Stockpiling Liquidity to Acquire Innovation

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Abstract

Cash utilization in U.S. merger and acquisition (M&A) transactions has increased over 50% since the early 1990s amidst a secular, global M&A boom. How does this cash-use relate to firms' cash stockpiles, and what are the aggregate implications for firm innovation, growth and monetary policy? To answer these questions, we pose a general equilibrium theory of R&D-intensive firm cash stockpiling and use in M&A transactions. M&A cash bids can close faster than those externally financed, hence reducing the hazard of competing offers and external risks of trade breakdown. A higher common-value component in M&A arising from transferable productivity of firms' intangible assets spurs increased M&A competition and serial acquirer cash-stockpiles. Despite sellers receiving a cash-premium as compensation, cash-use biases M&A rents and growth incentives towards serial acquirers. Higher nominal interest rates differentially impact internal and external growth incentives across firms, re-shaping the firm-size and productivity distribution. Calibrated to the U.S. economy, we find that increasing transferable productivity differences and M&A competition, not interest rates, can account for the majority of aggregate firm cash stockpiles since 1990.

Keywords: Mergers and Acquisitions (M&A), Intangible Assets, Firm Cash- Stockpiles, M&A cash-premium, Firm Dynamics, Endogenous Growth, Search & Matching, Monetary Policy

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1 Introduction

Information communication systems and intellectual property have proliferated and become critical to modern firms. These intangible assets fundamentally differ from physical capital in their non-rivalry, allowing for the same capital to be deployed across different firm business lines. This increased transferability of productivity encourages reallocation of sales towards “superstar” firms with the most productive intangible assets.

The merger and acquisition (M&A) market is a natural conduit for this reallocation, with global announced deal value rising from approximately $500 billion to $3.9 trillion over the past three decades.In particular, superstar firms seem to be among the most active in the M&A market with 5 of the highest-valued public tech giants alone disclosing they have acquired over 700 firms in the past three decades.[1] At the same time, these firms which are most active in M&A also appear amongst the largest holders of cash.[2] These cash stockpiles pose a puzzle to existing theories since these firms have the lowest cost of capital contrary to Falato et al. (2022), are highly productive and intensely scrutinzed firms inconsistent with an agency story such as Nikolov and Whited (2014), on aggregate only marginaly reduced their cash holdings with the reduced repatriation tax on foreign income by the Tax Cuts and Jobs Act (TCJA) as evaluated by Foley et al. (2007), Faulkender et al. (2019), Bennett and Wang (2021), and Garcia-Bernardo et al. (2022), nor has moved in a consistent manner with interest rate fluctuations (e.g. Azar et al. (2016), and Gao et al. (2021)).[3]

In this paper we examine the extent that these cash stockpiles of highly acquisitive, intangible superstars is not a coincidence, but are in fact driven by competitive threats in the M&A market. While firm-to-firm mergers are classically viewed as reflecting private synergies, we argue the transferability of intangible assets increases the common value component of target firms, raising the likelihood of competing bidders for the same target. Provided cash M&A offers can lower the hazard of competing offers through reduced public disclosure and shortening the time to close the deal, cash stockpiles may facilitate acquirers retaining a higher share of the acquisition surplus

[1]For details, see https://www.americanactionforum.org/insight/the-government-should-not-ban-mergers-and-buyouts/ and https://www.antitrustinstitute.org/wp-content/uploads/2019/07/Merger-Enforcement_Big-Tech_7.8.19.pdf. Many more acquisitions by these top 5 firms have very recently begun being investigated by a new FTC probe: https://www.crn.com/news/ftc-probing-past-apple-alphabet-amazon-facebook-microsoft-acquisitions.

[2]E.g. Apple, Microsoft and Alphabet alone accounted for one-quarter of the 1.9 trillion USD non-financial corporate US cash in 2016, with Apple’s cash/asset ratio around 33%, see https://www.spglobal.com/en/research-insights/articles/us-corporate-cash-reaches-19-trillion-but-rising-debt-and-tax-reform-pose-risk. For more discussion of the rise of non-financial corporate cash and its high levels of concentration amongst the top tech firms see Pinkowitz et al. (2013).

[3]Moreover, the debt capacity constraints argued by Falato et al. (2022) require collateral constraints tied to tangible capital, rather then debt capacity connected to operating cashflow which has been documented by Lian and Ma (2021) to be pervasive for modern public firms.

and probability of success.

First, we develop and provide empirical support, for a theory which captures this mechanism. Second, we embed it into a rich firm dynamics and endogenous growth model to examine the interplay between firm liquidity demand, firm concentration and growth. Here we demonstrate that this framework creates a novel link between firm-based innovative activities and monetary policy, with interest rates influencing the anticipated terms of trade in the M&A market and consequently the distribution of innovative activity across prospective M&A buyers and sellers. Third, we calibrate the model to the US and decompose the secular changes in firm cash stockpiles and market concentration observed between 1990 and 2015. Finally, we evaluate the aggregate implications, net the potentially asymmetric distributional effects, of various counterfactual monetary policy and M&A market interventions.

The model builds off the workhorse applied model of endogenous growth with firm dynamics developed by Klette and Kortum (2004).[4] In this model, firms own multiple product lines. Each firm follows a stochastic birth-death process governed by their internal rate of innovation and the aggregate rate of creative destruction. Similar to Acemoglu et al. (2018), we start our departure from their framework by introducing heterogeneity across firms in their (per-period) fixed cost to having a product on the market. These differences in fixed costs provide motive for re-allocation of product lines which we allow through a frictional M&A market. The M&A market features two-sided search between buyers and sellers akin to David (2021), however, we add the possibility of competing bidders, since Boone and Mulherin (2007) find that approximately 50% of M&A involve multiple competing buyers for the same seller. Competition amongst buyers for a given seller may occur due to physical delays in the closing of an agreed deal and limited commitment from the seller in not considering new bids (e.g. through go-shop provisions).[5] The threat of competition pins down the effective bargaining power of the seller in the model. Stockpiled liquidity can be used by a buyer to hasten the closing of a deal and lower the hazard of a competing bid. As a consequence, higher anticipated levels of competition increase the demand for cash by prospective buyers in the M&A market. Expected terms of trade in the M&A market then feedback into new entrants and other prospective sellers' incentives to innovate and thus can influence market concentration.

In addition to the model's ability to rationalize the increased cash demand by large, public innovative firms, the model also offers a new theory

[4]The model was shown to exhibit many patterns found in the micro-data by Lentz and Mortensen (2008). For examples of its applications, see Acemoglu et al. (2018), Lentz and Mortensen (2016) and Akcigit and Kerr (2018).

[5]A similar notion of speed providing an advantage in M&A was considered by Offenberg and Pirinsky (2015) in a partial equilibrium setting with one potential rival buyer for understanding tender offers, but in the presence of information frictions. Further to our knowledge this is the first paper to examine how this speed advantage fuels firm demand for liquidity stockpiles in general equilibrium.

of pricing and allocation in the M&A market. The theory is consistent with a variety of micro-evidence documented by Betton et al. (2008). The model yields a closed form surplus sharing rule for the initial bidder that depends on the financing choice of the initial bidder and a given level of competition (i.e. buyer-seller ratio). Furthermore, the theory generates a wedge between cash and externally financed offers (e.g. stock) which leads to a cash-premium that is increasing in the level of anticipated competition. This finding provides a possible rationalization of Malmendier et al. (2016) in which cash-offers provide on average a 15% cash-premium over stock-offers. The theory can also account for the co-existence of both cash and stock M&A offers as well as the correlation that stock offers tend to be larger than cash and are on average worse deals for acquirers. Finally, the model allows for real effects of monetary policy on firm's cash demand which can indirectly influence the incentives to innovate across buyers and sellers and therefore affect growth.

We calibrate the model to the US using moments on firm-level innovation, cash holdings and M&A activity, as well as aggregate census data on the entry rate of firms. We then re-calibrate the model to data from the 2010s but restrict adjustments in the parameters of only the entry costs, markups, holding cost of cash and fixed costs of the high productivity firms. We find that to account for the average cash/asset ratio rise observed in US firms, markups, entry costs and holding costs can only account for at maximum 24% of the increase. However, when allowing the transferable fixed production costs to vary, we can account for nearly the entire increase in cash holdings, that is, we can account for 94% of the 2015 level, with an 82% drop in the fixed costs of the high efficiency firms. This comes with a 7% increase in the concentration of firms and reduces aggregate innovation by 17% (although consumption growth itself remains fairly flat due to an increase in quality improvements).

Finally, to explore the role of policy in this model, we examine from our 2015 calibration, the effects of banning mergers (as proposed by Senator Amy Klobuchar) or cash use, as well as increasing inflation (equilvalently nominal interest rates). The results suggest to the extent that to the extent acquisitions are driven by gains from transferable productivity, cash-mergers facilitated re-allocation and increased welfare.[6] Moroever, monetary policy has a non-neutral, and significant effect short-run and long-run effects, including on the distribution of innovative activity, firm-size and aggregate growth through the M&A market. The results highlight how inflation can not only affect internal vs external growth incentives, but has heterogeneous impacts for small and large firms and the average terms of trade in M&A. The effects are non-monotonic, and can differ from standard monetary models where the marginal demanders of cash are households rather than large

[6]For details, see https://www.americanactionforum.org/insight/the-government-should-not-ban-mergers-and-buyouts/.

public firms who have outside options to rely on their own issued equity as a payment instrument. Perhaps most notably, the results demonstrate that the Friedman rule, or zero lower bound, is not in general optimal, since the speed advantage of cash can help facilitate more efficient reallocation of sales.

The remainder of the paper is organized as follows. We conclude this section with a review of the related literature. Section 2 documents some stylized facts which motivate the model. Section 3 describes the model while Section 4 presents the main theoretical results. Section 5 describes our model calibration and decomposes the secular changes. Section 6 quantitatively examines some policy counterfactuals, and we providing some concluding thoughts in Section 7.

Related literature: This paper builds off and contributes to several literatures. First, this paper contributes to the growing debate on the implications of the declining business dynamism observed across much of the developed world (e.g. Decker et al. (2017)). On one side, Covarrubias et al. (2019), Gutiérrez and Philippon (2017) and Grullon et al. (2019) argue that concentration has been the result of lax anti-trust and rising entrenchment of incumbent firms. Taking a less negative view, Autor et al. (2020) and Andrews et al. (2016) use micro panel data evidence from the US census and OECD nations respectively in support of a technological shift leading to winner-takes-most, 'superstar' firms. This paper tests and provides support for one potential driver of the superstar phenomenon with declining fixed costs of bringing on product to market, discussed by Bessen (2017) (e.g. Walmart's / Amazon's proprietary inventory management systems). Ma et al. (2016) finds that acquiring firms invest substantially in IT and hire less routine-intensive labour following an acquisition suggesting that these acquisitions facilitate a lowering / pooling of operating costs across pre and post-merged firms (or business lines).

This paper also contributes to the literature examining M&A market activity and its effect on misallocation in the macro-economy. Acquisitions can boost aggregate efficiency through re-allocating production inputs to higher productivity firms as in Jovanovic and Rousseau (2002), or achieving synergies in production like in Rhodes-Kropfe and Robinson (2008) or David (2021).[7] However, M&A can raise market power for incumbents, raising

[7]Study of the M&A market has increasingly been studied subject to search and matching frictions. Rhodes-Kropfe and Robinson (2008) study the assortative matching of firms in the merging of productivity, Levine (2017) studies the trade of seeds, but do not consider firm innovation and creative destruction. David (2021) examines the aggregate impact on growth of a real model of reallocation of firm productivity but without strategic considerations, opportunity for internal innovation, or financing frictions. Fons-Rosen et al. (2021) and Cortes et al. (2021) examines the substitution between acquisitions and internal innovative efforts and their anti-competitive and growth implications. Wang (2018) estimates anticipation effects embedded in merger premia, Celik et al. (2022) examine the role of equity M&A offers in mitigating adverse selection, particularly for more intangible, growth-oriented firms. Finally, Wright et al. (2018) study the aggregate implications of the interaction of frictional cap-

anti-trust concerns (e.g. Mermelstein et al. (2020)) and reducing the returns for a new innovator to bring a product to market (see Phillips and Zhdanov (2013)).

