are modelled in continuous time.[12] If well operations are suspended, production will be zero, but fixed costs are still incurred. After tax cash flow, denoted Π, in stages 2 (producing) and 3 (suspended) is given

[10]The Dirac function, $\mathcal{D}(t)$, is a generalized function which is zero except at $t=0$, and $\int_{-\infty}^{+\infty}\mathcal{D}(t)\,dt' = 1$.

[11]Equivalently, we could assume the firm borrows at rate ρ to finance the Cash Deposit. Since the deposit fully covers the cleanup cost, there is no risk to a lending agency.

[12]The decision problem will be solved numerically. Hence continuous time is approximated by small time steps of size Δt. Robustness checks are carried out for the chosen Δt.

by:

$$\Pi(P(t),S(t),\delta(t)) = \qquad (14)$$

$$\overbrace{\Big((1-\psi_R(P,q))P(t)-C_v\Big)q(t_p(K))\mathbb{1}_{\delta=\delta_2}-C_{f\delta_i}}^{\text{cash flows from operations}}+\overbrace{\rho\Omega(\delta,t)\mathbb{1}_{\delta\neq\delta_1}-\frac{d\Omega(\delta,t)}{dt}-X(t)}^{\text{financial assurance cash flows}}$$
$$-\overbrace{\max\left\{\psi_I\left(\Big((1-\psi_R(P,q))P(t)-C_v\Big)q(t_p(K))\mathbb{1}_{\delta=\delta_2}-C_{f\delta_i}-\psi_{\mathrm{D}}-X(t)-\frac{d\Omega(\delta,t)}{dt}\right),0\right\}}^{\text{income tax}}$$

$q(t_p)$ is gas production dependent on time in production, P is gas price, C_v is the constant variable cost of production, $C_{f\delta_i}$ is the constant fixed costs in stage δ_i, ψ_I is the income tax rate, $\psi_R(P,q)$ is the royalty rate, ψ_{D} refers to other available tax deductions, and $X(t)$ refers to any Surety Bond payments. Note that the royalty rate, $\psi_R(P,q)$, is shown to be a function of the price of gas and current production, as is the current case in Alberta.[13] It is assumed that interest payments received on the Cash Deposit are not taxed. In addition taxable income is reduced by any payments made to the bond. Note that $X(t)=0$ for the case of the Cash Deposit, while $\Omega(\delta,t)=0$ for the Surety Bond.

3.4 Discrete time cash flows

3.4.1 Costs for switching operating stages, δ_i, for $t\in\mathcal{T}_d$

Firms may incur a cost associated with the optimal control to switch from one stage to another. Let $C_{sw}(\delta_i,\delta_j)$ denote the cost of switching from stage δ_i to δ_j. Costs considered in this model include drilling costs required to begin production, $C_{sw}(\delta_1,\delta_2)$, well suspension costs, $C_{sw}(\delta_2,\delta_3)$, and reopening costs, $C_{sw}(\delta_3,\delta_2)$, all assumed to be fixed costs.

In practice, the cost of drilling a horizontal well depends on firm decisions about the length of lateral drilling and whether hydraulic fracturing will be undertaken. Firms can also increase production through multi-well drill pads whereby multiple wells are drilled from a single drill site, thereby accessing a larger portion of the resource for a given site disturbance.[14] For simplicity we assume only a single well is drilled at a fixed cost, but conceptually this is equivalent to drilling multiple wells at the same time from a single pad.[15]

3.4.2 Cash flows with bankruptcy or permanent well closure

The timing of well closure, T_c, and bankruptcy, T_b, are optimal stopping times chosen by the firm. Cash flows at T_c depend on the type of financial surety whereas if bankruptcy is

[13] Alberta royalties also depend on cumulative revenue from the a well. This is ignored in the model specification for simplicity, but will be addressed in the empirical example.

[14] See EIA, Sept 11 2012, "Pad drilling and rig mobility lead to more efficient drilling" https://www.eia.gov/todayinenergy/detail.php?id=7910.

[15] In [41], it is assumed that the drilling cost depends on the chosen initial rate of production.

chosen by the firm, cash flow is assumed to be zero:

Firm not bankrupt

$$\textit{Cash Deposit}: \; \Pi_{T_c} = \Omega(\delta, T_c) - C_c(\delta, T_c) = 0, \tag{15a}$$

$$\textit{Surety bond}: \; \Pi_{T_c} = -C_c(\delta, T_c), \quad \text{(will never be chosen)} \tag{15b}$$

Firm bankrupt

$$\Pi_{T_b} = 0 \tag{15c}$$

For the Cash Deposit, the firm receives a refund of the value the deposit, and also pays for site cleanup. Given that assumption that the Cash Deposit fully covers the cleanup cost, then $\Pi_{T_c} = 0$. In theory, a non-bankrupt firm with a Surety Bond pays for cleanup, but receives no offsetting cash flow. However as is noted in Section 3.6 below, it will always be optimal for the firm with a Surety Bond to declare bankruptcy. Hence this case is not relevant given the model assumptions.

3.5 Expected value of the drilling license

The value of the license to the firm can be determined as the expected value, in the risk neutral measure, of the sum of cash flows over the lifetime of the license. The value will depend on the state variables and time, denoted as $V^F(P, S, \delta, t)$. We search for the optimal feedback controls including $\delta^+(P, S, \delta, t)$, $T_c(P, S, \delta, t)$, and $T_b(P, S, \delta, t)$ with $t \in \mathcal{T}_d$. Recall the control set K is specified as follows:

$$K = \{\delta^+, T_c, T_b\}, \; t_m \in \mathcal{T}_d, \tag{16}$$

where t_m refers to times when impulse controls may be applied (see Equation (1)). Admissible values for control variables were detailed in Section 3.1.

$$\begin{aligned} V^F(p, s, \delta_i, t) \; &= \; \sup_K \mathbb{E}_K^Q \Bigg[\overbrace{\int_{t'=t}^{\hat{T}} e^{-r(t'-t)} \, \Pi(P(t'), S(t'), \delta(t')) \, dt'}^{\text{cash flow}, \; \delta_i, \; i=2,3} \\ &- \overbrace{\sum_{t' \in \mathcal{T}_d}^{\hat{T}} e^{-r(t'-t)} C_{sw}(\delta_i, \delta_j)}^{\text{switch cost}, \; \delta_i \to \delta_j, \; i,j,=1,2,3} \; + \; \overbrace{e^{-r(\hat{T}-t)} F(\hat{T})}^{\text{value at termination}} \; \Bigg| \; P(t) = p, S(t) = s, \delta(t) = \delta_i \Bigg]. \end{aligned} \tag{17}$$

where (p, s, δ_i) denotes realizations of the random and path dependent stochastic variables (P, S, δ). r is the risk-free interest rate, $\mathbb{E}_K[\cdot]$ is the expectation operator, and $\hat{T}$ is defined in Equation (2). $F(\hat{T})$ denotes the final payout at $\hat{T}$, which depends on whether or not the firm is bankrupt. The final payout at the optimal termination time $\hat{T}$ will be either Π_{T_c} or Π_{T_b} as defined in Equations (15a) and (15c), respectively.

Between decision dates, the firm remains in its current stage of operation, while V evolves in continuous time as the price of gas changes and as gas production occurs (if the firm is in stage 2). For a particular decision time, $t_m \in \mathcal{T}_d$, we define t_m^- and t_m^+ to represent the moments just before and after t_m. Specifically $t_m^- = t_m - \epsilon$ and $t_m^+ = t_m + \epsilon$, $\epsilon \to 0^+$.[16] Given any chosen operating stage, we can determine V between $t_{m+1}^- \to t_m^+$, going backward in time. Consider a time interval $h < (t_{m+1} - t_m)$. For t in between decision dates i.e, $t \in (t_m^+, t_{m+1}^- - h)$, the dynamic programming principle states that (for small h), $t < \hat{T}$,

$$V^F(p, s, \delta_i, t) = e^{-rh}\, \mathbb{E}^Q\Big[V^F(P(t+h), S(t+h), \delta(t), t+h)\Big| \\ P(t) = p, S(t) = s, \delta(t) = \delta_i\Big] + \Pi(p, s, \delta_i)h. \tag{18}$$

Letting $h \to 0$ and using Ito's Lemma,[17] the equation satisfied by the value, V, for all operating states is given as:

$$\frac{\partial V}{\partial t} + \frac{1}{2}\sigma^2 p^2 \frac{\partial^2 V}{\partial p^2} + \big(\eta(\bar{P} - p) - \lambda\sigma\big)\frac{\partial V}{\partial p} - q\,\frac{\partial V}{\partial s} + \Pi(p, s, \delta_i) - rV = 0, \text{ for } t < \hat{T}. \tag{19}$$

Note that if bankruptcy occurs then $V^F(\cdot) = 0$.

3.6 Determining the firm's optimal controls

Recall that the controls in this model are impulse controls taken in $t_m \in \mathcal{T}_d$ and comprise the choice of operating stage, δ^+, bankruptcy time, T_b, and closure time, T_c. In the numerical example, decision dates occur annually. Let $C_{sw}(\delta^-, \delta')$ denote the cost of switching from stage δ^- to δ'. We define the optimal stage of operations, denoted by δ^+:

$$\delta^+ \;=\; \arg\max_{\delta'}\, [V^F(p, s, \delta', q, t_m^+) - C_{sw}(\delta^-, \delta')]. \tag{20}$$

where t_m^+ refers to the instant after t_m. In the event of a tie (i.e. the value of remaining in the current stage equals the value of switching), it is assumed that the firm remains in the current stage, δ^-.

Now consider the decision to terminate the project prior to the license end date, either through well closure (which requires cleanup) or declaring bankruptcy. The firm compares the payout from terminating the project with the value of continuing operations, given the operation stage is chosen optimally.

- **Cash Deposit.** As noted in Equations (15a) and (15c), the terminal value is zero for the firm, whether or not it declares bankruptcy. For concreteness, we will assume in this case that the firm does not declare bankruptcy and uses the cash deposit refund to carry out the cleanup.

[16] As a visual aid, the times around t_m and t_{m+1} are depicted, going forward in time as: $t_m^- \to t_m \to t_m^+ \to t_{m+1}^- \to t_{m+1} \to t_{m+1}^+$.

[17] See [13] for an accessible treatment of Ito's Lemma in the context of optimal decision making under uncertainty with an Ito process such as Equation (8). [4] and [34] are more advanced treatments of the topic.

- **No financial surety.** It is optimal for the firm to declare bankruptcy rather than undertake site cleanup.
- **Surety bond.** There is no refund at project termination. The firm's optimal decision is to declare bankruptcy rather than incur $C_c(\delta, \hat{T})$

In short, for all cases considered, the value of termination for the firm will be zero. Only in the case of a 100 percent Cash Deposit will the firm undertake site cleanup.

The condition for the optimal terminal time is indicated in Equation (21) in which "terminate" refers to ceasing operations due to well closure or bankruptcy. For $t \in \mathcal{T}_d$:

$$V^F(p, s, \delta^-, q, t_m^-) \; = \; \max \Bigg[\overbrace{V^F(p, s, \delta^+, t_m^+) - C_{sw}(\delta^-, \delta^+)}^{\text{continue operations}}, \; \overbrace{0}^{\text{terminate}} \Bigg] \tag{21}$$

T_c and T_b are specified below.

Optimal well closure time, Cash Deposit:

$$T_c = \inf\{0 \le t < T; \;\; V^F(p, s, \delta^-, t_m^-) = \Omega(\delta_i, t_m^+) - C_c(\delta_i, t_m^+)\} = 0, \tag{22a}$$

Optimal bankruptcy time, Surety Bond or No Financial Surety cases:

$$T_b = \inf\{0 \le t < T; \;\; V^F(p, s, \delta^-, t_m^-) = 0\} \tag{22b}$$

The stochastic optimal control problem defined in Equations (19) through (22) is solved using a standard fully-implicit, finite-difference, semi-Lagrangian numerical approach described in Appendix C. Boundary conditions for the numerical solution are also described in Appendix C.