Interest has been growing in the determinants of innovation and potential misallocation in innovative capacity. Notable papers in this vein is Acemoglu et al. (2018) who introduce fixed costs and heterogeneous R&D capabilities to examine the misallocation of R&D inputs and Akcigit and Kerr (2018) who examine heterogeneity in the types and quality of innovation between large and small firms. To ourknowledge, the only papers to examine firms outsourcing or re-allocating growth opportunities through the M&A market are Phillips and Zhdanov (2013), and Levine (2017). Our paper examines a similar trade off of the former wherein bargaining power in the M&A market can influence small firms incentives to innovate, but in an endogenous growth, general equilibrium setting. In another related paper, Lentz and Mortensen (2016) who examine the social value of buyouts by new entrants of incumbent firms existing products. This paper currently abstracts from more pernicious aspects of M&A studied by Cunningham et al. (2021) of so-called 'killer acquisitions' in the pharmaceutical industry where innovation is stifled to protect incumbents existing products.

There is also some work examining linkages between cash holdings, concentration / growth and monetary policy. Liu et al. (2019) argue low long-term interest rates encourage market concentration by raising the benefit for industry leaders to gain a strategic advantage over followers. Our paper complements this work with the mechanism that lower opportunity costs of stockpiling liquidity increases the net benefits of being an acquirer. The only other equilibrium models linking monetary policy to innovation to our knowledge is Chu and Cozzi (2014) and Berentsen et al. (2012). The former imposes an exogenous cash-in-advance constraint on R&D and manufacturing expenditures. The latter assumes anonymity of entrepreneurs to induce a demand for cash. As such, their setting is not amenable to talk about the cash demand of large public firms who, by definition and in practice, have access to plethora of external financing options.

Finally, papers which examine the interaction of competition and cash holdings are Hoberg et al. (2014), Ma et al. (2014) and Galenianos and Kircher (2008).[8] The former two examine how cash provides strategic bene-

ital re-allocation with cash needed to facilitate trade for firms without access to alternative financing options tied to reputation.

[8]The corporate finance literature on the determinants of firm cash accumulation is extensive, beginning with Baumol (1952)-Tobin (1956) transaction cost motive, then extending to tax minimization (e.g., Foley et al. (2007), Faulkender and Petersen (2012)) or handling agency frictions (e.g., Jensen (1986), Dittmar and Mahrt-Smith (2007)). Explanations for the secular cash build-up focus on a selection effect of R&D intensive firms (e.g. Begenau and Palazzo (2021)), tax-based explanation (Faulkender and Petersen (2012)), precautionary balances driven by changing cost/production volatility (e.g. Zhao (2017)) or hybrids like tax-based explanation for IP-intensive firms (Faulkender et al. (2019)). Also explored are low carrying cost theories like Azar et al. (2016).

fits in terms of flexibility in the face of highly dynamic/ competitive product markets, while the latter shows how cash demand can be spurred by competition through auctions via a Burdett and Judd (1983) style mechanism. Our M&A market and cash demand can be thought as Galenianos and Kircher (2008) with the elimination of the assumption that firms cannot access credit to finance their trades but the addition of a speed advantage of cash which helps preclude competition. This relaxation is particularly important when trying to understand the demand of cash for large public firms like Google who have a demonstrated ability to receive credit / external finance to fund transactions. Further, unlike in Galenianos and Kircher (2008) buyers and sellers select search intensities affecting the buyer seller ratio within matches and the probability that a bidding opportunity is actually available to the buyer. In other words, firm chosen M&A search intensities change the expected amount of competition in the M&A market as well as the relevant buyer / seller market power.

2 Stylized facts on M&A and cash use

This paper examines a firm's demand for cash arising from a combination of competitive pressures amongst buyers in the M&A market and a speed advantage of cash. To support this thesis, we document several stylized facts linking M&A market activity with US firm cash holdings and other firm characteristics.

The data is comprised of balance-sheet data from Compustat, transaction level M&A data from Thompson Reuters SDC Platinum and pairwise firm product similarity scores obtained from Hoberg and Phillips (2016). We examine the sample period 1990 to 2015 inclusive. We restrict the sample of acquisitions to those which were completed, were for controlling shares (over 50% ownership ex-post) and involved US firms as targets yielding a sample of 69790 transactions. We remove all firms from Compustat not of US origin and with assets less than $10 million.

In Figure 1, we plot the value-weighted average share of M&A transactions involving cash, cash utilization in the M&A market has jumped from about 50% to nearly 80% since the start of the 90s.[9] Scouring primary SEC merger documents, Liu and Mulherin (2018) document that the average number of solicited bidders in the M&A market has risen by approximately

[9]The striking shift in 2001/2002 seems to have been driven by an accounting regulation change by the Financial Accounting Standards Board (FASB) in January 20 2001. FASB regulation No. 141 removed the 'pooled interest' accounting method for mergers which allowed the book values of the merging firms to be added together rather than the fair value 'purchase method.' As the pooling interest method meant that a merger had no effect on reported earnings while the purchase method adds additional liabilities (e.g. goodwill impairments) to the acquiring firm, stock acquisitions could benefit from using the pooled interest method yielding an advantage over cash acquisitions (which were constrained to use the purchase method).

70% and number of formal indications of interest has increased by roughly 40% in the same time interval.[10] Thus, given the roughly 60% increase in cash use within the M&A market observes over this period, there seems to be a roughly one-to-one increase in M&A cash usage share to an increase in number of solicited bidders prior to an M&A transaction (which corresponds to our notion of M&A competition in the paper).

We now move to our third piece of evidence motivating our model ingredients and subsequent analysis, that is linking cash holdings to mergers, product competition and innovation. From the above results, given the size of M&A transactions, with the higher cash usage in M&A and total M&A transactions increasing over time, a rise in cash holdings for firms active in M&A seems natural although not formally established to our knowledge.

To provide some empirical basis for this relationship, we estimate a logistic regression predicting a firm's likelihood to acquire based off of firm characteristics and competition from their closest product market rivals (based on Hoberg and Phillips (2016) product similarity data) to further inform our model ingredients. The results of the logit regression can be found in Table 1. There we see that firms are more likely to acquire if they (1) are more profitable firms in the high tech sector, (2) have higher cash growth and (3) lower (physical) investment are more likely to acquire. Further, higher product market competition strongly predicts future acquisition activity, first in terms of the the closeness of their competitors products and second by the percent of their top 10 closest rivals who acquired the previous year. These latter two, while unexamined by Hoberg et al. (2014), are consistent with their findings on the link between product market competition.

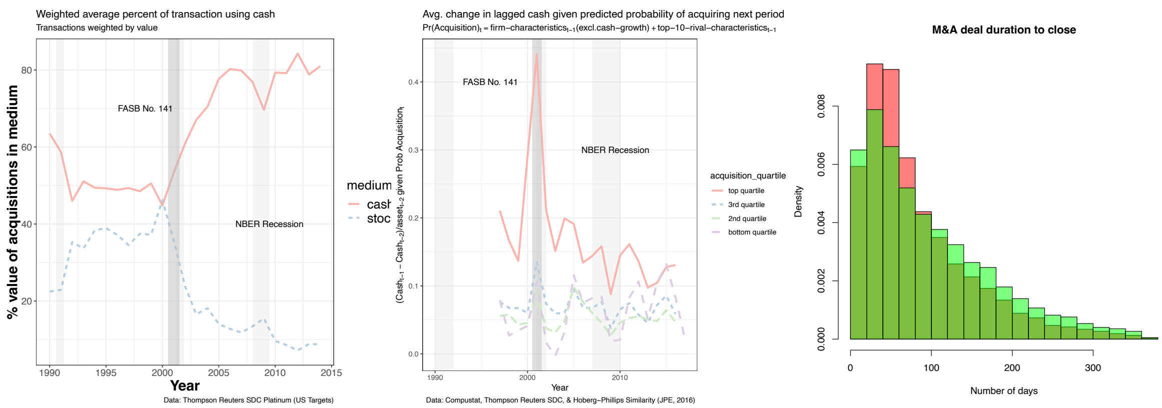
Next, in Figure 2, we plot the average cash holdings of firms within fitted quartiles of acquisition probabilities in the next year where we use the same specification as in Table 1, except excluding cash growth to prevent a mechanical relationship. Here we see that cash growth by quartile of acquisition probability is rank ordered, so that higher probability of acquiring implies higher cash growth the previous year. Further, due to the accounting regulation change (FASB reg 141) in 2001, there was a huge spike in the cash growth of firms in the sample, with greater spikes for higher quartiles in the acquisition likelihood. This suggests that cash accumulation is strategically done in anticipation of acquisition needs, not the other way around as suggested by Harford (1999) and Harford et al. (2008).

Finally, we give some support for the claim that cash offers in M&A generally provide a speed advantage over stock. In Figure 3, we plot the empirical distribution of the number of intervening days between date announced and date M&A deal is completed conditional on 100% cash or 100% stock offers (in red and green respectively). Crucially, we condition on there

[10] Notice that this is in stark contrast to the number of publicly reported bidders in SEC filings, which has fallen and the percent of M&A deals resulting in a publicly announced auction dropped by 75%.

being no competing bids recorded during the intervening time to avoid delays associated with competing offers, rather than driven by the payment method. Here we see that roughly speaking the cash offer duration distribution stochastically dominates the stock offer distribution (beyond the first 20 day window, where the two have similar probability). That is, cash offers probabilistically have a shorter duration, with the average deal being completed in roughly 10 fewer days.[11]

Figure 1: M&A and cash-use stylized facts



(a) M&A Payment Medium (b) M&A Cash Accumulation (c) M&A Closing Times

Left panel: Value-weighted average share of controlling M&A transactions of US targets by medium of exchange in cash or stock. Middle panel: Average lagged cash growth within fitted value quartiles from the logistic regression in Table 1 excluding the cash growth variable from the regression. Right panel: Time to closing of M&A transactions conditional on medium of payment being 100% cash (red) or 100% stock (green) offers. Duration is computed as difference between announced and completed date in SDC-platinum dataset. Sample is restricted to public parent targets with no competing bids in the window and a non-zero duration which is less than a year. Time to closing of M&A transactions conditional on medium of payment being 100% cash (red) or 100% stock (green) offers. Duration is computed as difference between announced and completed date in SDC-platinum dataset. Sample is restricted to public parent targets with no competing bids in the window and a non-zero duration which is less than a year. Source: Thompson-Reuters SDC Platinum (US targets) and Compustat Quarterly.

3 Model

We present the model in steps beginning with the investment-savings decision of an individual firm as well as the allocation and pricing process

[11]SEC regulations are likely the proximate cause. In particular, for US acquisitions SEC Rule 14d-1 requires a tender offer statement only on the day the offer is made and can be fully executed within 20 days, while according to SEC rule 14d-6 stock exchange offers / mergers must distribute a proxy statement at least 20 days before a vote. Furthermore, in general antitrust reviews for stocks are constrained to 30 days for stock and only 15 days for cash tender offers under the 1976 Hart-Scott-Rodino Antitrust Improvements Act. Finally, if firms wish to use cash, the funds must already effectively be put in place in advance of the offer since the 'prompt payment regulation' SEC Rule 14e-8(c) stipulates that the firm must pay for all tendered shares within three days of the tender close and SEC Rule 14e-8(c) deems any offer fraudulent if it fails to have a reasonable belief of being able to purchase the securities sought. For more in-depth discussion of the regulations and their links to a speed of execution advantage of cash, see Offenberg and Pirinsky (2015).

of the M&A market. We then describe entry and the birth-death process of firms generating a firm size, productivity and cash distribution, and the determination process of the aggregate value of money and innovation rate in general equilibrium.