3.7 Government's value function

During well production, the government receives taxes and royalty payments from the firm, denoted as Π^G, where:

$$\begin{aligned} \Pi^G(P, S, \delta_i, t) \; = \; & \overbrace{\psi_R(P, q) P(t) q(t_p) \mathbb{1}_{\delta=\delta_2}}^{\text{royalties}} \\ & + \overbrace{\max\left\{ \psi_I \Big((1 - \psi_R(P, q) P(t) - C_v) q(t_p) \mathbb{1}_{\delta=\delta_2} - C_{f\delta_i} - \psi_{\mathrm{D}} - X(t) - \frac{d\Omega}{dt} \Big), 0 \right\}}^{\text{income tax}} \end{aligned} \tag{23}$$

with terms defined after Equation (14). Note that carbon taxes are not considered part of the government's value function since these are intended to account for the negative environmental externality of gas production.

For the No Surety case the government will also incur cleanup costs in the event the firm goes bankrupt equal to $-C_c(\delta, \hat{T})$. For the Cash Deposit, the government incurs no cleanup costs and the present value cash flows associated with the deposit are exactly zero. For the Surety Bond case, the government also incurs zero cleanup costs since these are guaranteed by a third party. The government's value function, $V^G(\cdot, t)$, can be expressed as the expected value of these relevant cash flows:

$$\begin{aligned} V^G(p, s, \delta_i, t) =& \mathbb{E}^Q \left[\int_{t'=t}^{\hat{T}} e^{-r(\hat{T}-t)} \Pi^G(P, S, \delta_i, t) dt' - e^{-r(\hat{T}-t)} C_c(\hat{T})] \right], \textit{ No surety} \\ &= \mathbb{E}^Q \left[\int_{t'=t}^{\hat{T}} e^{-r(\hat{T}-t)} \Pi^G(P, S, \delta_i, t) dt'] \right], \textit{ Cash Deposit or Surety Bond.} \end{aligned} \tag{24}$$

where $\hat{T}$ is the optimal project termination time defined in Equation (2). These expectations are computed using Monte Carlo analysis based on the stored optimal controls from the numerical solution of the firm's HJB equation.

3.8 Surety bond value to the guarantor

As noted, the guarantor promises to pay the bond amount to the government in the event that the firm does not fulfill its cleanup obligations at the well termination time. The guarantor is aware of the risk of bankruptcy for a small firm and will only be willing to guarantee the cleanup cost if it is compensated for this risk. This amounts to the condition specified in Equations (13), where the annual fee payment is chosen such that the expected value of the payment in the risk neutral measure is equal to the expected value of cleanup costs, also in the risk neutral measure. It is assumed that markets are sufficiently complete that the guarantor can hedge the risk of the Surety Bond. It is further assumed there is no firm residual value after a bankruptcy declaration.

In the numerical example, the expected cleanup costs will be calculated for a range of values for $X(t)$ through Monte Carlo simulation. The present value of the Surety Bond, V^B, to the guarantor will be:

$$V^B(t) = \mathbb{E}^Q \left[\int_{t^s}^{T_b} e^{-rt} X(t) dt - [C_c(\delta, t)] \right]. \tag{25}$$

where t^s is the date of drilling the well. The value of $X(t)$ such that $V^B(t) = 0$ is referred to as the fair value. Recall that given our modelling assumptions, for the Surety Bond case the firm will always choose bankruptcy over incurring closure costs. Different values for $X(t)$ are tried until the condition in Equation (13) is met.

3.9 Total project value

The total value of the project V^T is calculated as the expected present value of the project to the firm plus the expected present value to the government. As noted, in the case of a

Surety Bond, it is assumed that under competitive markets the value to the guarantor is zero.

$$V^T = V^F(p, s, \delta_i, t) + V^G(p, s, \delta_i, t). \tag{26}$$

with V^F defined in Equation (17) and V^G defined in Equation (24).

4 Propositions regarding total project value with and without financial surety

This section presents some theoretical results comparing the total value of the gas well, as defined in Equation (26), under the Cash Deposit versus no surety scenarios. This comparison will be helpful for the interpretation of the subsequent numerical results. Specifically, we present two propositions comparing well value with and without the Cash Deposit. The first, Proposition 4.1, shows that the value of the well to the firm is higher when there is no Cash Deposit requirement. The second, Proposition 4.2, shows that the total value of the well, when there are no taxes or royalties, is higher when there is a Cash Deposit compared to No Financial Surety. Finally we show that when taxes and royalties are included, we cannot ascertain whether total project value will be higher or lower when there is a Cash Deposit versus when there is No Financial Surety.

Below is some simplified notation used in this section to refer to the components of total value. Recall that the control set is $K = \{\delta^+, \hat{T}\}$.

$f(K)$ is the expected present value of the firm's pre-tax, pre-royalty, cash flows from operations.

$$f(K) \equiv \mathbb{E}_K \int_{t'=t}^{\hat{T}} e^{-r(t'-t)} \Big((P(t') - C_v) q(t_p) \mathbb{1}_{\delta=\delta_2} - C_{f\delta_i} \Big) dt'$$

$h(K)$ is the expected present value of taxes and royalties.

$$h(K) \equiv \mathbb{E}_K \int_{t'=t}^{\hat{T}} e^{-r(t'-t)} \Pi^G(P, S, \delta_i, t') dt', \quad h(K) \geq 0.$$

Π^G is defined in Equation (23).

$g(K)$ is the expected present value of cleanup costs at the termination time $\hat{T}$, which also equals the present value of the Cash Deposit at $\hat{T}$, since by definition the deposit always just covers cleanup costs.

$$g(K) \equiv \mathbb{E}_K \, [e^{-r(\hat{T}-t)} C_c(\delta, \hat{T})] = \mathbb{E}_K \, [e^{-r(\hat{T}-t)} \Omega(\delta, \hat{T})], \quad g(K) \geq 0.$$

Cash flows at time t associated with the Cash Deposit (denote as $\Pi^\Omega(\delta, t)$) were defined in Equation (14). It is shown in Appendix A, that if the rate of interest paid on the Cash Deposit is equal to the firm's opportunity cost of capital, (i.e. $\rho = r$ under the risk neutral measure), then the expected net present value of cash flows from the deposit prior to drilling the well ($\delta(t) = \delta_1$) is:

$$\mathbb{E}_K \int_{t'=t}^{\hat{T}} \Pi^{\Omega} dt' = \mathbb{E}_K[-e^{-r(\hat{T}-t)}\Omega(\delta, \hat{T}) \mid \delta(t) = \delta_1]. \tag{27}$$

We now present the two propositions.

Proposition 4.1. *The value of the well to the firm under the Cash Deposit requirement given the firm's optimal control (denoted $V^{F\Omega^*}$) is less than or equal to the value of the well to the firm with No Financial Surety (denoted V^{FN^*}), i.e. $V^{F\Omega^*} \leq V^{FN^*}$. The '*' denotes value under the optimal control.*

Proof. By definition:

$$\begin{aligned} V^{F\Omega} =& f(K) - h(K) - g(K), \quad \text{and} \\ V^{FN} =& f(K) - h(K) \end{aligned}$$

Further,

$$\begin{aligned} V^{F\Omega *} - V^{FN *} &= \sup_K E_K[f(K) - h(K) - g(K)] \; - \; \sup_{K'} E_{K'}[f(K') - h(K')] \\ &\leq \sup_{K''} E_{K''}\big[\big(f(K'') - h(K'') - g(K'')\big) - \big(f(K'') - h(K'')\big)\big] \\ &\leq \sup_{K''} E_{K''}[-g(K'')] \; \leq \; 0. \end{aligned} \tag{28}$$

□

Proposition 4.2. *When taxes and royalties are excluded, the total value of the well including cleanup costs is greater under the Cash Deposit than with No Financial Surety, given the firm's optimal controls in each case.*

Proof. The following statement must be true by definition of the maximization operation.

$$\max_K [f(K) - g(K)] \geq f(K) - g(K). \tag{29}$$

Let $K^* = \underset{K \in Z_K}{\arg\max}\,[f(K) - g(K)]$ and $K^{**} = \underset{K \in Z_K}{\arg\max}\, f(K)$ where Z_K denotes the admissible values for K. It follows that:

$$\overbrace{f(K^*) - g(K^*)}^{V^{F\Omega^*}} \geq \overbrace{f(K^{**})}^{V^{F\Omega^{**}}} - g(K^{**}) \tag{30a}$$

$$V^{T\Omega}(K^*) \geq V^{TN}(K^{**}). \tag{30b}$$

The left hand side of Equation (30a) represents the value to the firm given the firm's optimal control under a Cash Deposit. Since there are no taxes or royalties, the left hand side also equals the total value of the well under the Cash Deposit requirement ($V^{T\Omega}(K^*)$). On the right hand side of Equation (30a), $f(K^{**})$ represents the maximized expected value of the well for the firm with no Cash Deposit requirement. $g(K^{**})$ represents the discounted expected value of cleanup costs that will be paid by government in the event of firm bankruptcy. Recall that in our model, with no Cash Deposit the firm will always choose bankruptcy rather than pay for cleanup costs. Hence the right hand side of Equation (30a) represents the total value of the well when there is no Cash Deposit, denoted as V^{TN}. It follows that $V^{T\Omega} \geq V^{TN}$ as per the proposition. $\square$

We are interested in comparing the total value of the well, with and without the Cash Deposit ($V^{T\Omega}$ and V^{TN}), when taxes and royalties are included. Let K^* denote the optimal control for the firm with a Cash Deposit requirement and K^{**} denoted the firm's optimal control with No Financial Surety when taxes and royalties are included:

$$\begin{aligned} K^* &\equiv \underset{K \in Z_K}{\arg\max}\, [f(K) - h(K) - g(K)] \\ K^{**} &\equiv \underset{K \in Z_K}{\arg\max}\, [f(K) - h(K)] \end{aligned}$$

Then,

$$\begin{aligned} V^{T\Omega} &= \overbrace{f(K^*) - h(K^*) - g(K^*)}^{V^{F\Omega^*}} + \overbrace{h(K^*)}^{V^{G\Omega}} \\ &= f(K^*) - g(K^*), \end{aligned}$$

and

$$\begin{aligned} V^{TN} &= \overbrace{f(K^{**}) - h(K^{**})}^{V^{FN^*}} + \overbrace{h(K^{**}) - g(K^{**})}^{V^{GM}} \\ &= f(K^{**}) - g(K^{**}). \end{aligned}$$

Therefore,

$$V^{T\Omega} - V^{TN} = \overbrace{[f(K^*) - f(K^{**})]}^{\text{Difference in PV firm cash flow}} + \overbrace{[g(K^{**}) - g(K^*)]}^{\text{Difference in PV cleanup costs}} \lesseqqgtr 0. \tag{31}$$

We cannot say *a priori* if $V^{TD} - V^{TN}$ is positive or negative. This will be explored in the numerical example.

5 Application to a hypothetical gas well

5.1 Project specification

We study the optimal decisions of a firm with a license to extract natural gas from a reservoir in Alberta in the PSAC 2 area[18] from the Montney formation. Data published by the Alberta Energy Regulator and industry sources are used to characterize the hypothetical well. The reserves are assumed to be developed via horizontal drilling, which accounts for the majority of wells drilled in Alberta over the past decade and projected for the future according to [2]. Horizontal drilling has has greatly increased well productivity compared the traditional vertical wells. Key parameters and cost assumptions are detailed in Table 1 and described below.

- Initial well productivity is set at at 58.2 $10^3 m^3/d$ [2] and then follows the decline curve given in Equation (5). The parameters of Equation (5) are chosen to approximately match the AER's forecast productivity declines:[19] $b = 0.6$ and the initial decline rate (on an annual basis) is $d_s = 0.3$.

- Initial gas reserves, S_0, are set at 110 $\times 10^6 m^3$. If a firm produces gas according to the assumed decline curve in every year up to the lease end, reserves would be just 95% exhausted after 50 years. Hence, reserves are not a constraint on production.