The economy consists of a unit continuum of differentiated goods. Consumers have symmetric Cobb-Douglas preferences across the goods so that their expenditure on each good is the same. Household's can borrow or lend at interest rate r_t and maximize their path of consumption given their present-value flow of labour income and profits from firms. We set the numeraire so that household expenditure is constant at one ($E_t = P_t C_t = 1$). Since time is continuous there is thus a unit flow of expenditure on each good. [12]

The production of the final consumption good is determined by the quantity and productivity of the economy's intermediate inputs as well as the total fixed costs of production, υ. Aggregate final good production is given by

$$\log C_t = \int_0^1 \log(q^{J_t(j)} y_t(j))dj - \upsilon = E[J_t(j)] \log q + \int_0^1 \log y_t(j)dj - \upsilon$$

where $x_t(j)$ is the quantity of input $j \in [0, 1]$ at time t, $q^{J_t(j)}$ is the level of productivity in input j which is described by the step size $q > 1$ as well as the number of innovations (steps) in this input up to date t $J_t(j)$. Innovations have Poisson arrival at the endogenous rate δ for all intermediate inputs so that $E[J_t(j)] = \delta t$. As a result, the steady state growth rate of consumption is δ.

3.1 A firm and it's growth technologies

A firm is defined by the portfolio of n goods that they produce, their firm type τ and their accumulated internal savings (cash), m. Due to competition between firms (i.e. limit pricing), each good is produced by a single firm and yields a variable profit flow $0 < \pi_\tau < 1$ before fixed costs so that net profit flow is $\pi_\tau - \tau$, where in an overload of notation τ is the fixed costs to operating each product line.[13] Since the fixed cost τ and variable profit flow per good π_τ is the same for a given firm only the number of goods n produced by a firm, not the identity of those goods matter in their decision-making. Their static firm profits are then $n[\pi_\tau - \tau]$.

To add new product lines to its portfolio, a firm may have each product team invest in R&D with innovation intensity ι or attempt to purchase

[12] See Klette and Kortum (2004), Lentz and Mortensen (2005) and Grossman and Helpman (1991) for more details on the household problem.

[13] In particular, labour is the only variable factor of production for each input, and with unit labour productivity the limit price paid between the quality leader and mimicker's with the previous vintage of the input is $p = qw$. With total expenditures normalized to 1, revenues per product must also be unity, $px = 1$. Thus, $\pi_\tau = px - wx = \frac{q_\tau - 1}{q_\tau} \in (0, 1)$. See Grossman and Helpman (1991) for more detail.

a related product line by accessing the M&A market at rate γ. On the flip-side a firm wishing to sell a product line accesses the M&A market for that product line at rate λ. Both are associated with convex labour costs $wc(\iota), wc_S(\lambda)$ where w is the competitive wage.

In the case of investing in R&D, with a successful innovation for a particular good, the innovating firm can price out the previous incumbent in that market and takeover the entire market for that good. The incumbent firm of that product line consequently loses that product from their portfolio and so shrinks in size. Expenditures in R&D can yield an innovation for any good (not held already by the firm) with equal probability; that is a uniform draw on $[0, 1]$. The Poisson hazard rate of losing a given product line to an outsiders innovation is $\delta > 0$ and assumed to be constant over time. δ captures the rate of creative destruction in the model and is the fundamental source of aggregate growth (and is equal to household consumption growth discussed in the previous subsection).

Besides innovating on an existing good, a firm desiring to grow may purchase the exclusive rights to sell a good from another firm through accessing the M&A market as a buyer. While each firm earns positive profits on a given product line, with heterogeneity in the fixed cost of operating each product line, a higher cost firm $\bar{\tau}$ can sell their product line to a lower cost firm $\underline{\tau}$ for profit given the static surplus of $\pi_{\underline{\tau}} - \underline{\tau} - [\pi_{\bar{\tau}} - \bar{\tau}] = \pi_{\underline{\tau}} - \pi_{\bar{\tau}} + \bar{\tau} - \underline{\tau}$. [14]

We assume that due to bundling of products, the markup on acquired products is the same as the markup charged on a firms own internally created products.[15] In this way, high type firms $\underline{\tau}$ are able to charge markup $q_{\underline{\tau}} - 1$ despite the actual quality improvement of the product being $q_{\bar{\tau}}$.[16] With this assumption, markups are not all associated with positive quality improvements, leading to lower average consumer surplus with reallocation (however, because of a representative household owning the profits of the firm, this lower split of surplus to consumers is not costly in terms of welfare holding all else constant).

Access to trading opportunities in the M&A market is stochastic with a matching opportunity at rate γn for acquirers and λn for targets. Details on the allocation and pricing in the M&A market is given in the next section. As will be also discussed in the next section, cash can be used by a buyer in the M&A market instead of external financing to provide better terms of trade, we assume no pecuniary costs of external financing so that the only benefit of cash savings is in use for M&A transactions. Cash can be

[14]Note if the quality jump size of innovations is the same for high and low τ firms then we have $\pi_{\underline{\tau}} = \pi_{\bar{\tau}}$ so static surplus reduces to $\bar{\tau} - \underline{\tau}$.

[15]See work suggesting the optimality of bundling for multi-product monopolists Bakos and Brynjolfsson (1999) and Nalebuff (2004).

[16]Suppose for instance that the $\underline{\tau}$, productive firms through better targeting of consumers and bundling of products, can charge a limit price for their pool of goods at the quality of their known innovations.

exchanged to the numeraire at price φ with inflation such that $\frac{\varphi_t}{\varphi_{t+1}} = 1+\phi$. Firms discount the future at the interest rate $r > 0$, so the nominal interest rate on money (in terms of numeraire of agg. expenditures) is $1 + i = (1 + r)(1 + \phi)$.

3.2 Firm's investment-savings decision

For expositional simplicity, we present the firm problem in discrete time but formulated to be consistent with expressions obtained when taking the continuous time limit.[17]

Let $x = (m, \tau)$ denote the state of the firm beside the number of product lines n. In an overload of notation. We will denote $\tilde{x} = (x, n)$. The standalone value of a firm is then

$$\begin{gathered}(1+r)V_{n,t}(x) = \max_{m',\iota,\gamma,\lambda} n[\pi - \tau] - b \\ +n\big(\iota E\big[V_{n+1,t+1}(x') - V_{n,t+1}(x')\big] - c(\iota)w\big) \\ +n\big(\gamma E\big[W_n^A(\gamma, x', x_T) - V_{n,t+1}(x')\big] - c_A(\gamma)w\big) \\ +n\big(\lambda E\big[W_n^T(\lambda, x', x_T) - V_n(x')\big] - c_T(\lambda)w\big) \\ +n\delta\big[V_{n-1,t}(x') - V_{n,t+1}(x')\big] + V_{n,t+1}(x')\end{gathered}$$

s/t

$$b = \varphi_t(m' - m)$$

and $x' = (m', \tau)$.

Substituting in b yields

$$\begin{gathered}(1+r)V_{n,t}(x) = \varphi m + n[\pi - \tau] \qquad (1) \\ +\max_{m'\geq 0}\Big\{ -\varphi m' + \max_{\iota}\big\{n\big(\iota E\big[V_{n+1,t+1}(x') - V_{n,t+1}(x')\big] - c(\iota)w\big)\big\} \\ +\max_{\gamma\geq 0}\big\{n\big(\gamma E\big[W_n^A(x', \tilde{x}_T) - V_{n,t+1}(x')\big] - c_A(\gamma)w\big)\big\} \\ +\max_{\lambda\geq 0}\big\{n\big(\lambda E\big[W_n^T(x') - V_{n,t+1}(x')\big] - c_T(\lambda)w\big)\big\} \\ +n\delta\big[V_{n-1,t+1}(x') - V_{n,t+1}(x')\big] + V_{n,t+1}(x')\Big\}.\end{gathered}$$

[17] That is, in the discrete time formulation, we assume the arrival probabilities ι, λ, γ are such that the joint occurrence of two events (innovation / acquisition opportunity) simultaneously may be taken to be zero (consistent with being Poisson arrival rates in continuous time). To facilitate consistent expressions across continuous and discrete time, we implicitly re-scale all of the contemporaneous variables in the firm problem, e.g. $\tilde{\pi} = \beta\pi$, and $\beta = (1 + r)^{-1}$. While Choi and Rocheteau (2020) shows in general formal equilvalence between the discrete time and continuous formulations is not assured, with risk neutral agents and linear payoffs discrepancies vanish (see for instance Choi and Rocheteau (2020)).

3.3 Trade in the merger and acquisition (M&A) market

Firms that gain access into the M&A market at time t are randomly matched with buyers urn-ball paired to sellers. With this matching assumption (and a continuum of, the number of buyers that visit a given seller is random and will follow a Poisson distribution with parameter θ (where θ is the buyer seller ratio of firms with access to the M&A market). Each product line is marketed separately and so only a single product line of a given firm can be sold at a location / point in time. Consequently if multiple buyers arrive at the same location, some will be rationed out from acquiring the product line.

While the number of prospective buyers matched to a given seller in the M&A market is determined by urn-ball matching, the timing of these buyers bidding opportunities are stochastic. After the arrival of the first bidder, each seller has a fixed intra-period time-window $\hat{T}$ over which they may consider additional bids and after which they exit the market (with or without a deal). Subsequent buyers who arrive prior to $\hat{T}$ make competing bids against each-other given their surplus and that of their competitors in a second price auction.[18] Due to frictions in the transaction process and a lack of commitment by the seller, any agreed upon deal prior $\hat{T}$ can be overturned by another round of bidding upon arrival of new bidders. There is a stochastic exogenous shock $\chi(\hat{T})$ which dissolves the match between the seller and all realized / potential buyers in that period. We assume that this trade breakdown probability is increasing in the bidding window $\hat{T}$.

Their state in the M&A market is $\tilde{x} = (\tau, m, n)$. As in Galenianos and Kircher (2008), let $q^{b\ell}(s)$ denote the probability of out of b competing buyers, ℓ buyers having exactly $s = s(\tilde{x}, \tilde{x}_T)$ level of surplus and no buyers having more than s. With this, given b realized competitors (and conditional on no trade breakdown), the value of a given buyer paired with a target is, for $b \geq 1$:

$$C_n^{A,b}(\tilde{x}, \tilde{x}_T; p_0) = [1-\widehat{H}_b(s)]V_{n,t+1}(x) + \int_0^{m^-} [V_{n+1,t+1}(m - p_1(s, \tilde{s}))]\{p_1 > p_0\} dH_b(\tilde{s})$$

$$+\widehat{H}^b(s) \sum_{\ell=1}^{b} q^{b\ell}(s) \left[\frac{1}{\ell+1} [V_{n+1,t+1}(m - p_1(s, s), \underline{\tau})] + \frac{\ell}{\ell+1} \{p_1 > p_0\} V_{n,t+1}(x) \right].$$

where p_0 is the initial offer given to the seller (constitutes a reserve price in the auction), p_1 is auction price as a function of the highest two surpluses and $\widehat{H}_b(\cdot)$ is the max order statistic cumulative distribution of b draws of buyer surplus for x_T.[19]

[18] I can allow for private information if using dominant strategies in a second-price auction as in Galenianos and Kircher (2008).