- A lease end date, T, of 50 years is assumed after which time the firm (if not bankrupt) must close and properly abandon the well. As per Alberta regulations, it is further assumed that the lease is nullified if the well is not drilled within five years of the lease start date. The mandatory drilling date is denoted as T_D.

- Well cleanup costs are based on information provided in [38].[20] The assumed value of \$135 thousand for the base case reflects the present value in the year of closure of a five year cost stream, discounted at 2%. Included are the well closure cost of \$70,500 in the year the well is closed, reclamation cost of \$28,231 in the following year, well inspection (\$661) and lease payments (\$7,750) incurred annually in the five year period until the well is deemed to be fully closed and land reclaimed. In the base case, it is assumed that well cleanup costs are constant over time, implying $\gamma = 0$ in Equation (9b).

  In an analysis of data from up to 19,500 oil and gas wells in the U.S., [36] find that compared to wells over 60 years old, costs for wells decommissioned at ages 40 to 60 were 9% less expensive and wells aged 0 to 40 were 20% less expensive. Median

[18] PSAC is an abbreviation for the Petroleum Services Association of Alberta. PSAC 2 is in the western part of the province in the Foothills Front.

[19] According to [1] initial productivity in 2023 for horizontal gas wells drilled in the PSAC2 region is 58.2 $10^3 m^3/d$, with decline rates in the first three years of 43%, 29%, and 23%, followed by 13% declines for years 4 through 10.

[20] [38], Online Appendices D and E.

decommissioning costs were US$75,000 for plugging and surface reclamation[21], and in rare cases costs exceeded US$1 million. The authors note that higher costs for older wells "are likely caused by degradation of steel and cement casing over time, which can create multiple challenges for plugging operations" (page 10228). In the numerical example, a case will be presented in which clean up costs start to grow rapidly after 30 years.

- Fixed and variable costs when operating (stage 2) are taken from [2] for PSAC2, Montney formation.[22] Fixed costs when mothballed reflect a lease payment and well inspection cost as estimated in [38].

- Income tax rates and the carbon tax rate are reported in Table 1. The carbon tax rate is the 2024 rate set by the Canadian Federal Government. Royalties are base on Alberta's Modernized Royalty Framework introduced in 2016 as reported on the Alberta Government website.[23] Royalty rates increase with the price of natural gas.

- Lacking information on suspension and reopening costs, the author made an arbitrary assumption of $5000.

- Drilling cost information is taken from some industry sources, including [42, 12]. Drilling and well completion is assumed to cost $5 million.

5.2 Price process parameters

Figure 1 plots the monthly Alberta natural gas reference price since 1988. This reference price is a monthly weighted average field price of all Alberta gas sales, as determined by the Alberta Department of Energy and Minerals through a survey of actual sales transactions. The parameters of Equation (8) are obtained through an ordinary lease squares regression on this data. The data was converted to annual averages to eliminate seasonality and deflated by the Canadian consumer price index. An ordinary least squares regression resulted in the parameter estimates in Table 2.

The market price of risk, λ, is estimated using a very simple approach described in [21], which relies on the capital asset pricing model (CAPM). The market price of risk is estimated as $\lambda = (E(r_m) - r)\beta/s$, where $E(r_m)$ is the expected of a broad market index, r is the risk free rate, β and s are, respectively, the beta from the CAPM and the volatility of an asset whose value depends solely on the price of natural gas. We assume $E(r_m) = 8.4$ %, $r = 2\%$, $s = 0.31$, $\beta = 0.93$, which gives a λ of 0.19. Using data provided by Aswath Damordaran,[24]

[21] This median cost lower than the cleanup cost of C$135k assumed in our numerical example. The C$135k estimate is from [38]. It is based on Canadian data and includes lease payment and well inspection costs over five years when closure and reclamation occurs.

[22] Table S5.6 Alberta Natural Gas Supply Costs by PSAC area, 2022.

[23] Government of Alberta, Modernized Royalty Framework: formulas.
https://open.alberta.ca/publications/modernized-royalty-framework-formulas

[24] Stern School of Business at New York University (https://pages.stern.nyu.edu/~adamodar/).

Well parameters	
Initial well productivity, $10^3 m^3$/day**	$q_s = 58.15$
Initial reserves, $10^6 m^3$*	$S_0 = 110$
Hyperbolic decline curve:	
b factor*	$b = 0.6$
Initial effective decline rate (annual)*	$d_s = 0.35$
CO2 emission intensity, tonnes/$10^3 m^3$*	0.1567
Costs	
Variable operating costs C$/$10^3 m^3$**	$C^V = 60.36$
Fixed operating costs C$000/year**	$C^F_{\delta_2} = 38.44$
Fixed costs when mothballed C$000/year**	$C^F_{\delta_3} = 8.411$
Drilling cost (C$000)**	$C_{sw}(\delta_1, \delta_2) = \$4000$
Cost of well closure and cleanup, Base case	
(C$000)**, Eqn (9a)	$C_{sw}(\delta_2, \delta_4) = C_{sw}(\delta_3, \delta_4) = C_c(\delta, t) = \$135$
Cleanup cost growth rate*, γ, Eqn (9b)	$\gamma = 0$ (base case)
Cost to mothball and reactivate (C$000)*	$C_{sw}(\delta_2, \delta_3) = C_{sw}(\delta_3, \delta_2) = \$5$
Taxes	
Carbon tax C$/tonne of CO2**	80
Income tax rate (combined federal and provincial)**	27%
Canadian Development Expense Deduction Rate**	30%
Price model parameters	
Speed of mean reversion, η*	0.12
Long run mean of price, $\bar{P}$*	173 $/$10^3 m^3$ (or 4.63 $/GJ)
Volatility, σ*	0.31
Market price of risk*, λ	0.18
Other	
Risk-free rate (annual)*	$r = 0.02$
Max. price in computational domain ($/$10^3 m^3$)*	$p_{max} = 1730$
Lease end date, years *	$T = 50$
Max. time allowed prior to drilling, years*	$T_D = 5$
Project stages*	δ_1, δ_2, δ_3
Fixed decision times*	t_m in τ_d spaced annually

Table 1: *Parameter values for the hypothetical gas well.* **Assumed or estimated by the author.* ***Initial productivity and fixed and variable costs are taken from Alberta Energy Outlook ST98:2023, Natural Gas Supply Demand data. Data is for a horizontal well in area PSAC2 in the Montney formation. Well cleanup costs are based on information provided in [38] . Fixed costs when mothballed reflect lease payments and well inspection costs provided in [38]. Taxes are based on government data.*

the expected real return of the market is computed as the arithmetic average of real returns on the S&P 500 index, including dividends, from 1927 to 2023. β is the reported value for Oil and Gas, Production and Exploration Sector as of January 2024. $s = 0.31$ is the calculated

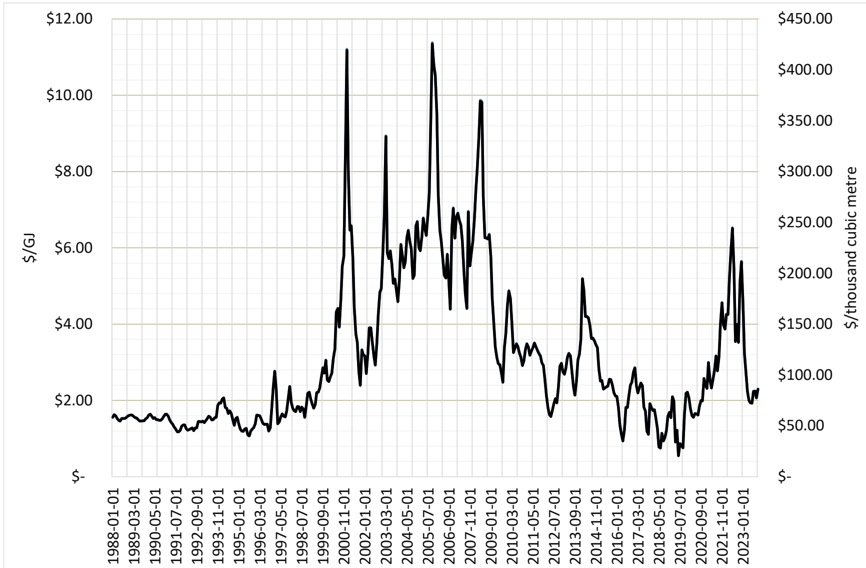


Figure 1: Alberta natural gas reference price, C$/gigajoule (left axis) and C$ per thousand cubic metre (right axis). Source: Government of Alberta, https://economicdashboard.alberta.ca/dashboard/natural-gas-price#

η	0.12	speed of mean reversion
$\bar{P}$	173 $\$/10^3 m^3$ (or 4.63 \$/GJ)	long run mean of P
σ	0.31	volatility
λ	0.18	market price of risk

Table 2: *Parameter estimates for Equation* (8) *and the market price of risk.*

volatility of real natural gas prices (the reference price) in Alberta.

These parameter values are estimated in a simplistic fashion. A more rigorous approach would use natural gas futures prices to estimate the natural gas price model under the risk neutral measure without the need to determine a market price of risk. However futures markets do not provide good information for the fifty year time frame of interest in this paper.

6 Numerical results analysis

The optimal controls that maximize the value function (Equation (17)) are determined by solving Equations (19) to (22) over a grid of possible reserve levels, price levels and time steps using the numerical technique described in Appendix C. Recall that the optimal controls are the decisions to drill (and open) the well (switch from stage 1 to 2), temporarily mothball the well (stage 2 to 3), reopen the well (stage 3 to 2), permanently close the well (stages 2 or

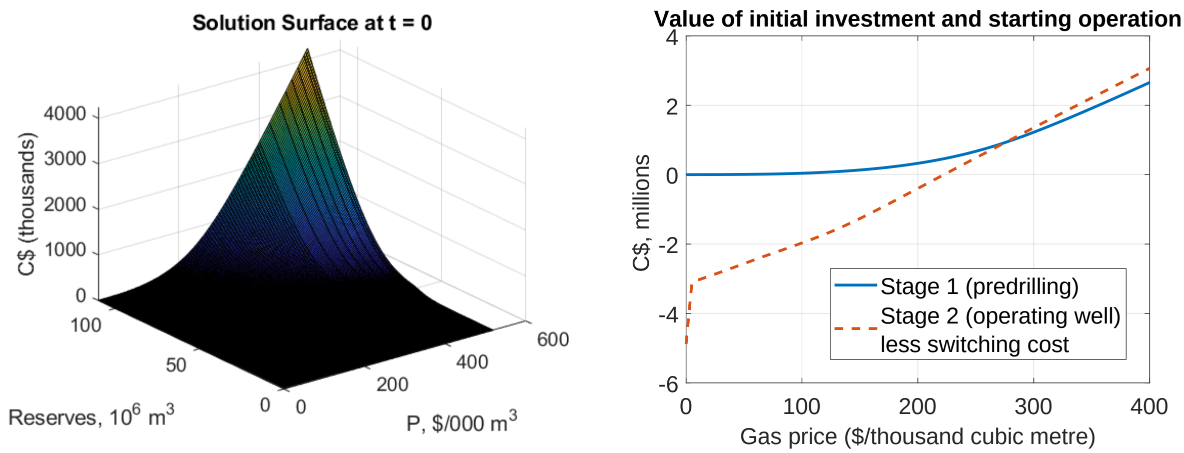
3 to 4), or go bankrupt (stages 2 or 3 to 4). The optimal controls from the HJB solution are stored and then used in Monte Carlo simulation to depict the distribution of key variables of interest. 3×10^4 simulations of the stochastic natural gas model are undertaken. For each of those simulations, the optimal controls from the solution of the HJB equation are used to determine the expected path of key variables over time, such as project value, production levels, and the timing of closure or bankruptcy. Linear interpolations is used for the controls based on the stored HJB solution. We are also able to infer the value of the project to the government by computing the expected value of taxes and royalties collected versus cleanup costs imposed on the government when a firm goes bankrupt.