[19] To reduce on clutter, we omit dependence of the value-functions and prices on the distributional

3.4 The initial bidder payment / financing choice

Buyers with a bidding opportunity can choose to make a bid using internal funds (cash) or have it be directly externally financed. We assume that the financing choice or payment method can affect the duration of the remaining bidding window. In particular, cash offers allow deals to close faster than externally financed offers i.e. internal offers have a bidding window T_c which is less than the external financed offer T_s which is itself weakly shorter than the seller's bidding window when they have rejected the initial bidder's offer ($T_c < T_s \leq T_R$). For simplicity. We will assume that only the payment / financing choice of the initial bidder can affect the bidding window. A seller is willing to accept a shorter bidding window to avoid the higher probability of no trade $\chi(T_R) \geq \chi(T_s) > \chi(T_c)$. Notice that we assume that there is no pecuniary cost to external financing so the only benefit of cash is changing the expected amount of competition and the probability of trade in the M&A market.[20]

As is derived in the appendix, the number of competing bidders conditional on the initial offer type ω is $B|\omega$ is distributed as a poisson with parameter $\tilde{\theta}(\omega) = \theta(1 - \exp(-\psi \widehat{T}_\omega))$ where θ is the buyer/seller ratio (market tightness) and $\widehat{T}_\omega$ is the duration of the bidding window. The value of a the initial bidder of type (x, n) matched with a target of type (x_T, n_T) and making a cash offer is

$$B_0^A(1, \tilde{x}, \tilde{x}_T, p_0) = \sum_{b=0}^{\infty} (1 - \chi_1) \frac{e^{-\theta_1} \theta_1^b}{b!} C^{A,b}(\tilde{x}, \tilde{x}_T; p_0)$$

while the value of an externally financed offer is

$$B_0^A(0, \tilde{x}, \tilde{x}_T; p_0) = \sum_{b=0}^{\infty} (1 - \chi_0) \frac{e^{-\theta_0} \theta_0^b}{b!} C^{A,b}(\tilde{x}, \tilde{x}_T; p_0).$$

The initial bidder makes the discrete choice

$$d = \arg\max\{B^A(0), B^A(1)\}. \tag{2}$$

Consequently, denoting $B_{\underline{b}}^A(d, \tilde{x}, \tilde{x}_T)$ as the expected value of an acquirer given $b \geq \underline{b}$ realized bidders, the expected value of a buyer entering the M&A market is

$$W_n^A(x, \tilde{x}_T) = \nu \max\left\{B^A(0; \tilde{x}, \tilde{x}_T, p_0(0, \tilde{x}, \tilde{x}_T), B^A(1; \tilde{x}, \tilde{x}_T, p_0(1, \tilde{x}, \tilde{x}_T))\right\} +$$

objects - equilibrium buyer-seller ratio in M&A θ, the equilibrium distribution of cash holdings conditional on type τ and size n, $\hat{F}_{n,\tau}$.

[20] This contrasts with Galenianos and Kircher (2008) and the standard channel in the new-monetarist literature in that we do not assume that firms are anonymous and so do not have access to credit/debt, instead firms can bid any amount to acquire the product line.

$$(1-\nu)\Big[d^A(\tilde{x}_T)B_1^A(1,\tilde{x},\tilde{x}_T,p_0(1,\tilde{x},\tilde{x}_T))+(1-d^A(\tilde{x}_T))B_1^A(0,\tilde{x},\tilde{x}_T,p_0(0,\tilde{x},\tilde{x}_T))\Big]$$

where $d^A(x_T)$ denotes the financing decision of the initial bidder by some rival acquirer for this given target.

3.5 The seller's value in the M&A market

The seller's value function in the M&A market takes a similar form as above. Like the initial bidder, we assume that the seller does not know how many bidders will be able to bid on them. Let $P^T_{b,d}$ denote the probability of a seller receiving b bidders given bidding window d chosen by the initial bidder (if any). As is shown in the Appendix B.1,

$$P^T_{b,d} = \begin{cases} (1-e^{-\theta})(1-\chi_d)\frac{e^{-\tilde{\theta}_d}\tilde{\theta}_d^{b-1}}{(b-1)!} & b \geq 1 \\ e^{-\theta} & b = 0 \end{cases}$$

where with probability $(1-e^{-\theta})$ at least one buyer is matched with the seller and hence an initial bid is realized, and conditional on there being an initial bidder, the probability distribution of subsequent bidders is Poisson with parameter $\tilde{\theta}_d$, $d \in \{0, 1, R\}$ depending on whether the initial accepted offer is cash or external financed, or that the offer is rejected. Finally with probability $(1-\chi_d)$ the M&A trade is not destroyed exogenously.

As with the acquirer define $B^T_{\underline{b}-1}(d, \tilde{x}_T)$ as the expected value from having at least $\underline{b} \geq 1$ bidders for the seller of type x_T, given at least one buyer is matched with the seller. It follows that $B^T_{\underline{b}-1}(d, \tilde{x}_T)$ is given by

$$B^T_{\underline{b}-1}(d,\tilde{x}_T) = (1-\chi(T_d))\sum_{b=\underline{b}-1}^{\infty}\frac{e^{-\tilde{\theta}_d}\tilde{\theta}_d^b}{b!}C^{T,b}(d,\tilde{x}_T)$$

where for $b \geq 2$ there is an auction between buyers with ex-post price $p_1(\tilde{s})$ which is determined by the trade-surplus (= cash holdings) of the second highest acquirer and so,

$$C^{T,b}(\tilde{x}_T) = \int_{\tilde{s}} V_{n-1}(m_T + p_1(\tilde{s}), \tilde{x}_T)d\widehat{H}_{b,b-1}(\tilde{s})$$

for $b = 1$, no additional buyers arrive besides the initial bidder who offered p_0 before

$$C^{T,1}(d,\tilde{x},\tilde{x}_T) = V_{n-1}(m_T + p_0(d;\tilde{x},\tilde{x}_T),\tau_T)$$

and for $b = 0$ no bidders show up and so the seller keeps their standalone value into the next investment-savings period,

$$C^{T,b}(d,\tilde{x}_T) = V_n(m_T,\tau_T).$$

With this, (given that in equilibrium an offer will always be such that the seller accepts) the expected value of a target in the M&A market is:

$$W_n^T(x_T) = \mathbb{P}(d)(1-e^{-\theta})B_s^T(1,\tilde{x}_T) + (1-\mathbb{P}(d))(1-e^{-\theta})B_s^T(0,\tilde{x}_T)$$
$$+[e^{-\theta} + ((1-\mathbb{P}(d))\chi_0 + \mathbb{P}(d)\chi_1)(1-e^{-\theta})]V_n(x_T)$$

where $\mathbb{P}(d)$ is the probability of a cash initial bid.

The first two terms capture the expected value of selling to some buyer given the expected probability d of the initial bidder financing choice being cash or external financed. The last term captures the event in which no buyers are matched with the seller in the M&A market and hence the seller simply exits the M&A market, and the event in which some buyer is matched with the seller but the trade breaks down.

3.6 Determination of M&A prices

The initial bid price, given acquirer type x, target type x_T and medium of exchange implied bidding window d is $p_0(d;\tilde{x},\tilde{x}_T)$ and is assumed to be a tioli offer by the buyer, hence is determined by the seller's outside option, so $p_0(d;\tilde{x},\tilde{x}_T) = p_0(d,\tilde{x}_T)$.

If the seller accepts the offer she sells the product line with probability $1-\chi_d$ (χ_d, the trade-breakdown prob), but with probability $e^{-\theta_d}$ she sells it to the initial bidder, and with probability $1-e^{-\theta_d}$ at least one other bidder shows up, leading to an auction yielding price p_1:

$$Accept(d,p_0;\tilde{x},\tilde{x}_T) = (1-\chi_d)B_0^{T,d}(\tilde{x},\tilde{x}_T)$$

while if the seller rejects the initial bid, the seller refuses to shorten the bidding window from the default but loses the reserve price $p_0(d)$ offered by the initial bidder. The value of rejecting an offer for the seller is thus

$$Reject = (1-\chi^R)B^{T,R}(x,x_T) = (1-\chi^R)\sum_{b=0}^{\infty}\frac{e^{-\theta_R}\theta_R^b}{b!}C^{T,b}(x_T,\varnothing)$$

where χ^R is the trade-breakdown probability from the standalone window, and $\theta_R = \theta(1-e^{-\psi T_R})$.

Consequently, for $d \in \{0,1\}$, $p_0(d)$ solves

$$p_0(\omega) = \inf\{p : Accept(\omega,p) = Reject\}. \tag{3}$$

The determination of the auction price $p_1(x,x_A;x_T)$ is simpler. With competitive bidding between buyers in full information (or via a second price auction with private information), the equilibrium offer sets the surplus of the second highest acquirer to zero. That is, denoting $S^A(p;x) = V_{n+1}(m - p, \tau) - V_n(m, \tau)$ as the trade-surplus for an acquirer,

$$p_1(x_A) = \inf\{p : S^A(p;x) = 0\}. \tag{4}$$

3.7 Free-entry, aggregate innovation and the firm-size distribution

The entry of a new firm requires an innovation of some product which arrives at intensity h. The cost of entry is amount of labour hired. Thus, the free entry condition for intermediate producers is:

$$\sum_{\tau} V_1(0, \tau)\Upsilon(\tau) = \frac{w}{h} \tag{5}$$

where $\Upsilon(\tau)$ is the proportion of entrants who have cost type τ (realized after entry).

New product arrival rate of an incumbent firm of type x with n products is $\iota(x, n)$. Integrating out over the distribution of cash holdings for a given type and taking $M_n(\tau)$ as the measure of firms with cost τ and n products, we have the equilibrium

$$\delta = \eta + \sum_{\tau} \sum_{n=1}^{\infty} \int_{\tilde{m}} \iota(\tilde{x}) n M_n(\tau) d\widehat{F}(\tilde{m}; \tau, n) \tag{6}$$

where $\widehat{F}$ is the distribution of cash holdings conditional on firm type τ and size n.

Denote $\Lambda(\tau, n)$ as the equilibrium rate of a firm of type τ, and size n selling a product line in the M&A market and $\Gamma(\tau, n)$ as the equilibrium rate of a firm of type τ, size n buying a product line and $\widehat{\iota}(\tau, n)$ as the expected rate of innovation for a firm of type τ, size n. Since firms can only move up/down one product line at a time, no net in/out-flows implies the steady state firm distribution satisfies for $n \geq 2$:

$$\begin{aligned} &[\widehat{\iota}(\tau, n-1) + \widehat{\Gamma}(\tau, n-1)](n-1)M_{n-1}(\tau) + [\delta + \Lambda(\tau, n+1)]M_{n+1}(\tau) \\ &= (\widehat{\iota}(\tau, n) + \delta + \widehat{\Gamma}(\tau, n) + \Lambda(\tau, n))nM_n(\tau). \end{aligned} \tag{7}$$

As when an incumbent loses his last product line, he dies, while new entrants flow in at rate η and probability of being type τ is $\Upsilon(\tau)$, so $n = 1$:

$$\Upsilon(\tau)\eta + [\delta + \Lambda(\tau, 2)]2M_2(\tau) = (\Gamma(\tau, 1) + \Lambda(\tau, 1) + \iota(\tau, 1) + \delta)M_1(\tau) \tag{8}$$

since births equal deaths in steady state $\Upsilon(\tau)\eta = [\delta + \Lambda(\tau, 1)]M_1(\tau)$.

3.8 Market clearing conditions

Money is the only storable object for firms (although short-term credit / external finance is allowed). The stock of (perfectly divisible) money at a point of time is M_t^S. The monetary authority prints money at a growth rate μ, so $M_{t+1}^S = M_t^S \mu$ and injects the new money lump-sum to households. Since final consumption good expenditures are normalized to 1, profits are expressed in terms of expenditures and φ_t is the value of money in terms of final consumption good expenditures. Assuming constant real money demand in terms of the numeraire, we have

$$\varphi_{t+1} M_{t+1}^D = \varphi_t M_t^D$$

and market clearing is

$$M_t^S = M_t^D.$$

As can be seen intuitively, provided the target firm has non-negative cash, the surplus from a sale is independent of the target's cash holdings and hence given that the low type has zero surplus from acquiring, target's cash demand is zero. Thus, the demand for money, M_t^D is given by the acquirer's cash demand, which with no transfers, is $M_t^D = \sum_{n=1}^{\infty} M_n(\underline{\tau}) \int_0^{\infty} \tilde{m} d\widehat{F}_t(\tilde{m})$.

There is a fixed labour pool L which is allocated across production, R&D and M&A activities. Denote $L_X(n, x)$ as the amount of labour demanded by the firm for production the intermediate goods by firm size n and cost type τ. Denote $L_R(n, x) = nc(\iota(x))$ as the amount of research demanded and $L_A(n, x) = nc_A(\gamma(x)), L_T(n, x) = nc_T(\gamma(x))$. Finally, let $L_E = \frac{\eta}{h}$ denote the number of researchers in new startups. Thus,

$$L = \sum_{\tau} \sum_{n=1}^{\infty} \int_{\tilde{m}} [L_X(n, x) + L_R(n, x) + L_A(n, x) + L_T(n, x)] d\widehat{F}(\tilde{m}) M_n(\tau) + L_E \tag{9}$$

Total fixed costs of production υ act as a drag on total consumption, and is given by

$$\upsilon = \sum_n M_n(\bar{\tau})\bar{\tau} + \sum_n M_n(\underline{\tau})\underline{\tau}. \tag{10}$$

Finally, the equilibrium market tightness is given by

$$\theta = \frac{\sum_{n=1}^{\infty} \sum_{\tau} \int_m \gamma_n(x) n M_n(\tau) d\widehat{F}(m)}{\sum_{n=1}^{\infty} \sum_{\tau} \int_m \lambda_n(m, \tau) n M_n(\tau) d\widehat{F}(m)}. \tag{11}$$

3.9 Equilibrium definition

A steady-state equilibrium is a list

$$\{V_n(x), W_n^A, W_n^T, d(x, x_T), \mathbb{P}(d), p_0, p_1, M_n(\tau), \widehat{F}, \widehat{H}, \varphi, \theta, \Upsilon(\tau)\}$$

which is characterized by the tuple $(w, \eta, \delta, \theta, \varphi)$ such that

1. Given prices, inflation ϕ and market tightness, $m' \in \text{supp}\widehat{F}_t$, $\iota_n(x), \gamma_n(x), \lambda_n(x)$ solves firm's investment savings problem (55)
2. Initial bidder payment choice $d(x, x_T)$ solves (2)
3. M&A initial bid and auction prices $p_0(0; \tilde{x}, \tilde{x}_T), p_0(1; \tilde{x}, \tilde{x}_T), p_1(s)$ solve (3) and (4)
4. φ ensures money market clearing satisfied, $M_t^S = M_t^D(\varphi)$
5. Labour market clearing (9) and free-entry holds (5)
6. Beliefs about market tightness θ satisfies (11), creative destruction rate (6), cash distribution $\widehat{F}$, buyer surplus distribution $\widehat{H}$, are consistent.

We restrict attention to equilibria where (i) $\iota(n,\tau) + \Gamma(n,\tau) < \delta$ for all types (x, n) (to have a finite firm size distribution) and (ii) real-balances are constant over time, i.e. $\varphi_t M_t^D = \varphi_{t+1} M_{t+1}^D$.

4 Theoretical results

To ease the analysis and facilitate closed form solutions, we will restrict attention to two cost types, $\tau \in \{\underline{\tau}, \bar{\tau}\}$ so that in equilibrium there is perfect sorting into buyers and sellers by type τ. To see this note that the static surplus from an acquisition of an acquiring firm with τ_A and a selling firm of type τ_T is $\pi - \tau_A - (\pi - \tau_T) = \tau_T - \tau_A$ which is positive if and only if $\tau_T > \tau_A$.

Observe that since there is no pecuniary cost to external financing and no evolution of productivity over time, firms have no incentive to accumulate precautionary savings as in standard corporate finance models. Consequently, firms have no motive to sell a product line to slacken financial constraints. Thus, intuitively no static surplus will imply no future surplus from the transaction either. From hereon we will refer to the high cost firms $\bar{\tau}$ as the targets or sellers and the low cost firms $\underline{\tau}$ as buyers or acquirers.[21]

By inspection of (55) it is clear that firms have quasi-linear preferences in cash m. That is,

$$V_n(x) = \frac{\varphi}{1+r} m + R_n(\tau).^{22}$$

Following similar logic to Lentz and Mortensen (2005) we conjecture that in addition the non-monetary portion of the value-function takes the following form

$$R_n(\tau) = \frac{n[\pi - \tau]}{r + \delta} + \frac{R_0(\tau)}{r} + n\Delta R(\tau), \tag{12}$$

[21] The fact that they sort into buyers / sellers will be confirmed in a later subsection.

[22] To see this formally, take the derivative of $V_n(x)$ in (55) with respect to current cash m and re-arrange, noting that x' is independent of m by the Envelope theorem.

and will verify in the following sections that a solution indeed takes this form.[23] The solution taking this homothetic form in n simplifies the analysis substantially since it collapses differences in buyer surplus $\widehat{H}$ to be equivalent to differences in accumulated cash m', $\widehat{F}$.

4.1 M&A trade surplus

Using the conjecture, the target surplus for a price p of selling one product line is

$$S_T(p) = V_{n-1}(m_T + p, \bar{\tau}) - V_n(m_T, \bar{\tau}) = \frac{\varphi' p}{1+r} - \Delta R(\bar{\tau}) - \frac{\pi - \bar{\tau}}{r+\delta}. \quad (13)$$

Consequently, the expected surplus from accessing the M&A market as a seller (target) $W^T(x) - V_n(x)$ simplifies to

$$\begin{aligned} &\sum_{b=1}^{\infty} P_b^T \int_{\tilde{m}} [V_{n-1}(m_T + p_1(\tilde{s}), \bar{\tau}) - V_n(m_T, \bar{\tau})] d\widehat{H}_{b,b-1}(\tilde{s}) \\ &\qquad + P_1^T [V_{n-1}(m_T + p_0, \bar{\tau}) - V_n(m_T, \bar{\tau})] \\ &= \sum_{b=1}^{\infty} P_b^T \int_{\tilde{s}} S_T(p_1(\tilde{s})) d\widehat{H}_{b,b-1}(\tilde{s}) + P_1^T S_T(p_0) \end{aligned}$$

where $P_b^T = (1 - e^{-\theta}) \left(\mathbb{P}(d)(1 - \chi_1) \frac{e^{-\tilde{\theta}_1} \tilde{\theta}_1^{b-1}}{(b-1)!} + (1 - \mathbb{P}(d))(1 - \chi_0) \frac{e^{-\tilde{\theta}_0} \tilde{\theta}_0^{b-1}}{(b-1)!} \right)$ for $b \geq 1$.

Plugging in $S_T(p)$ under p_0 and p_1 we get

$$W^T(x) - V_n(x) = (1 - e^{-\theta}) \sum_{d \in \{0,1\}} \mathbb{P}(d)(1 - \chi_d) \left(-\Sigma(\bar{\tau}) + e^{-\theta_d} \frac{\varphi' p_0(d)}{1+r} + (1 - e^{-\theta_d}) \mathbb{E}\left[\frac{\varphi' p_1(\tilde{s})}{1+r} \right] \right) \quad (14)$$

where we have defined $\Sigma(\tau) \equiv \Delta R(\bar{\tau}) + \frac{\pi - \bar{\tau}}{r+\delta}$ as the fundamental surplus of a product line for firm type τ.

In other words, for a given anticipated payment choice d, the expected surplus of a target in the M&A market is the probability of selling a product line $(1 - e^{-\theta})(1 - \chi_d)$ times the conditional surplus after the sale with a loss of the product line $-\Sigma(\bar{\tau})$ plus the expected payment which is the initial bid $p_0(d)$ with probability $e^{-\tilde{\theta}_d}$ and an auction of two-plus bidders with $(1 - e^{-\tilde{\theta}_d})$.

[23] Uniqueness of this value-function without the M&A market / cash is established in Lentz and Mortensen (2005) by first conjecturing that $R_n(\tau) = \frac{n[\pi - \tau]}{r+\delta} + R_0(\tau) + \tilde{R}_n(\tau)$ and show that the value function simplifies is a functional difference equation in $\tilde{R}_n(\tau)$ which satisfies Blackwell's sufficient conditions for a contraction and that $\tilde{R}_n(\tau)$ is strictly increasing in n and that $\tau' > \tau$ implies $R_n(\tau') < R_n(\tau)$ as in LM. Naturally, provided the M&A market keeps the same homogeneity in n (as is built into the setup), the same result extends to this setting.

Following similar logic for the low cost firm (acquirer) as to the target above, we have that the expected surplus from being a buyer in the M&A market is $W^A(x) - V_n(x)$, or

$$\nu \max\{B_0^A(0,x) - V_n(x), B_0^A(1,x) - V_n(x)\} + (1-\nu)\left[\sum_d \mathbb{P}(d)\big(B_1^A(d) - V_n(x)\big)\right]$$

where we have dropped the dependence on $\tilde{x}_T$ and n in $B^A(\cdot)$ since with the form of surplus S^A, S^T the target's cash-holdings are irrelevant.

Using the definition of $B_0^A(d)$, and that the acquirer trade surplus $S^A(p)$,

$$V_{n+1}(m-p,\underline{\tau}) - V_n(m,\underline{\tau}) = \Delta R(\underline{\tau}) + \frac{\pi - \underline{\tau}}{r+\delta} - \frac{\varphi' p}{1+r} \tag{15}$$

I have that the surplus of the acquirer for a given realized number of competitors b is

$$C^{A,b}(x,x_T) - V_n(x') = \begin{cases} \int_0^{s^-} S^A(p_1(\tilde{s}))d\widehat{H}_b(\tilde{s}) & b \geq 1 \\ S^A(p_0) & b = 0 \end{cases}.$$

It thus follows that the acquirer's expected surplus is

$$W^A - V_n = \nu(1-\chi_d)\left[\Sigma(\underline{\tau}) - e^{-\tilde{\theta}_d}\frac{\varphi' p_0(d)}{1+r} - (1-e^{-\tilde{\theta}_d})\frac{\varphi' p_1}{1+r}\right]$$

$$+(1-\nu)\sum_{\hat{d}} \mathbb{P}(\hat{d})(1-\chi_{\hat{d}})\left[\Sigma(\underline{\tau}) - \sum_{b=1}^{\infty}\int_{\tilde{s}\leq m} P_b^A \frac{\varphi' p_1(\tilde{s})}{1+r} d\widehat{H}_b(\tilde{s})\right] \tag{16}$$

where $P_b^A = \frac{e^{-\tilde{\theta}_{\hat{d}}}\tilde{\theta}_{\hat{d}}^b}{b!}$.

In light of the above ((13) and (15)) we have that total M&A surplus is independent of the level of the seller's cash. Since prices are determined here by take-it-or-leave-it offers, it follows that prices are simply expectations over the surplus with different numbers of bidders governed by θ. This yields the next lemma.