6.1 Constant cleanup costs

6.1.1 100% Cash Deposit

Consider first the results for a Cash Deposit covering 100 percent of estimated cleanup costs. The solution of Equation (17) allows us to determine the value of the asset (the right to drill at a particular location) at time zero for various starting values for the natural gas price and reserve levels, given the optimal controls. This value surface is depicted in Figure 2(a) over a range of initial reserve levels and natural gas prices prior to drilling the well. Value is increasing in both the starting price and initial reserves. The blue curve in Figure 2(b) depicts a slice of the value surface from Figure 2(a) at a starting reserve level of 110 million cubic meters. The red dashed curve shows the value of the asset just after the well has been drilled, less the cost of drilling. The intersection of these two curves represents a threshold price of about \$275 per thousand cubic meter. If the current natural gas price is below \$275 per thousand cubic meter it is optimal to delay drilling the well. If the current price exceeds \$275 per thousand cubic meter then it is optimal to drill the well immediately. Note the threshold price is time dependent since there is a fixed lease end date. In general the threshold price to begin drilling declines as time passes and the lease end date approaches.

Threshold prices versus reserve levels at time zero are depicted in Figure 3 for the all of the controls. Recall that with a 100% Cash Deposit, the firm is indifferent to declaring bankruptcy or permanently closing the well (which requires site remediation); it is assumed that the latter is chosen. Figure 3(a) shows that the threshold price to drill the well falls as the assumed initial reserve level is increased, implying the well is more likely to be drilled with higher initial reserve estimates. The solid red line in 3(b) shows threshold prices for mothballing the active well. Given that the well is actively producing at prices above this line, if the price drops to or below the red line, the well will be temporarily mothballed. In that same figure, the blue dashed line shows critical prices to restart the mothballed well. Given that the well is mothballed below the blue line, if the price rises to or above blue the line, the well will be restarted. Figure 3(c) shows threshold prices to permanently close the well from the mothballed or active stages. It will be observed that at time zero (i.e. with 50 years remaining in the lease) the well will not be closed if reserves are above 30 million m^3. For easier comparison, Figure 3(d) includes the four curves from Figures 3(b) and 3(c), which allows us to see that at low reserve levels (below about 17.5 million m^3), the threshold



(a) Value versus resource stock & gas price prior to drilling.

(b) Value in stages 1 and 2 versus gas price.

Figure 2: Project value prior to drilling (Stage 1), benchmark case, 100% Cash Deposit at starting time of $t = t$. In the right-hand panel, starting reserves are 110 million cubic meters.

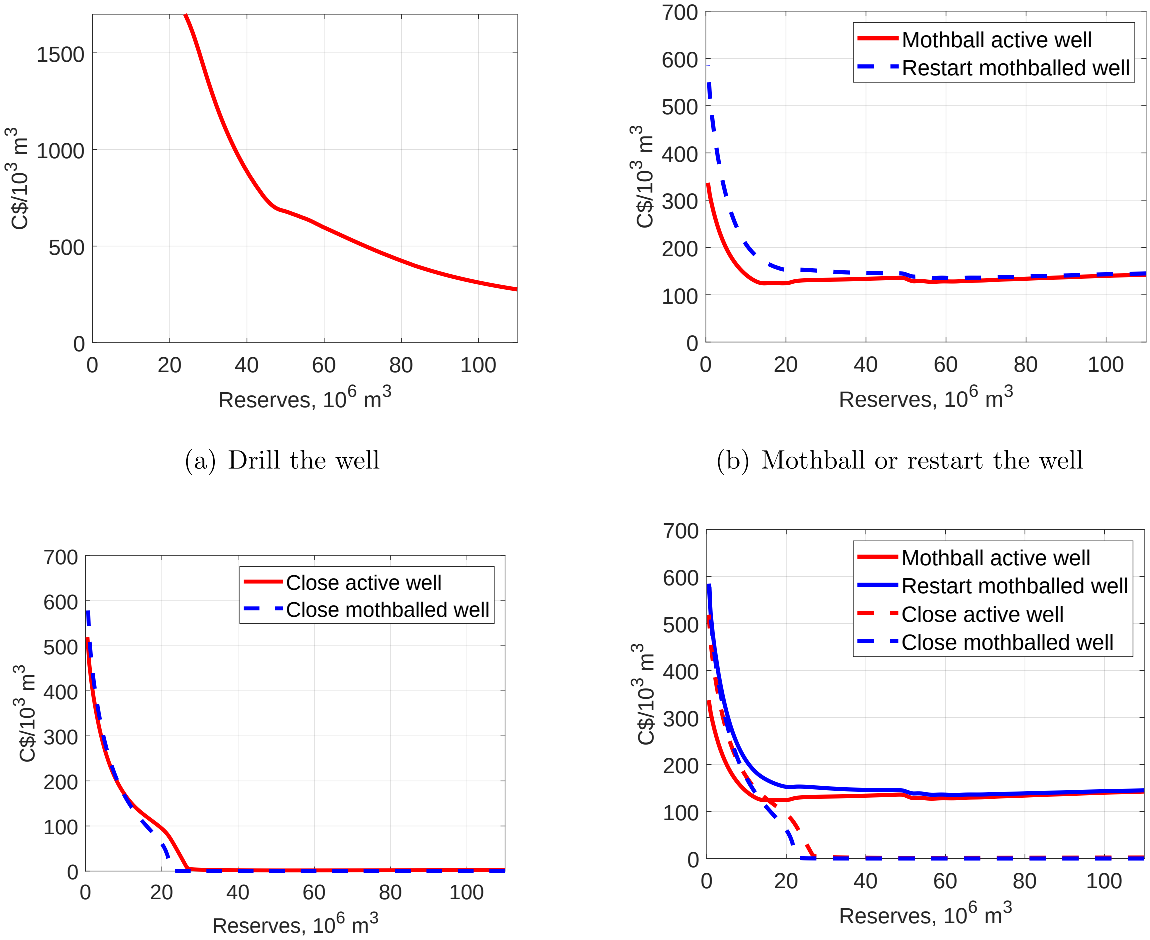
prices for closing the well (dashed red and blue lines) are higher than the threshold prices for mothballing the well (red solid line), implying the mothballing option will not be chosen.

Figure 4 shows some of the Monte Carlo results for this case. The assumed starting price at time zero is \$173 per $10^3 m^3$ which is the estimate for the long run mean price $\bar{P}$ from Table 2. Figure 4(a) shows the probability of being in the different stages over time. We observe that the probability of remaining in Stage one falls in the first five years and then remains at 80%. This reflects the assumption that the well must be drilled by the fifth year or the lease is relinquished. Given the assumptions for this example, there is a significant likelihood that the option to drill the well will not be exercised. Figure 4(b) shows various percentiles for cumulative production, conditional on the well being drilled. The 95th percentile indicates that by the lease end date, cumulative production will be 95 million cubic meters or below (with 95% probability), which represents 90% of maximum possible production.[25] Median cumulative production is around 80 million m^3.

6.1.2 Cash Deposit and No Financial Surety compared

Optimal controls and project values for the benchmark case may be contrasted with the case when No Financial Surety is imposed. Table 3 shows the expected value at time zero for the Cash Deposit and No Surety cases. The firm does better (by 5%) when No Financial Surety is imposed. Government is also slightly better off by 1.3% , while the total value of

[25]Maximum possible production after 50 years given the assumed decline curve is approximately 105 million m^3.



(a) Drill the well

(b) Mothball or restart the well

(c) Permanently close the well

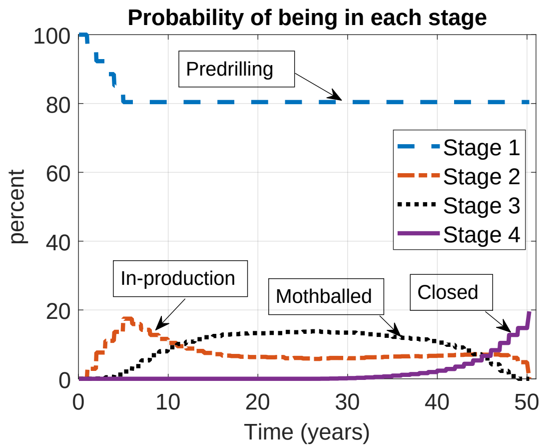
(d) Mothball, restart, or close the well

Figure 3: Threshold prices at time zero, benchmark case (100% cash bond). Gas price at time 0 = 173 dollars per thousand cubic metre.

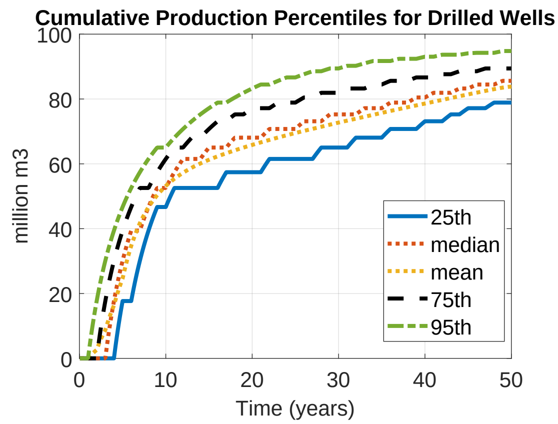
the project is higher by 1.9% when there is no surety. Overall the differences between these two cases are quite small and it appears not to be beneficial to require the Cash Deposit.

Also shown for comparison (columns 3 and 4) are values when no income taxes or royalties are imposed, which we will refer to as the “No Tax Case”. (Note that the carbon tax is still imposed.) As discussed above, the efficient extraction path is represented by the Cash Deposit case with no income taxes or royalties. In the No Tax Case the firm is just slightly better (1.9%) off with no surety. The expected government value when there is no surety is the expected present value of the cleanup cost to be incurred. The total value is essentially the same for the Cash Deposit and No Surety cases.

The Cash Deposit and No Surety cases are close in value because their optimal controls are nearly the same. Figure 5(a) shows that expected cumulative production levels with and



(a) Probability of being in each stage



(b) Cumulative production percentiles conditional on a well being drilled.

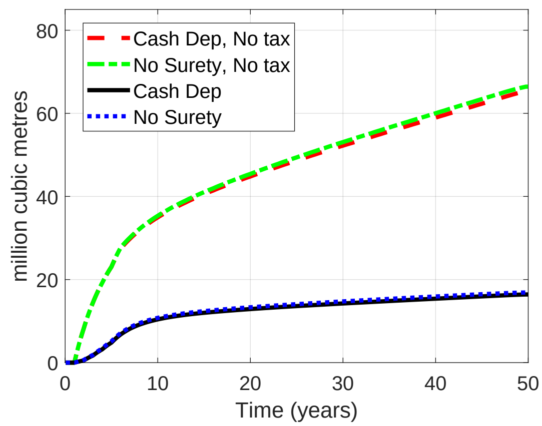
Figure 4: Benchmark case (100% cash bond). Starting gas price = 173 dollars per thousand cubic metre, Initial reserves 110 million cubic meters. Base case cleanup cost of C$135 thousand.

	(Including taxes and royalties)		(No income tax or royalties)	
	Cash Deposit	**No surety**	**Cash Deposit**	**No surety**
Firm value	215	226	2,205	2,247
Government value	1038	1051	0	-43
Total value	1253	1277	2,205	2,204

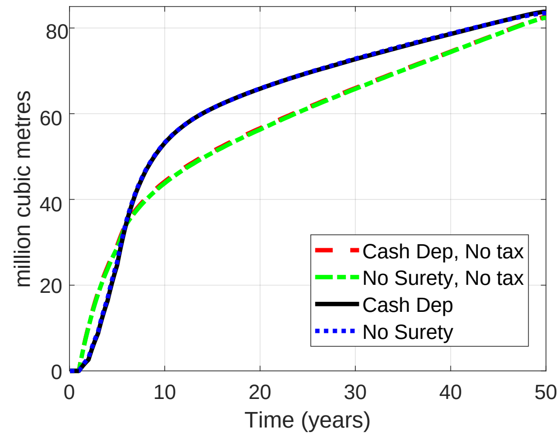
Table 3: Comparing Cash Deposit versus No Surety cases. Expected value at time zero in C$'000. Starting gas price = C$173 per thousand cubic metre, Initial reserves 110 $\times 10^6$ cubic meters. Base case cleanup cost of C$135 thousand.

without the Cash Deposit for all leases (i.e. whether or not a well is drilled) are very close for the cases when taxes are included (lower black and blue lines) and in the No Tax cases (upper green and red lines). Note that that expected cumulative production is substantially higher when no income taxes or royalties are included, indicating the significant distortions caused by these levies. Figure 5(b) shows the same information, but only for leases where wells are drilled. Figure 13 in Appendix B shows the probability of being in each stage for these same cases and confirms that the well is much more likely to be drilled if no taxes or royalties are imposed.