Lemma 4.1. *With no pecuniary costs of external financing, the M&A surplus and expected value of participating in the M&A market is independent of the level of the seller's internal funds.*

4.2 M&A pricing

From earlier we have that the acquirer surplus is

$$S^A(p) = \Delta R(\underline{\tau}) + \frac{\pi - \underline{\tau}}{r+\delta} - \frac{\varphi' p}{1+r}$$

where from hereon we will define the fundamental acquirer surplus

$$\Sigma(\underline{\tau}) = \Delta R(\underline{\tau}) + \frac{\pi - \underline{\tau}}{r+\delta}$$

and so the price paid when at least two acquirers are present is:

$$p_1(m', m^A) = \frac{(1+r)}{\varphi'}\Sigma(\underline{\tau}). \tag{17}$$

In contrast, p_0 makes the target who has met an initial bidder indifferent between accepting the deferred offer (which does not forgo the opportunity of new bidders, but rather changes the probability distribution over the arrival of competing bidders and the floor price at which trade occurs) and rejecting the offer (in which case the bidding window is the standalone length T_R, but without a reserve price if no new bidders arrive in time to provide tioli offers). As in the latter case, the target has no outside option / reserve price if only one bidder shows up, the target then receives zero surplus and so positive surplus is only achieved for the target in the event that at least one more new bidders show up. Consequently, the value of rejecting the offer is

$$B_1^T(R, \tilde{x}_T, \varnothing) - V_n(x_T) = (1-\chi_R)\sum_{b=1}^{\infty}\frac{e^{-\theta_R}\theta_R^b}{b!}\int_0^{\infty} S^T(p_1(\tilde{s}))d\widehat{H}_{b,b-1}(\tilde{s}).$$

Noting from (17) that $p_1(\cdot)$ is independent of any of the bidders / sellers cash holdings and that the buyer / seller types are homogenous, every buyer / seller pair has the same surplus and hence p_1 is deterministic. In other words, $\widehat{H}$ is degenerate and the surplus of rejecting the initial bid by the seller simplifies to $(1-\chi_R)(1-e^{-\tilde{\theta}_R})S^T(p_1)$.

The value of accepting is simply the lost value of the product plus the payment of p_0 in the event that no other bidders arrive plus the payment p_1 in the event another bidder arrives, where the arrival probability $\tilde{\theta}_d$ is conditioned on the offered medium of exchange. That is, the surplus to the target of accepting the offer in payment type d is $B_1^T(d, \tilde{x}_T, \varnothing) - V_n(x_T)$ or

$$(1-\chi_d)\Big[-(\Delta R(\bar{\tau}) + \frac{\pi - \bar{\tau}}{r+\delta}) + \frac{p_0(d)\varphi'}{1+r}e^{-\tilde{\theta}_d} + \frac{p_1\varphi'}{1+r}(1-e^{-\tilde{\theta}_d})\Big] \tag{18}$$

$$= (1-\chi_d)\Big[e^{-\theta_d}S^T(p_0(d)) + (1-e^{-\theta_d})S^T(p_1)\Big]$$

Solving for p_0 by making the target indifferent between accepting and rejecting the offer of payment type d, we get

$$p_0(d) = \frac{(1+r)}{\varphi'}\left(\Sigma(\underline{\tau}) + \left[(1-e^{-\theta_R})\left(\frac{1-\chi_R}{1-\chi_d}\right) - 1\right]e^{\theta_d}S\right) \tag{19}$$

where $S = \Sigma(\underline{\tau}) - \Sigma(\bar{\tau})$ is the total surplus from the transfer of the product line.

Define

$$\beta_d(\theta) \equiv \left[\frac{(1-e^{-\theta_R})}{e^{-\tilde{\theta}_d}} \left(\frac{1-\chi_R}{1-\chi_d} \right) - \frac{(1-e^{-\theta_d})}{e^{-\tilde{\theta}_d}} \right]$$

and observe we can re-write the price as

$$p_0(d) = \frac{(1+r)}{\varphi'} \Bigg((1-\beta_d)\Sigma(\bar{\tau}) + \beta_d \Sigma(\underline{\tau}) \Bigg). \tag{20}$$

That is, provided $\beta_d \in [0,1]$ the initial bid price is a weighted average between the surplus of the buyer and seller from the product line. The following lemma characterizes the parameter space in which β can be interpreted as a surplus sharing rule

Lemma 4.2 (β bounds)**.** *$\beta_d(\theta) \in [0,1]$ for $\theta \in [0, \bar{\theta}_\beta]$ for some $\bar{\theta}_\beta$ provided*

$$\frac{1-\chi_R}{\chi_R - \chi_d} \big(\frac{1-e^{-\psi T_R}}{1-e^{-\psi T_d}} - 1 \big) \geq 1.$$

Proof. Observe that since trade-breakdown is non-decreasing in the bidding horizon then since $\chi_R \geq \chi_d$ and $\theta_R > \theta_d$ we have

$$\beta_d \leq 1 - e^{-(\theta_R - \theta_d)} < 1.$$

Further we have directly that $\beta_d(0) = 0$ and $\frac{\partial \beta_d}{\partial \theta} > 0$ for

$$\theta < \theta^*_\beta \equiv \frac{1}{1-e^{-\psi T_R}} \log \left(\frac{1-\chi_R}{\chi_R - \chi_d} \big(\frac{1-e^{-\psi T_R}}{1-e^{-\psi T_d}} - 1 \big) \right),$$

and so $\beta_d \in [0,1]$ for all θ up to $\bar{\theta}^\beta := \{\theta > 0 : \beta_d(\theta) = 0\}$. Notice that this upper bound exists provided $\frac{1-\chi_R}{\chi_R - \chi_d} \big(\frac{1-e^{-\psi T_R}}{1-e^{-\psi T_d}} - 1 \big) \geq 1$. □

With this, we can now summarize the equilibrium M&A prices in the next theorem.

Theorem 4.3 (equilibrium M&A prices)**.** *Assuming no costly external financing, in the M&A market equilibrium initial bidder price $p_0(d)$ is contingent on the payment choice d but is otherwise independent of the cash holdings of both the buyer / seller and is given by*

$$p_0(d) = \frac{(1+r)}{\varphi'} \Bigg(\Sigma(\bar{\tau}) + \beta_d S \Bigg) \tag{21}$$

where

$$\beta_d(\theta) \equiv \left[(1-e^{-\theta_R}) \left(\frac{1-\chi_R}{1-\chi_d} \right) - (1-e^{-\theta_d}) \right] e^{\theta_d}. \tag{22}$$

Further, p_1 is invariant of the payment choice of the initial bidder and given by

$$p_1 = \frac{(1+r)}{\varphi'}\Sigma(\underline{\tau}). \tag{23}$$

An immediate corollary which is nevertheless important to highlight is that prices are independent of the cash holdings of the buyer and seller conditional on the payment method d selected for the initial bid.

Corollary. *M&A prices $p_0(d), p_1$ are independent of the level of cash held by either the buyers / sellers.*

Next notice that there is a cash-premium $p_0(1) - p_0(0) > 0$ when the trade-breakdown probability difference between cash and external financing is small, ie $\chi_1 \approx \chi_0$.

Theorem 4.4 (cash premium)**.**
If $T_1 < T_0$ then cash premium, $p_0(1;\theta) - p_0(0;\theta) > 0$, holds for any $\theta > 0$.

Proof. By (20) $\frac{\varphi'}{1+r}[p_0(1) - p_0(0)] = [\beta_1 - \beta_0]S$.
Now differentiating (22) with respect to T_d we have

$$\frac{\partial \beta_d(\theta)}{\partial T_d} = (\beta_d - 1)\frac{\partial \tilde{\theta}_d}{\partial T_d}$$

Finally as $\frac{\partial \tilde{\theta}_d}{\partial T_d} = \theta\psi e^{-\psi T_d} > 0$ for any $\theta > 0$ and from Lemma 4.2 $\beta_d < 1\ \forall \theta \geq 0$ we have $\frac{\partial \beta_d(\theta)}{\partial T_d} < 0 \forall \theta > 0$. Thus, since $T_1 < T_0$ $p_0(1;\theta) > p_0(0;\theta) \forall \theta > 0$.

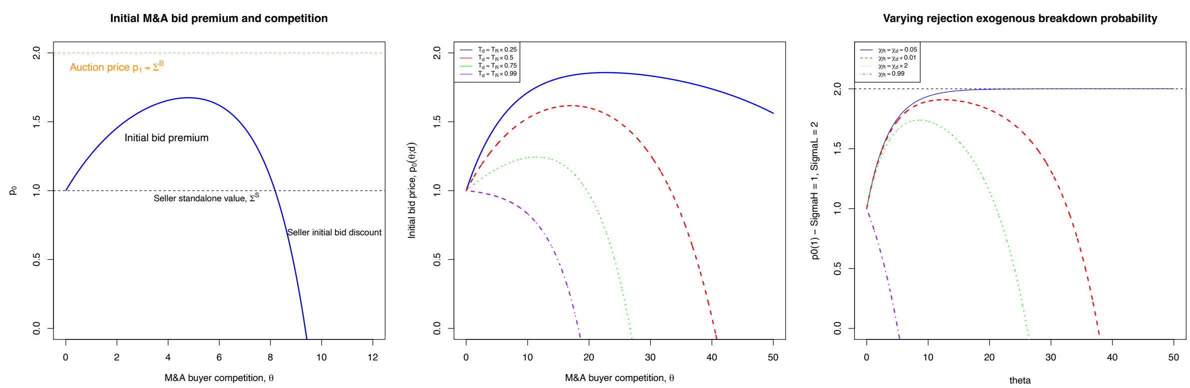
□

Simple comparative statics exercises give rise to the following corollary.

Corollary (M&A cash-bid comparative statics)**.** *For θ not too large, provided cash has a speed of execution advantage, a cash premium exists for initial offers and the cash-bid, $p_0(1)$ is (i) increasing in the amount of competition amongst buyers θ, (ii) decreasing in the exchange rate of goods for money φ', (iii) increasing in the interest rate r, (iv) increasing in both the acquirer surplus and the seller's value of the innovation.*

The dynamics of the initial cash offer $p_0(1;\theta)$ as a function of the buyer-seller ratio θ is depicted in Figure 2a. There we see that the bid increases in competition for levels of θ not too high, thereby leading to an implied bargaining power coefficient between 0 and 1, but for sufficiently high levels of competition the initial offer can actual decrease in competition as the seller deems the odds of the initial deal being consummated goes to zero, but having an offer on hand with a shorter bidding window will reduce the likelihood of an exogenous breakdown. For sufficiently high levels of competition this reduction in the likelihood of exogenous breakdown is sufficiently large that the target would actually be willing to pay some positive amount to get the lower breakdown probability.[24]

Figure 2: M&A Initial Bid Comparative Statics



(a) Pricing & competition (b) Speed advantage T_d (c) Breakdown advantage χ_R

Left panel: Initial cash bid price $p_0(1;\theta)$ as a function of competition. Right panel: Comparative statics of initial bid price as a function of competition, varying the relative bidding window, breakdown probability of cash versus the rejection window. Expected M&A competition is the buyer-seller ratio θ. Target firm surplus Σ^T is normalized to 1, buyer surplus is set to $\Sigma^B = 2$ which is also the auction price p_1. Left panel: $\psi = 1, T_d = 1, T_R = 3, \chi_d = 0.050, \chi_R = 0.055$ Middle panel: $\psi = 1, T_R = .54, \chi_R = 0.08, \chi(d) = 0.05$. Right panel: $\psi = 1, T_d = 1, T_R = .54, \chi_d = 0.05$

4.3 Initial bid payment choice

Assuming that all bidders bring symmetric amounts of cash, the expected surplus for the initial bidder is simply the probability that no other bidders arrive (since otherwise the surplus is totally bid away in competition) $\widehat{B}_d = (1-\chi_\omega)e^{-\tilde{\theta}_d}S^A(p_0(d))$. Although the initial bidder using cash reduces the likelihood of additional competitive bids, from the above corollary we know that the seller will demand a higher level of compensation accounting for the effective preclusion of additional bids. Thus, it is not immediate that the initial bidder will prefer to make a cash offer over a stock. Nevertheless, we show in the next proposition that initial bid cash offers are always preferable to stock offers.