Given the assumptions thus far, the quantitative results indicate that imposing a strong form of surety such as the Cash Deposit does not increase total project value. With No Financial Surety imposed the government ends up having to pay the firm's cleanup costs but this is offset by the higher receipts from income taxes and royalties compared to the Cash Deposit case.



(a) Expected cumulative production, all leases



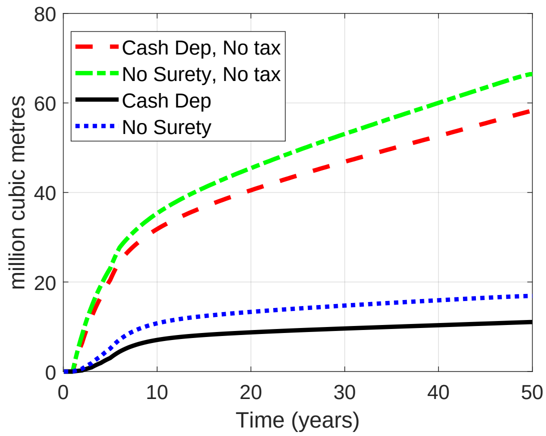
(b) Expected cumulative production, drilled leases

Figure 5: Expected cumulative production, base case cleanup cost. Starting gas price = 173 dollars per thousand cubic metre, Initial reserves 110 million cubic meters. Base case cleanup cost is C$135 thousand.

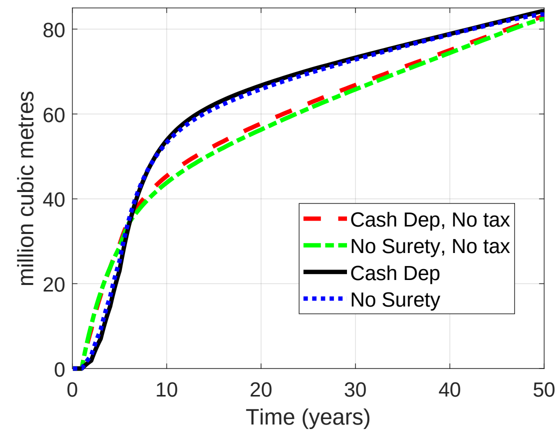
6.1.3 High cleanup costs

This section explores the differences between the Cash Deposit and No Surety cases assuming cleanup costs are increased by a factor of 10 to C$1.353 million. This is of interest given the variability of cleanup costs for oil and gas wells [35] and other types of natural resource extraction projects where cleanup costs are a much larger portion of project value. Figure 6(a) shows that expected cumulative production for all leases under the Cash Deposit policy (black solid curve) is significantly lower than for the No Surety policy (blue dotted line). The figure also indicates that expected cumulative production is substantially higher if no taxes (except the carbon tax) are imposed. Figure 6(b) provides the same information for drilled leases, which gives a similar result as for base case costs. We conclude that the impact of imposing the Cash Deposit requirement is largely due to the reduced probability of a well being drilled. Similarly the impact of taxes is seen through a lower likelihood of drilling the well. Graphs showing the probability of a well being in each stage in the high cost case are contained in Appendix B.

Figure 7(a) shows the expected value at time zero for the firm, government and the total project value for the high cost case. The difference between the Cash Deposit and No Surety policies is now more evident. Both the firm and government are better off under the No Surety policy, and the total expected project value is significantly higher. This is due to fewer projects being undertaken when a Cash Deposit is imposed. Figure 7(b) shows presents the same information when no taxes are imposed (except the carbon tax). In this case we see that the Cash Deposit is now the optimal policy. This is as expected since, as was shown in Proposition 4.2. Without distortionary taxes, the Cash Deposit causes the



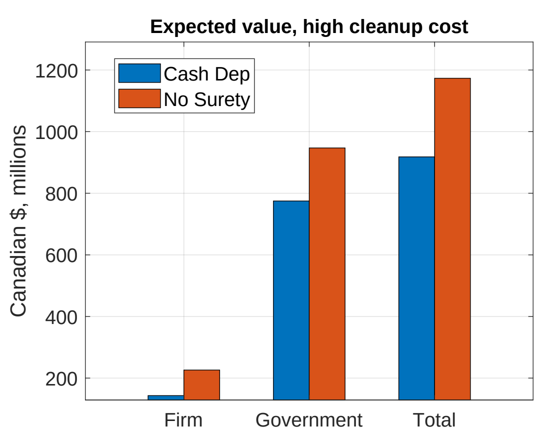
(a) Expected cumulative production, all leases, high cost



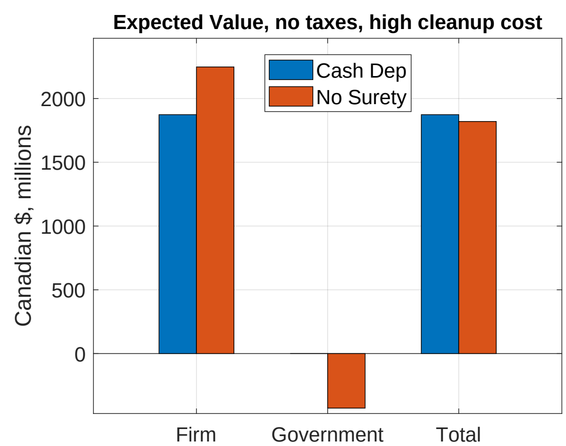
(b) Expected cumulative production, drilled leases, high cost

Figure 6: **High cleanup cost**, Expected cumulative production. Starting gas price = 173 dollars per thousand cubic metre, Initial reserves 110 million cubic meters. Base case cleanup cost is C$135 thousand.

firm to choose the efficient optimal controls for resource extraction.



(a) Expected value, time zero, high cleanup cost



(b) Expected value time zero, No Taxes, high cleanup cost.

Figure 7: **High cleanup costs**, Expected value at time zero. Starting gas price = 173 dollars per thousand cubic metre, Initial reserves 110 million cubic meters. High cleanup cost is C$1.353 million

6.2 Cleanup costs rising over time

This section considers a case in which cleanup costs grow over time. For simplicity it is assumed the cost increase occurs exogenously with the passage of time, regardless of how much time has passed since the well was drilled. This serves to demonstrate some key main points about the desirability of a Cash Deposits versus No Surety without the need to add another path dependent variable to the model.[26] We might rationalize the assumption that costs are growing due to increasingly strict regulations about cleanup, reclamation and monitoring requirements as well as aging well infrastructure.

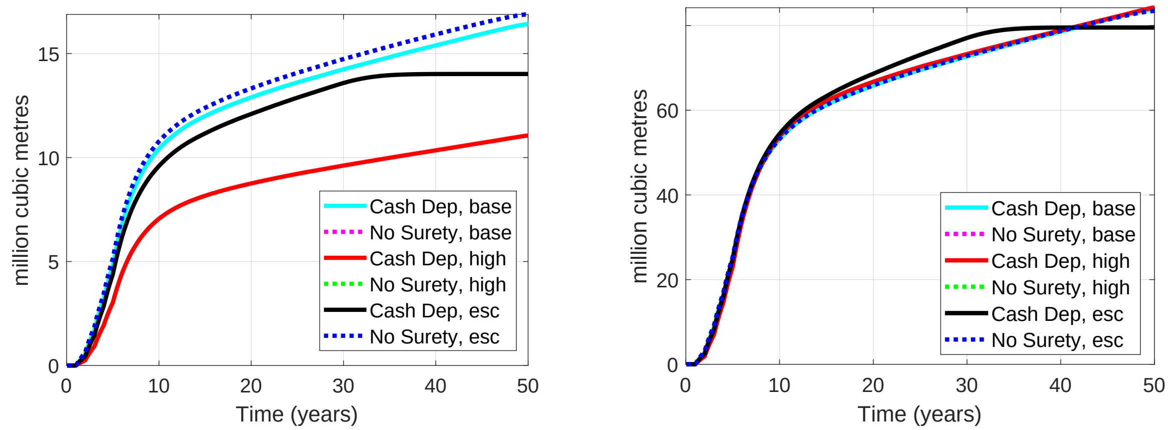
Several different cost escalation assumptions were tried. For example, assuming a moderate increase in cleanup costs of 1% per year beginning in the 20th year after the well is drilled did not have a significant effect on results compared to the base case cleanup cost.

We also considered an extreme case whereby cleanup costs rise by 18% annually beginning in year 30. Figure 8 compares expected cumulative production for the base case, high cost case, and escalating cost cases, with the left hand diagram showing all leases and the right hand diagram showing only drilled leases. Cumulative production for the No Surety policy is the same for all cases; the firm is unaffected by cleanup costs since it is optimal to declare bankruptcy rather than cleanup. In the left diagram, we observe that for the Cash Deposit, the escalating cost and high cost cases both have lower cumulative production than the No Surety cases. The constant high cost cases shows significantly lower cumulative production throughout much of the lease life, while the escalating cost case, not surprisingly, shows cumulative production tailing off significantly after year 30.

The optimal controls for the escalating cost case can be observed more directly Figure 14 in Appendix B. This figure confirms that with a Cash Deposit in the high cost and escalating cost cases, the firm is more likely to remain in stage 1 (i.e. the well is never drilled). The firm in the escalating cost case is much more likely to close early (move to stage 4) than the other cases.

Figure 9 compares expected value at time zero for the three main cases. In Figure 9(a), it is clear that the firm's value is invariant to the different policies in the No Surety cases. Under the Cash Deposit, the firm is worse off particularly in the high cost and escalating cost cases. Looking at Figure 9(b), it will be observed that for the escalating cost case the government is significantly better off with a Cash Deposit, than if No Surety is imposed. This is contrary to the results in the Base and High Cost cases. Total expected value in Figure 9(c) indicates that the Cash Deposit is the optimal policy for the escalating cost case, unlike for the other two cost cases. With the sharply increasing cleanup costs, the Cash Deposit induces the firm to produce the gas more quickly and shut down the well sooner when cleanup costs are lower. Without a Cash Deposit, the firm is content to leave the escalating cleanup costs for the government. As in other cases, the Cash Deposit reduces the likelihood that the well will be drilled, but this is offset by the positive effect on total

[26] The needed extra variable would be "well age" which is dependent on the price of natural gas, a stochastic state variable.



(a) Cumulative expected production, whether or not well is drilled

(b) Cumulative expected production, conditional on well being drilled.

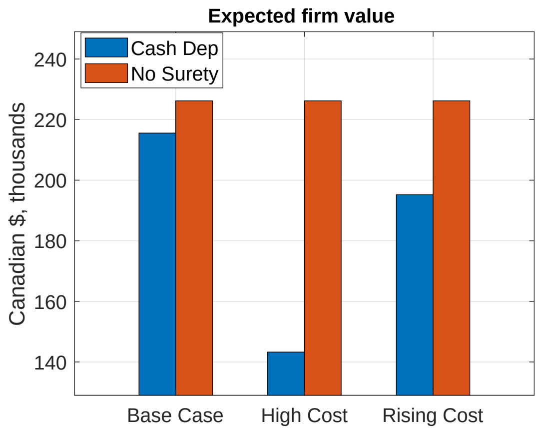
Figure 8: **Expected cumulative production for various cleanup cost cases.** No Surety cases all coincide so that only one dashed curved can be seen. Base cleanup cost is \$135k, high cleanup cost is C\$1.353 million, escalating cleanup cost starts at C\$135k and then escalates at 18% per year starting in year 30 after the well is drilled. Starting gas price is C\$173 per thousand cubic metre, Initial reserves 110 million cubic meters.

cleanup costs. In contrast to the previous cases, even when taxes are royalties are included, the Cash Deposit now induces more efficient behaviour on the part of the firm.

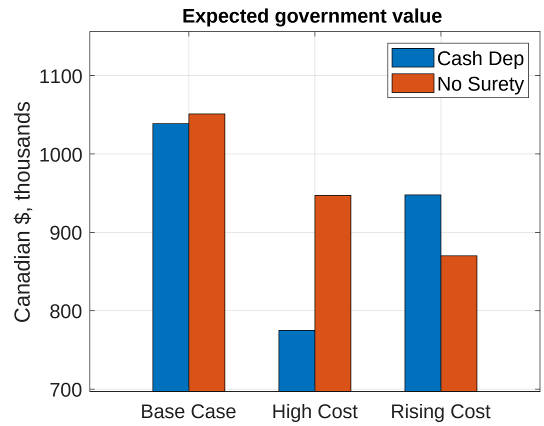
6.3 Surety bond

In this section we determine the fair fee that would be charged to the owner of our hypothetical gas project for the Surety Bond, as described in Sections 3.2 and 3.8, for the High Cleanup cost case. We also consider the impact of a range of different bond fees on the expected value of the project. It is assumed that when the firm goes bankrupt there is no residual value and so in the event of bankruptcy of the gas firm, the guarantor is not reimbursed for any amount of the bond.

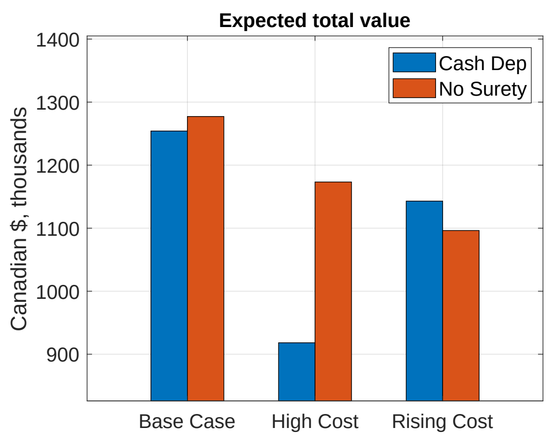
Figure 10(a) plots the present value of expected cleanup costs and the present value of surety payments for an annual fee ranging from zero to C\$170k. The two curves cross at an annual fee of C\$145k, which is 11% of assumed cleanup costs of C\$1.35 million. In Figure 10(b), expected firm value declines precipitously as the annual fee is increased. Figure 10(c) shows that the expected government value is highest with an annual fee of C\$2k per year. In Figure 10(d) total expected value includes the present value of net receipts to the guarantor (Equation (25)). Competitive markets for guarantor providers and assuming no arbitrage opportunities would result in rate of \$145,000 per year. Rates less that \$145,000 imply the guarantor would be losing money on average, and hence would not offer this product. It will



(a) Expected firm value, time zero, all cases



(b) Expected value time zero, government.

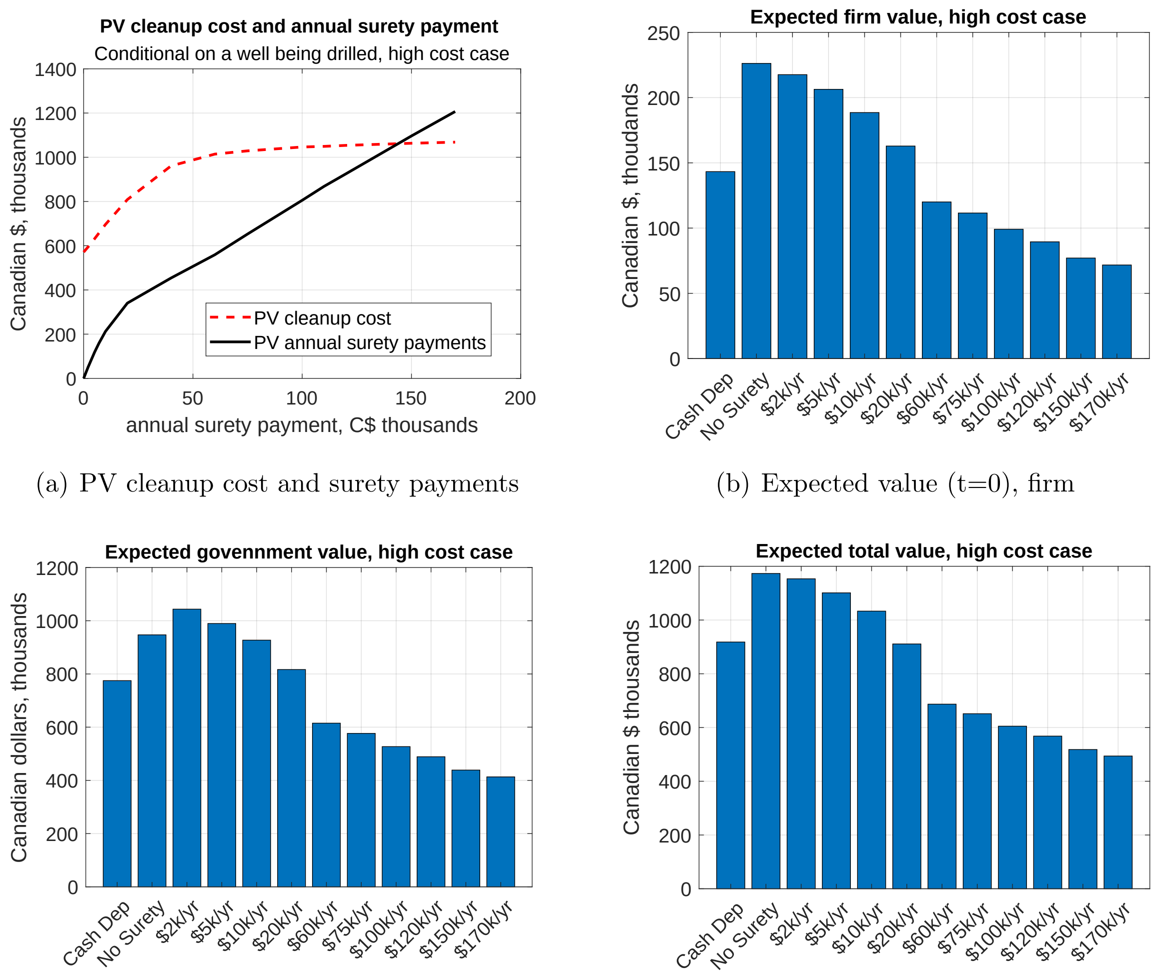


(c) Expected total value time zero.

Figure 9: **All cleanup cost cases**, Expected value at time zero. Starting gas price = 173 dollars per thousand cubic metre, Initial reserves 110 million cubic meters. Base case cleanup costs are C\$135 thousand. High case cleanup costs are C\$1.35 million. Rising cleanup cost starts at C\$135 thousand and escalates at 18% per year beginning in year 30.

be observed that total expected value is highest for the No Surety case and decreases as the annual surety payment rises.

Requiring firms to buy oil and gas surety bonds serves insulates government from liability, but reduces the value of the gas well overall. The reason for this result can be gleaned from Figure 11. Expected cumulative production falls as the annual surety payment rises, causing a loss of value for the gas producer and the government.



(a) PV cleanup cost and surety payments

(b) Expected value (t=0), firm

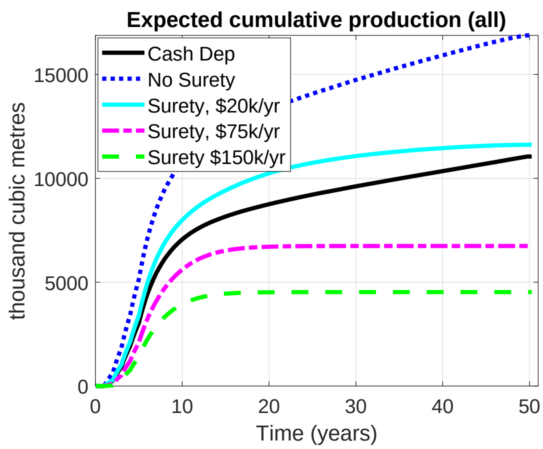
(c) Expected value (t=0), government

(d) Expected value (t=0), total

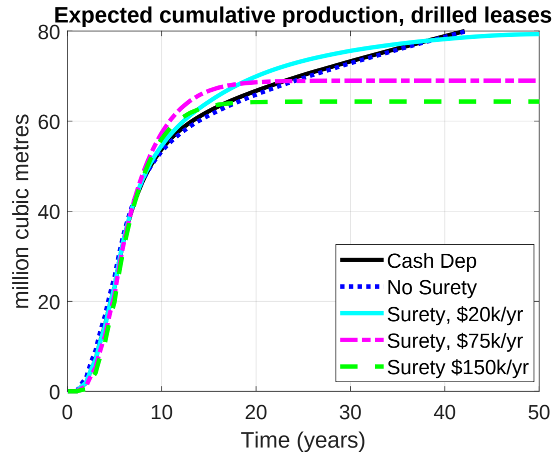
Figure 10: **High Cleanup Cost Case**, Comparing Cash Deposit, No Surety, and various annual Surety Bond payments. Starting gas price = 173 dollars per thousand cubic metre, Initial reserves 110 million cubic meters. High cleanup cost is C$135 million.

7 Concluding Comments

In the presence of taxes and royalties, it is not possible to conclude *a priori* whether strict financial surety in the form of a Cash Deposit will increase the total value of a resource extraction project. This paper has demonstrated numerically a plausible case in which it is better for the government to fund site cleanup out of tax and royalty revenues, rather than impose additional financial assurance requirements on firms. This numerical example uses parameters and data for a natural gas lease in Alberta, Canada. In the presence of distortionary taxes on resource firms, the imposition of financial surety that fully indemnifies



(a) Expected cumulative production, all leases, high cost case



(b) Expected cumulative production, drilled leases, high cost case

Figure 11: **High Cleanup Cost Case**, Comparing expected cumulative production for various annual surety payments. Starting gas price = 173 dollars per thousand cubic metre, Initial reserves 110 million cubic meters. High cleanup cost is C$135 million.

the government reduces the expected cumulative production from the lease. Government receipts may be reduced by more than expected cleanup costs, thereby decreasing the total value of the lease to society.

This conclusion depends on the assumption of known, constant well cleanup cost. The paper also presents a case in which cleanup costs rise sharply towards the end of the lease and the result is that a strong financial surety in the form of a Cash Deposit increases the expected total value of the natural gas lease. In this case the firm produces most of the resource prior to the rise in cleanup costs so that the financial surety requirement does not change firm choices significantly relative to the efficient path and motivates the firm to undertake well cleanup in a timely fashion - i.e. before cleanup costs rise to an exorbitant level.

It should also be noted that the model in this paper assumes the government pays the firm interest on a cash bond at the firm's opportunity cost of capital. If this were not the case, the cash bond would further distort the firm's decisions away from the efficient path.

The paper also considered a case in which a firm is required to purchase an annual Surety Bond guaranty from a third party willing to guarantee the cleanup will be undertaken. It is assumed in this case that markets are sufficiently complete that a guarantor can hedge the risk of a Surety Bond. If this is not the case then the fee charged to firms judged to be a poor financial risk would be higher than the rate calculated in this paper. This would increase the distortionary effect of the Surety Bond.

This paper highlights the importance of considering the impact of any financial assurance requirement on firm behaviour and government resource revenue receipts. Policy recommendations should be specific to a particular industry and tax regime. For the natural gas industry, well cleanup costs are generally well understood, their magnitude is relatively small compared to the value of gas production, and firm operation decisions do not have a significant effect on their size. This contrasts with the mining industry, examined in [3], where cleanup costs are large and grow with the quantity of ore produced (via tailings ponds). In addition, ongoing remediation decisions affect the level of future cleanup costs. A strong form of financial surety is more likely to increase total resource value in the mining industry.