Theorem 4.5 (initial bidder cash preference)**.** *The initial bidder strictly prefers a cash offer over a stock offer, $d(x, x_T) = 1$ for any $\theta \geq 0$ if $\chi_0 \geq \chi_1, T_1 < T_0$.*

Proof. Since the surplus of a buyer is zero in the event that another bidder shows up, only the initial bidder receives any premium. The expected surplus from an initial bid for the initial bidder is simply the probability of no

[24]In general, absent some costly adjustment or regulatory restriction, this negative premium would be eliminated in equilibrium if the seller could choose a priori the length of their outside bidding window (and thereby the implicit probability of exogenous trade breakdown).

competitors showing up and the trade not breaking down. Thus, a cash bid preference occurs if $\hat{B}_1 - \hat{B}_0 > 0$.

Define $\omega_d = (1-\chi_d)e^{-\theta_d}$ as the likelihood of a successful initial bid then

$$\hat{B}_1 - \hat{B}_0 = \omega_1 S^A(p_0(1)) - \omega_0 S^A(p_0(0)) = \Big[\omega_1 - \omega_0 - (\omega_1\beta_1 - \omega_0\beta_0)\Big] S.$$

Observe that $\omega_d\beta_d = (1-\chi_R)(1-e^{-\theta_R}) - (1-\chi_d)(1-e^{-\theta_d})$ so that,

$$\omega_1\beta_1 - \omega_0\beta_0 \leq (1-\chi_1)[e^{-\theta_1} - e^{-\theta_0}]$$

where the inequality follows from $\chi_0 \geq \chi_1$. As $T_1 < T_0$ the right-hand-side is positive and thus,

$$\omega_1 - \omega_0 - (\omega_1\beta_1 - \omega_0\beta_0) \geq \omega_1 - \omega_0 - \Big((1-\chi_1)[e^{-\theta_1} - e^{-\theta_0}]\Big) = (\chi_0 - \chi_1)e^{-\theta_0} \geq 0.$$

Since $S > 0$ we have the result.

□

4.4 Investment-savings behaviour for the high-cost firm

We now move back from the M&A market to the firm investment-savings problem. For the high cost firm, the expected surplus from acquiring is zero, $W^A(\bar{\tau},\cdot) - V_{n,t+1}(\bar{\tau},\cdot) = 0$, while the expected surplus from entering the M&A market as a seller is

$$W_n^T(\bar{\tau},\cdot) - V_{n,t+1}(\bar{\tau},\cdot) = (1-e^{-\theta})(1-\chi_d)\Big(-\Sigma(\bar{\tau}) + e^{-\tilde{\theta}_d}\frac{\varphi' p_0(d)}{1+r} + (1-e^{-\tilde{\theta}_d})\frac{\varphi' p_1}{1+r}\Big)$$

which using the solutions of (21) and (17) it simplifies to

$$W_n^T(\bar{\tau},\cdot) - V_{n,t+1}(\bar{\tau},\cdot) = (1-e^{-\tilde{\theta}_d})(1-\chi_d)\Big(e^{-\tilde{\theta}_d}\beta_d + (1-e^{-\tilde{\theta}_d})\Big) S \equiv \tilde{\Lambda}_d\tilde{\beta}_d S$$

where the high cost firm's expected probability of receiving surplus as a seller in the M&A market is

$$\tilde{\Lambda}_d \equiv (1-e^{-\tilde{\theta}_d})(1-\chi_d) \tag{24}$$

and their expected share of the total surplus S is

$$\tilde{\beta}_d \equiv e^{-\tilde{\theta}_d}\beta_d + (1-e^{-\tilde{\theta}_d}). \tag{25}$$

Note that here we have assumed that M&A buyers all follow the same financing choice $d \in \{0,1\}$. In other words, the seller's expected surplus in the M&A market is the probability of a successful trade $(1-e^{-\theta})(1-\chi_d)$ times the expected share of the surplus received from trade which with probability $e^{-\tilde{\theta}_d}$ the initial bidder is the only realized buyer whereupon the seller receives share β_d of the total surplus while with probability $(1 - e^{-\tilde{\theta}_d})$ an auction occurs and the seller receives the entire surplus S.

Substituting the conjecture of the value function (12) and the expected surplus in the M&A market for the high cost type above into (55) we have

$$R_0(\bar{\tau}) + (r+\delta)n\Delta R(\bar{\tau}) = \max_{m' \geq 0} \Bigg\{ -\varphi_t m' + \frac{\varphi_{t+1}}{1+r} m'$$
$$+n \max_{\iota \geq 0} \Bigg\{ \iota \Sigma(\bar{\tau}) - wc(\iota) \Bigg\} + n \max_{\lambda \geq 0} \Bigg\{ \lambda \tilde{\Lambda} \tilde{\beta}_d S - wc(\lambda) \Bigg\} \Bigg\}.$$

Taking the FOC wrt m' we have $-\varphi_t + \frac{\varphi_{t+1}}{1+r} = -(1 - \frac{1+\phi}{1+r})\varphi < 0$ and so the no-borrowing constraint is binding. Hence $m' = 0$. Applying this policy over time, we have that with probability 1 (ie only in discrete instants whereupon they receive internal funds from an acquirer), the seller has zero internal funds, $m = 0$. It hence follows that we have $R_0(\bar{\tau}) = 0$ and so $\Delta R(\bar{\tau})$ solves

$$(r+\delta)\Delta R(\bar{\tau}) = \max_{\iota \geq 0} \Bigg\{ \iota[\Sigma(\bar{\tau})] - wc(\iota) \Bigg\}$$
$$+ \max_{\lambda \geq 0} \Bigg\{ \lambda \tilde{\Lambda}_d \tilde{\beta}_d S - wc_\lambda(\lambda) \Bigg\} \Bigg\}. \tag{26}$$

or in terms of the surplus $\Sigma(\bar{\tau})$,

$$(r+\delta)\Sigma(\bar{\tau}) = \pi - \bar{\tau} + \max_{\iota \geq 0} \Bigg\{ \iota \Sigma(\bar{\tau}) - wc(\iota) \Bigg\}$$
$$+ \max_{\lambda \geq 0} \Bigg\{ \lambda \tilde{\Lambda}_d \tilde{\beta}_d S - wc_\lambda(\lambda) \Bigg\} \Bigg\}. \tag{27}$$

Finally, we have directly that the optimal R&D intensity and acquisition sale intensity is

$$\Sigma(\bar{\tau}) = wc'(\bar{\iota}) \tag{28}$$
$$\tilde{\Lambda}_d \tilde{\beta}_d S = wc'_\lambda(\lambda). \tag{29}$$

That is, innovation intensity $\bar{\iota}$ is monotonically increasing in the surplus from a product line Σ and M&A sale intensity λ is monotonically increasing in the expected surplus S from an M&A transaction.

4.5 Investment behaviour for the low-cost firm

Moving to the low cost ($\underline{\tau}$) firm's investment-savings problem, we have that the expected benefit as a seller in the M&A market is zero, $W^T(\underline{\tau}, \cdot) - V_{n,t+1}(\underline{\tau}, \cdot) = 0$. On the other hand, the expected surplus of accessing the

M&A market as a buyer in this market, $W_n^A(\underline{\tau},\cdot) - V_{n,t+1}(\underline{\tau},\cdot)$ which combining (16) with the solved for prices of (21) and (17) it simplifies to

$$W_n^A(\underline{\tau},\cdot) - V_{n,t+1}(\underline{\tau},\cdot) = \nu(\theta)(1-\chi_d)e^{-\tilde{\theta}_d}(1-\beta_d)S \equiv \tilde{\Gamma}_d(1-\beta_d)S. \quad (30)$$

Plugging the above (and the conjectured value-function) into (55) for the low cost firm type $\underline{\tau}$ yields

$$\begin{aligned} R_0(\underline{\tau}) + (r+\delta)n\Delta R(\underline{\tau}) &= n \max_{\iota \geq 0}\left\{\iota\Big(\Sigma(\underline{\tau}) - wc(\iota)\right\} \\ + \max_{\hat{m}\geq 0}\left\{-\varphi\hat{m} + \frac{\varphi'}{1+r}\hat{m} + n\max_{\gamma\geq 0}\left\{\gamma\tilde{\Gamma}_d(1-\beta_d)S - wc_\gamma(\gamma)\right\}\right\}. \end{aligned} \quad (31)$$

where $d = 1$ is feasible if and only if $\hat{m} \geq p_0(1)$.

Taking the optimal $\hat{m} = \underline{m} \geq 0$ (and hence $d(\hat{m})$) as given, by inspection taking the equilibrium constant value for a low cost firm to be

$$R_0(\underline{\tau}) = -\varphi\underline{m} + \frac{\varphi'}{1+r}\underline{m} = -i\frac{\varphi'\hat{m}}{1+r}$$

where $i = (1+r)(1+\phi) - 1$, we then have the equilibrium net return from a new product line given by

$$\begin{aligned} (r+\delta)\Delta R(\underline{\tau}) &= \max_{\iota\geq 0}\left\{\iota\Sigma(\underline{\tau}) - wc(\iota)\right\} \\ &+ \max_{\gamma\geq 0}\left\{\gamma\tilde{\Gamma}_d(1-\beta_d)S - wc_\gamma(\gamma)\right\}\Big\}. \end{aligned} \quad (32)$$

or in terms of the surplus from owning an innovation

$$\begin{aligned} (r+\delta)\Sigma(\underline{\tau}) &= \pi - \underline{\tau} + \max_{\iota\geq 0}\left\{\iota\Sigma(\underline{\tau}) - wc(\iota)\right\} \\ &+ \max_{\gamma\geq 0}\left\{\gamma\tilde{\Gamma}_d S - wc_\gamma(\gamma)\right\}\Big\}. \end{aligned} \quad (33)$$

Again taking FOCs for ι and γ we have symmetrically

$$\Sigma(\underline{\tau}) = wc'(\underline{\iota}) \quad (34)$$

$$\tilde{\Gamma}_d(1-\beta_d)S = wc'_\gamma(\gamma). \quad (35)$$

That is, the low cost firm innovation intensity $\underline{\iota}$ takes the identical functional form as the high-cost with respect to their surplus value from owning an innovation Σ, while their buyer search intensity γ is a function of their adapted probability of receiving trade surplus $\tilde{\Gamma}_d$, their share of surplus received as a winning initial bidder $(1-\beta_d)$) and the actual total surplus S from an M&A transaction.

4.6 Savings behaviour for the low-cost firm

The solution for the optimal level of cash $\hat{m} \geq 0$ is given by

$$\max_{\hat{m}\geq 0}\left\{ -\frac{\varphi'}{1+r}\hat{m}i + n\gamma(\hat{m})S\left(\tilde{\Gamma}_1\{\hat{m} \geq p_0(1)\} + \tilde{\Gamma}_0(1-\{\hat{m} \geq p_0(1)\})\right)\right\}$$

where

$$\gamma(\hat{m}) = \arg\max_{\tilde{\gamma}\geq 0} \tilde{\gamma}S\left[\tilde{\Gamma}_1(1-\beta_1)\{\hat{m} \geq p_0(1)\}+\tilde{\Gamma}_0(1-\beta_0)(1-\{\hat{m} \geq p_0(1)\})\right]-wc_B(\tilde{\gamma}).$$

First, note, that the upper-bound of cash is $\Sigma(\underline{\tau})$ since increasing cash will never yield higher benefit in the M&A market but carries a positive holding cost. Second note that with external financing costs at zero then there is no benefit to cash above $p_0(1)$ while a positive holding cost. Third, similarly any level of cash $\hat{m} < p_0(1)$ does not change the expected surplus from the M&A market but also is subject to a holding cost. Thus $\hat{m} \in \{0, p_0(1)\}$ when there are no external financing costs.