The suggestion that under certain circumstances governments should pay for the cleanup of oil and gas wells is contrary to the polluter pay principle, may be perceived as being unfair to the tax payer and may be politically unpalatable. There may also be other unintended consequences of such a policy that are not considered here. For example, would such a policy cause firms to take less precaution and care in their drilling and production activity such that cleanup costs would be significantly affected. In order to make policy recommendations for individual industries, solid research is needed on the determinants of cleanup costs as well as the potential public health and environmental consequences of improper cleanup and remediation. A better understanding is also needed of potential firm responses to various financial surety instruments.

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A Expected present value of Cash Deposit cash flows

This appendix shows the derivation of Equation (27). Cash flows from the Cash Deposit for the firm consist of the initial deposit when the well is drilled, fully covering cleanup costs, any subsequent deposits in the event that cleanup costs increase over time, and interest paid on the bond by the government to the firm. (This excludes the refund of the deposit upon well closure, as the firm's closure and cleanup costs will exactly match the amount of the refund.)

A.1 Prior to drilling the well, $\delta = \delta_1$

Starting in the pre-drilling state ($\delta = \delta_1$), the expected value of bond cash flows given the control set K is

$$\begin{aligned}\mathbb{E}_K\left[\int_{t'=t}^{\hat{T}} e^{-r(t'-t)}\pi^{\Omega}(\delta,t')dt'\middle|\ \delta(t)=\delta_1,\ t<t_s\right] \\ =\mathbb{E}_K\left[\int_{t'=t}^{\hat{T}} e^{-r(\hat{T}-t)}\left(\rho\Omega(\delta,t')\mathbb{1}_{\delta\neq\delta_1}-\frac{d\Omega(\delta,t')}{dt}\right)dt'\ \middle|\ \delta(t)=\delta_1,\ t<t_s\right].\end{aligned} \tag{32}$$

Note that the term reflecting interest earned on the bond (at rate ρ) is present only once the well has been drilled, and hence not when $\delta = \delta_1$. We include the indicator function, $\mathbb{1}_{\delta\neq\delta_1}$ for clarity where $\mathbb{1}_{\delta\neq\delta_1}$ equals 1 if the condition $\delta \neq \delta_1$ is true and equals zero otherwise. Recall from Equation (11),

$$\frac{d\Omega(\delta,t)}{dt} = \mathbb{D}(t-t_s)\zeta_c + \frac{dC_c(t)}{dt}\mathbb{1}_{\delta\neq\delta_1} \tag{33}$$

where $\mathbb{D}$ is the Dirac function also known as the Dirac delta function, ζ is the initial Cash Deposit required to start drilling, t_s is the drilling date, and $\frac{dC_c(t)}{dt}\mathbb{1}_{\delta\neq\delta_1}$ accounts for any change in cleanup cost (which requires further payments into the Cash Deposit) after drilling is completed.

Substituting Equation (33) into Equation (32) gives:

$$\begin{aligned}\mathbb{E}_K\left[\int_{t'=t}^{\hat{T}} e^{-r(t'-t)}\pi^{\Omega}(\delta,t')dt'\middle|\ \delta(t)=\delta_1,\ t<t_s\right] = \mathbb{E}_K\Bigg[\Bigg(\overbrace{\int_{t'=t}^{\hat{T}} e^{-r(t'-t)}\rho\Omega(\delta,t')\mathbb{1}_{\delta\neq\delta_1}dt'}^{\text{interest payments}} \\ -\overbrace{e^{-r(t_{s^+}-t)}\Omega(\delta,t_{s^+})}^{\text{payment at well start}}-\overbrace{\int_{t'=t_s^+}^{\hat{T}} e^{-r(t'-t)}\frac{dC_c(t')}{dt'}\mathbb{1}_{\delta\neq\delta_1}dt'}^{\text{payments with rising cleanup costs}}\Bigg)\ \Bigg|\ \delta(t)=\delta_1,\ t<t_s\Bigg].\end{aligned} \tag{34}$$

where t_{s^+} refers to the instant after the well has been drilled. This uses the fact that the integral associated with the initial Cash Deposit is $\int_{t'=t}^{\hat{T}}\mathbb{D}(t'-t_s)\zeta dt' = \zeta = \Omega(\delta,t_{s^+})$.

The first term on the right hand side of Equation (34) may be integrated by parts resulting in:

$$\int_{t'=t}^{\hat{T}} e^{-r(t'-t)} \rho \Omega(\delta, t') \mathbb{1}_{\delta \neq \delta_1} dt' = \tag{35}$$
$$-\frac{\rho}{r}\Omega(\delta, \hat{T}) e^{-r(\hat{T}-t)} \mathbb{1}_{\delta \neq \delta_1} + \frac{\rho}{r}\Omega(\delta, t_{s^+}) e^{-r(t_{s^+}-t)} \mathbb{1}_{\delta \neq \delta_1} + \frac{\rho}{r}\int_{t'=t_{s^+}}^{\hat{T}} e^{-r(t'-t)} \frac{dC_t(t')}{dt'} \mathbb{1}_{\delta \neq \delta_1} dt'.$$

Substituting Equation (35) into Equation (34) gives:

$$\mathbb{E}_K\left[\int_{t'=t}^{\hat{T}} e^{-r(t'-t)} \pi^{\Omega}(\delta, t') dt' \,\middle|\, \delta(t) = \delta_1,\ t < t_s \right] = \tag{36}$$
$$\mathbb{E}_K\Bigg[\Bigg(- e^{-r(t_{s^+}-t)} \Omega(\delta, t_{s^+}) + \left(\frac{\rho}{r} - 1\right) \int_{t'=t_{s^+}}^{\hat{T}} e^{-r(t'-t)} \frac{dC_c(t')}{dt'} dt' \, \mathbb{1}_{\delta \neq \delta_1}$$
$$+ \frac{\rho}{r}\left(e^{-r(t_{s^+}-t)} \Omega(\delta, t_{s^+}) - e^{-r(\hat{T}-t)} \Omega(\delta, \hat{T}) \right) \mathbb{1}_{\delta \neq \delta_1} \Bigg) \,\Bigg|\, \delta(t) = \delta_1,\ t < t_s \Bigg]$$

If $\rho = r$, this simplifies to:

$$\mathbb{E}_K\Big[\int_{t'=t}^{\hat{T}} e^{-r(t'-t)} \pi^{\Omega}(\delta, t') dt' \,\Big|\, \delta(t) = \delta_1,\ t < t_s \Big] = \mathbb{E}_K\Big[- e^{-r(\hat{T}-t)} \Omega(\delta, \hat{T}), \,\Big|\, \delta(t) = \delta_1,\ t > t_s \Big]. \tag{37}$$

A.2 After the well start date, $\delta \neq \delta_1$

For the convenience of the reader (and to tie in with Lemma 4.1 in [3]), we derive the expected value of cash flows for the Cash Deposit after the well is drilled. After the well start date $t > t_s$, the initial cash payment will have been made. The expected value of bond cash flows from t onward, where the starting stage is $\delta \neq \delta_1$ is given as:

$$\mathbb{E}_K\left[\int_{t'=t}^{\hat{T}} e^{-r(t'-t)} \pi^{\Omega}(\delta, t') dt' \,\middle|\, \delta(t) \neq \delta_1,\ t > t_s \right] \tag{38}$$
$$= \mathbb{E}_K\left[\int_{t'=t}^{\hat{T}} e^{-r(t'-t)} \left(\rho \Omega(\delta, t') \mathbb{1}_{\delta \neq \delta_1} - \frac{dC_c(t')}{dt'} \right) dt' \,\middle|\, \delta(t) \neq \delta_1,\ t > t_s \right].$$

The term $e^{-r(t'-t)}\rho(\Omega(\delta, t))$ can be integrated by parts giving a result similar to Equation (35), except that the lower limit of integration is now $t > t_s$ Substituting the result into

Equation (38):

$$\mathbb{E}_K\left[\int_{t'=t}^{\hat{T}} e^{-r(t'-t)}\pi^{\Omega}(\delta,t')dt' \,\middle|\, \delta(t)\neq\delta_1,\ t>t_s\right] = \mathbb{E}_K\Bigg[\Bigg(\left(\frac{\rho}{r}-1\right)\int_{t'=t}^{\hat{T}} e^{-r(t'-t)}\frac{dC_c(t')}{dt'}dt' \tag{39}$$

$$+\frac{\rho}{r}\left(\Omega(t)-e^{-r(\hat{T}-t)}\Omega(\delta,\hat{T})\right)\Bigg)\,\Bigg|\,\delta(t)\neq\delta_1,\ t>t_s\Bigg]$$

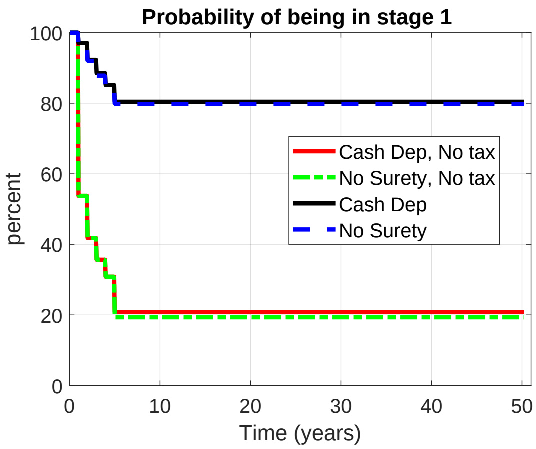
In this case, if $\rho = r$, then

$$\mathbb{E}_K\Big[\int_{t'=t}^{\hat{T}} e^{-r(t'-t)}\pi^{\Omega}(\delta,t)dt' \,\Big|\, \delta(t)\neq\delta_1,\ t>t_s\Big] = \mathbb{E}_K\Big[\Big(-e^{-r(\hat{T}-t)}\Omega(\delta,\hat{T})+\Omega(t)\Big)\,\Big|\,\delta(t)\neq\delta_1,\ t>t_s\Big]. \tag{40}$$

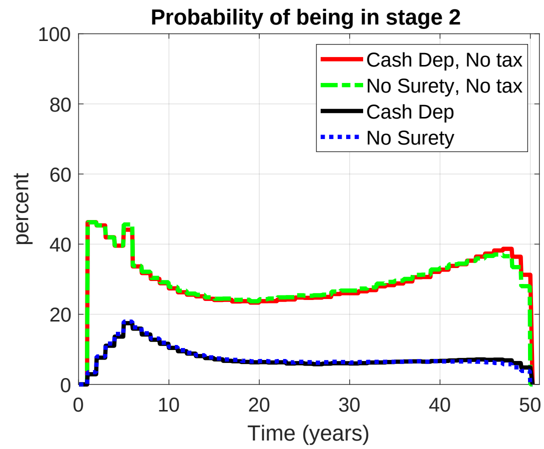
For $t < t_s$, $\Omega(t) = 0$ and we are left with the result in Equation (37). Equation (40) matches the expression in Lemma 4.1 of [3].

B Extra figures

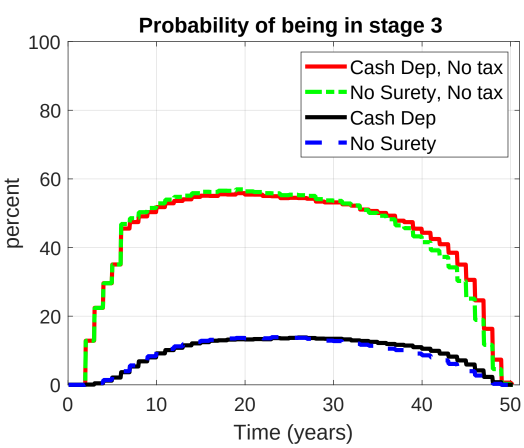
B.1 Base Case Cleanup Cost



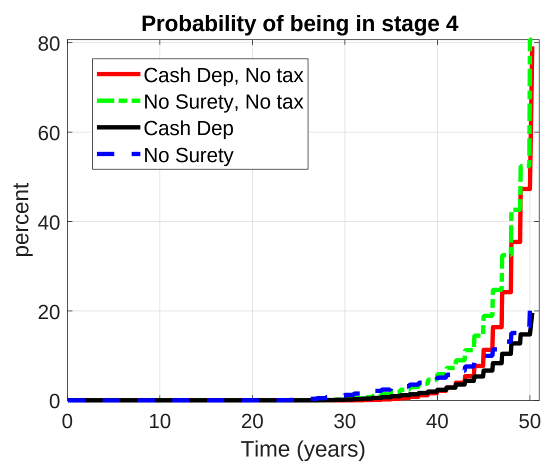
(a) Probability of being in stage 1



(b) Probability of being in stage 2



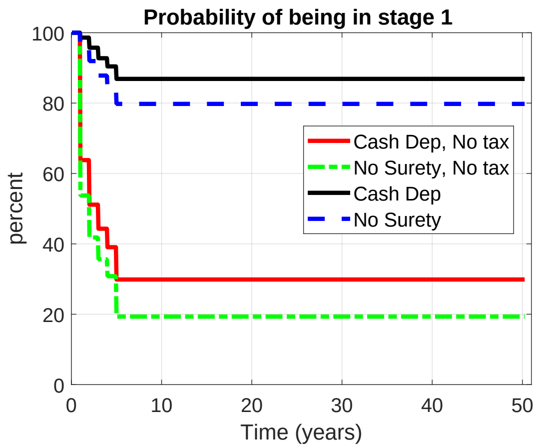
(c) Probability of being in stage 3



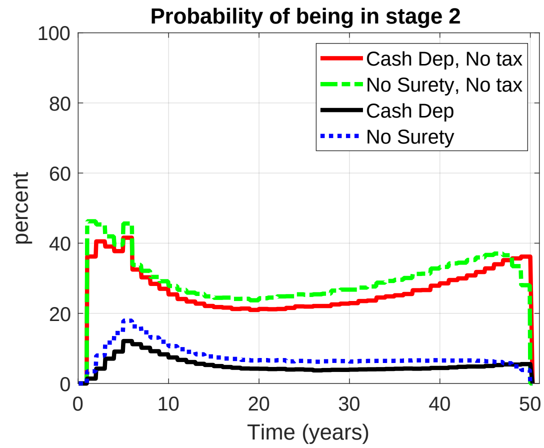
(d) Probability of being in stage 4

Figure 12: Probability of being in different stages for the cases: Cash Deposit; No Surety; No Tax: Cash Deposit; and No Tax: No Surety. Starting gas price = 173 dollars per thousand cubic metre, Initial reserves 110 million cubic meters. Base case cleanup cost of C$135 thousand.

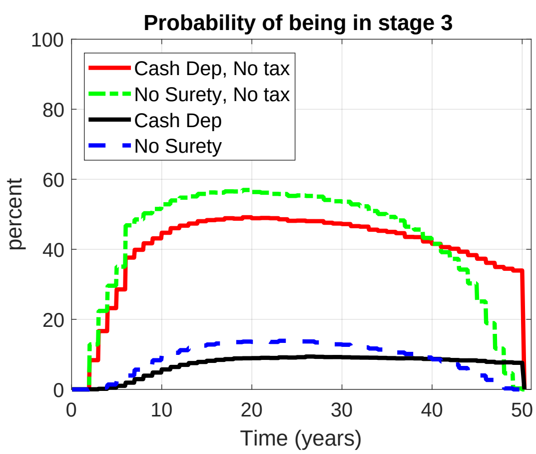
B.2 High Cleanup Costs



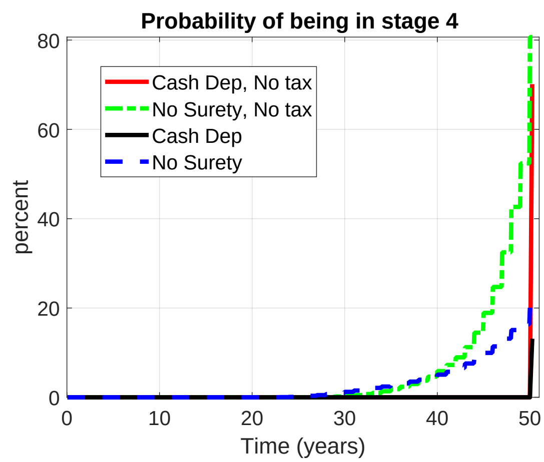
(a) Probability of being in stage 1



(b) Probability of being in stage 2



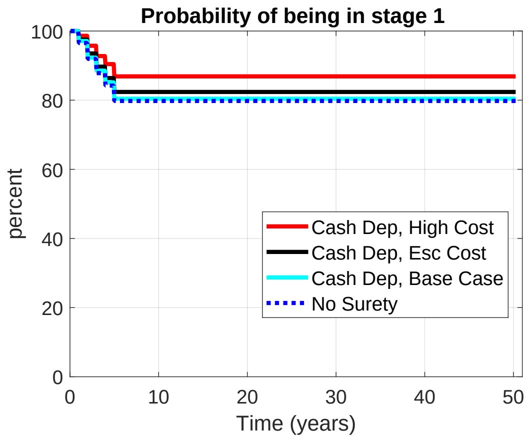
(c) Probability of being in stage 3



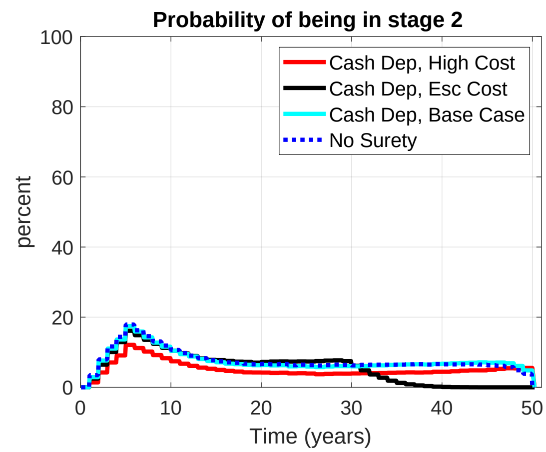
(d) Probability of being in stage 4

Figure 13: Probability of being in different stages for the cases: Cash Deposit; No Surety; No Tax: Cash Deposit; and No Tax: No Surety. Starting gas price = 173 dollars per thousand cubic metre, Initial reserves 110 million cubic meters. High cleanup cost of C$1.353 million.

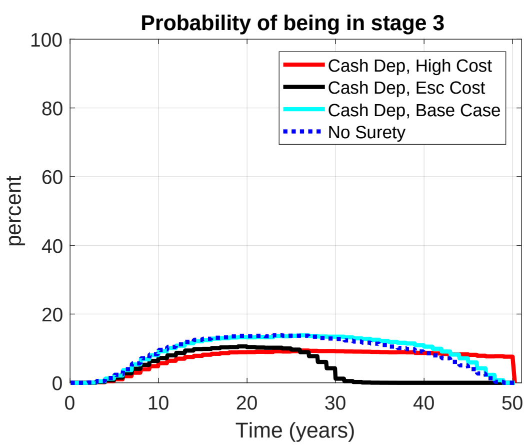
B.3 Escalating cleanup costs



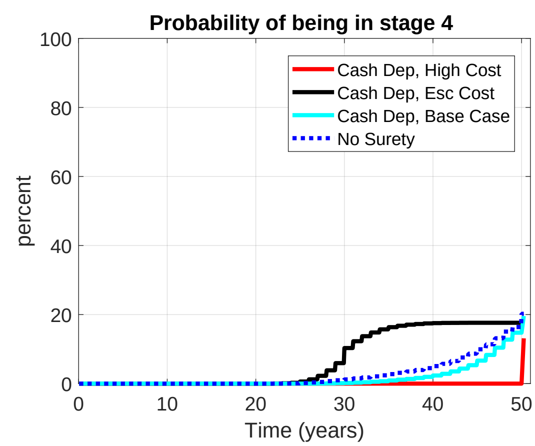
(a) Probability of being in stage 1



(b) Probability of being in stage 2



(c) Probability of being in stage 3



(d) Probability of being in stage 4

Figure 14: Probability of being in different stages for all cases. Starting gas price = 173 dollars per thousand cubic metre, Initial reserves 110 million cubic meters. Base case cleanup cost of C$135 thousand,, high cleanup cost is C$1.353 million, escalating cleanup cost starts at $135k and then escalates at 18% per year starting in year 30 after the well is drilled.

C Numerical solution of the optimal control problem and Monte Carlo analysis

This section briefly describes the numerical approach to solving the stochastic optimal control problem given by Equations (19)–(22). The computational domain of Equation (19) is

$(p, s, \delta_i, t) \in \Gamma$ where $\Gamma \equiv [0, p_{max}] \times [0, s_0] \times [\delta_1, \delta_2, \delta_3] \times [0, T]$. p_{max} is chosen large enough to approximate an infinite domain. s_0 are initial reserves. In Equation (19), the terms

$$\frac{1}{2}\sigma^2 p^2 \; \frac{\partial^2 V}{\partial p^2} + \big(\eta(\bar{P} - p) - \lambda\sigma\big) \; \frac{\partial V}{\partial p} - rV, \tag{41}$$

can be discretized using a standard finite difference approach while the other terms in the equation are discretized using a semi-Lagrangian scheme. These approaches are described in [10] and references therein. The numerical computations increase in accuracy as the solution grid is refined, but at the cost of a significant increase in computation time. We chose a level of grid refinement such that the maximum error from the approximation of the PDE is 2%.

The numerical solution of Equations (19)–(22) specifies the firm's strategy in terms of the operating stage and time of bankruptcy or closure. These are feedback controls and depend on the state variables, including the natural gas price, natural gas reserves and time. We compute and store the optimal controls as a function of state variables. We then integrate the stochastic differential equation for natural gas prices (Equation (8)) using a Monte Carlo analysis for a given starting price, as well as chosen starting values for other state variables. At each time step we look up the stored optimal controls associated with the new price level and current state variables, and then update all other state variables. This proceeds until the lease end date at year 50. We do this for a large number of possible price paths (30000), and then calculate the expected value, median and other percentiles of key variables of interest as they evolve over time. The expected values calculated from the Monte Carlo analysis should agree with the values from the solution of the HJB equation, to an acceptable level of accuracy. In effect, the Monte Carlo analysis provides a check on the HJB equation solution. Our Monte Carlo results are within 0.5% of the PDE results, indicating a good level of accuracy.

C.1 Boundary conditions

Boundary conditions at upper and lower bounds of p, r, and t are described in this section.

- Evaluation of Equation (19) as the commodity price $\boldsymbol{p \to 0}$ implies the following.

$$\frac{\partial V}{\partial t} + \big(\eta\bar{P} - \lambda\sigma\big)\frac{\partial V}{\partial p} + \Pi - q \; \frac{\partial V}{\partial s} - rV = 0. \tag{42}$$

  No special boundary condition is needed at $p = 0$ as the PDE reduces to a first order hyperbolic equation, with outgoing characteristics. No information from outside the computational domain is needed; the PDE supplies the required information.[27]

- As $\boldsymbol{p \to p_{max}}$, we assume $\frac{\partial^2 V}{\partial p^2} \to 0$, which from Equation (19) implies:

$$0 = \frac{\partial V}{\partial t} + \big(\eta(\bar{P} - p) - \lambda\sigma\big)\frac{\partial V}{\partial p} \; - rV + \Pi - q \; \frac{\partial V}{\partial s} \tag{43}$$

[27] See [14] for a discussion of boundary conditions.

The assumption that V is linear in p is common in the literature [25]. The value chosen for p_{max} is intended to approximate an infinite upper limit.

- As $\boldsymbol{s \to 0}$, the admissible set of values for q collapses to zero as shown in Equation (5). No boundary condition is needed.
- As $\boldsymbol{s \to s_{max}}$, no special boundary conditions is required as Equation (19) has outgoing characteristics in the s direction.
- At $\boldsymbol{t = \hat{T}}$, the value of the well is the discounted final payout at the optimal termination time as indicated in Equation (17).