Now by inspection of the cash objective function, we see that the benefit of cash for M&A is non-decreasing in the number of business units since the probability of a team accessing the M&A market as an acquirer in a given instant is increasing in the number of units. Since with no cost of external financing optimal cash is either 0 or $p_0(1)$, there will be threshold level of N such that $\forall n < N$ cash demand is zero while for $n \geq N$, cash demand is $p_0(1)$. This is intriguing as it relates to the cash policies of private vs public firms and the dynamics of cash to asset ratios of firms post going public. In particular, Gao et al. (2013) find that comparable private firms hold less cash than public firms, and since private firms typically have less product lines under their control than public firms, we can interpret this N as governing the split between private / public. Furthermore, Begenau and Palazzo (2017) show that cash to asset ratios are highest upon entry as a public firm and decline afterwards, which is consistent with firms which jump to N and suddenly have $p_1(1)$ of cash, but as they grow this ratio will decline.

Theorem 4.6 (optimal cash demand with no external financing)**.** *With no pecuniary external financing costs, cash demand $\widehat{m}$ is zero for non-acquirers and is either* 0 *or* $p_0(1)$ *with cash demand positive for $n \geq \hat{N}$ for some $\hat{N}$ whenever the optimal initial bidder medium is cash, where $\forall n \geq \hat{N}$:*

$$n\nu\left[\gamma_1\hat{B}_1 - \gamma_0\hat{B}_0\right] \geq i\left[\beta_1\Sigma(\underline{\tau}) + (1-\beta_1)\Sigma(\bar{\tau})\right] \tag{36}$$

where we denote $\gamma_1 = \gamma(p_0(1))$, $\gamma_0 = \gamma(0)$.

Furthermore, all $n \geq 1$ acquirer types accumulate cash if

$$\nu(\bar{\theta}_0)e^{-\bar{\theta}_0(1-e^{-\psi T_1})}(\chi_0-\chi_1)\underline{\gamma} \geq i\frac{\bar{\Sigma}}{S} \tag{37}$$

where

$$\underline{\gamma} = \arg\max_{\gamma} \gamma\nu(\bar{\theta}_0)(\pi-\underline{\tau})(1-\chi_0)e^{-\bar{\theta}_0(1-e^{-\psi T_0})} - wc_B(\gamma). \tag{38}$$

and $\bar{\theta}_0$ *is* θ *s.t.* $p_0(0;\theta) = 0$.

Before proving the above, the following lemma will be useful.

Lemma 4.7 (Initial stock bid value is declining in θ)**.** *For any* $\chi_0 < 1$, $\hat{B}_0(\theta)$ *is strictly decreasing in* θ *for* $\theta \in [0, \bar{\theta}^\beta]$

Proof of Lemma 4.7.

$$\hat{B}_0 = w_0(1-\beta_0)S$$

Taking derivatives we have

$$\frac{\partial \hat{B}_0}{\partial \theta} = -(1-\chi_0)e^{-\theta_0}S\left[\frac{\partial \theta_0}{\partial \theta}(1-\beta_0) + \frac{\partial \beta_0}{\partial \theta}\right]$$

By direct computation

$$\frac{\partial \beta_0}{\partial \theta} = \frac{\partial \theta_0}{\partial \theta}\beta_0 + e^{\theta_0}\left[\frac{\partial \theta_R}{\partial \theta}\frac{1-\chi_R}{1-\chi_0}e^{-\theta_R} - \frac{\partial \theta_0}{\partial \theta}e^{-\theta_0}\right]$$

and so we have

$$\frac{\partial \hat{B}_0}{\partial \theta} = -(1-\chi_0)e^{-\theta_R}S\left[\frac{\partial \theta_R}{\partial \theta}\frac{1-\chi_R}{1-\chi_0}\right] < 0$$

where the last inequality follows from $\frac{\partial \theta_R}{\partial \theta} = (1-e^{-\psi T_R}) > 0$.

□

I now return to proving the theorem above.

Proof of Theorem 4.6. Computing the two candidate cash levels, and subtracting them we have

$$n\nu\left[\gamma_1\hat{B}_1 - \gamma_0\hat{B}_0\right] \geq \frac{\varphi'}{1+r}p_0(1)i.$$

Now since $\hat{B}_1 \geq \hat{B}_0$ (from 4.5) we have $\gamma_1 \geq \gamma_0$ and so (since $n \geq 1$)

$$n\nu\left[\gamma_1\hat{B}_1 - \gamma_0\hat{B}_0\right] \geq \nu\gamma_0\left[\hat{B}_1 - \hat{B}_0\right] \geq \nu\gamma_0(\chi_0-\chi_1)e^{-\theta_0}$$

where the last inequality follows from the last line of the proof of Theorem 4.5. Finally, noting that $\gamma_0 \geq \underline{\gamma}$ since $\Sigma(\underline{\tau}) \geq \frac{\pi-\underline{\tau}}{r+\delta} \geq \pi - \underline{\tau}$ (assuming $\Delta R \geq 0$ and $r+\delta < 1$) and that for θ restricted to range where $p_0(0) \geq 0$, the $\arg\min_{\theta \leq \bar{\theta}_0} \hat{B}_0(\theta) = \bar{\theta}_0$ since $\hat{B}_0(\theta)$ is strictly declining.

□

The fact that cash demand is degenerate with no cost of external financing is unsurprising, the fact that it may nevertheless be demanded in equilibrium for a speed of execution advantage is to my knowledge a novel channel.

If we add costly external financing then a non-degenerate cash distribution for buyers will arise akin to Galenianos and Kircher (2008). In particular their setting can be obtained by assuming that a fixed cost to external finance is arbitrarily large, necessitating cash to be used in acquisitions, and those with higher cash having higher surplus. Furthermore, even without adding a pecuniary external financing friction, if $N > 1$ s.t. for $n = 1$, no cash demand is optimal, then there will be a mix of acquirers, small without cash and large with cash.[25]

An interesting implied feature of the current model setup is that since $p_1 > p_0(1)$, for low holding costs, we generate endogenously that cash will be used when the payment is low, while external financing will be used when the payment is large, consistent with the predictions of Myers and Majluf (1984) pecking order theory, but for very different reasons.

Corollary (Equilibrium medium of exchange)**.** *In equilibrium, initial cash bids will have a cash-premium, while external financed deals will be for larger deal values,*

$$0 < p_0(0) < p_0(1) < p_1, \tag{39}$$

and cash deal volume is $\Gamma \sum_n nM_n(\underline{\tau})$ while stock volume is only on p_1 so that the observed average stock value is larger than the average cash value.

The following result is immediate given the value of $p_0(1)$ obtained above.

Corollary (Firm specific value)**.** *In the 'all-cash' equilibrium, the firm-type specific adjustment in the value function is $R_0(\bar{\tau}) = 0 \geq R_0(\underline{\tau})$ where*

$$R_0(\underline{\tau}) = -i[\beta_1(\theta)\Sigma(\underline{\tau}) + (1 - \beta_1)\Sigma(\bar{\tau})] \tag{40}$$

4.7 Equilibrium surplus from firm-growth

Theorem 4.8 (Eq. surplus values)**.** *In equilibrium, conditional on the R&D / search intensities, the high / low cost surplus from innovations can be obtained in closed form, with the low cost surplus $\Sigma(\underline{\tau})$ given by*

$$\Sigma(\underline{\tau}) = \frac{\pi - \underline{\tau} - w[c(\underline{\tau}) + c_B(\underline{\tau})] + \widehat{\Gamma} S}{r + \delta - \underline{\iota}} \tag{41}$$

[25] In this case, the small acquirer type may also optimally put some search intensity into being a target and selling to these already established acquirer firms. To avoid this un-necessary, and ancillary complication one can introduce a small fixed cost of M&A that will preclude this low surplus acquirer to acquirer transaction).

the high cost firm surplus $\Sigma(\bar{\tau})$ *given by*

$$\Sigma(\bar{\tau}) = \frac{\pi - \bar{\tau} - w[c(\bar{\tau}) + c_S(\bar{\tau})] + \widehat{\Lambda} S}{r + \delta - \bar{\iota}} \tag{42}$$

and total surplus S *given by*

$$S = \frac{\bar{\tau} - \underline{\tau} - w[(c(\underline{\tau}) - c(\bar{\tau}))(1 - \frac{\alpha}{a}) + c_B(\gamma) - c_S(\lambda)]}{r + \delta - \widehat{\Gamma} + \widehat{\Lambda}} \tag{43}$$

where $\widehat{\Gamma} = \gamma\tilde{\Gamma}(1 - \beta)$, *and* $\widehat{\Lambda} = \lambda\tilde{\Lambda}\tilde{\beta}$.

Proof. See Appendix, section B.3 for proof. □

Thus we see that the surplus for the buyer is increasing in their static operating profits $\pi - \underline{\tau}$ increasing in the trade-surplus $\bar{\tau} - \underline{\tau}$ and decreasing in their total costs.

It is easy to see that $\Sigma(\underline{\tau}) > 0$ provided labour expenditures are higher for the low cost firm than the high cost (which is easily verified by the FOCs for ι, γ, λ).

4.8 Steady state firm-size distribution

From our solution above, we have that in equilibrium the rate of low cost firms buying a high cost firm product line is

$$\Gamma_d = \gamma_d(1 - \chi_d)\nu \tag{44}$$

which is different from their internalized probability of gaining a product line $\tilde{\Gamma}_d$, while the total probability of a target selling an innovation is

$$\Lambda = \lambda(1 - \chi_1)(1 - e^{-\theta}).$$

Since firm search and R&D intensities (that is after scaling for firm size n) is independent of n, the rate of growth / decline given by Λ, Γ, ι are independent of n and hence for $n \geq 2$:

$$[\iota(\tau)+\Gamma(\tau)](n-1)M_{n-1}(\tau)+[\delta+\Lambda(\tau)]M_{n+1}(\tau)-(\iota(\tau)+\delta+\Gamma(\tau)+\Lambda(\tau))nM_n(\tau)$$

and for $n = 1$:

$$\Upsilon(\tau)\eta + [\delta + \Lambda(\tau)]2M_2(\tau) = (\Gamma(\tau) + \Lambda(\tau) + \iota(\tau) + \delta)M_1(\tau).$$

Consequently, since births equal deaths in steady state $\Upsilon(\tau)\eta = [\delta + \Lambda(\tau)]M_1(\tau)$. By induction, we have

$$M_n(\tau) = \frac{n-1}{n}\frac{(\iota(\tau) + \Gamma(\tau))}{\delta + \Lambda(\tau)}M_{n-1}(\tau)$$

and so

$$M_n(\tau) = \frac{\Upsilon(\tau)\eta}{n(\delta+\Lambda(\tau))}\left(\frac{\iota(\tau)+\Gamma(\tau)}{\delta+\Lambda(\tau)}\right)^{n-1}.$$

Aggregating over firm size, the equilibrium mass of a given firm type τ is

$$M(\tau) = \sum_{n=1}^{\infty} M_n(\tau) = \frac{\eta}{\delta+\Lambda(\tau)}\log\left(\frac{\delta+\Lambda}{\delta+\Lambda-\iota-\Gamma}\right)\frac{\delta+\Lambda}{\iota+\Gamma}\Upsilon(\tau)$$

provided the sum is finite. With this, we have that the fraction of firm type τ with n products is

$$\frac{M_n(\tau)}{M(\tau)} = \frac{\frac{1}{n}\left(\frac{\iota+\Gamma}{\delta+\Lambda}\right)^n}{\log\left(\frac{\delta+\Lambda}{\delta+\Lambda-\iota-\Gamma}\right)}$$

which is logarithmic with parameter $0 < \frac{\iota+\Gamma}{\delta+\Lambda} < 1$ which is the types innovation rate relative to their depreciation rate. Intuitively for the acquiring distribution, this distribution will be more skewed rightward than in LM and the lower type will be more skewed leftward than seen in LM.

Plugging in that $\Gamma(\bar{\tau}) = 0 = \Lambda(\underline{\tau})$ we have that the size distribution of low cost firms is

$$\frac{M_n(\underline{\tau})}{M(\underline{\tau})} = \frac{\frac{1}{n}\left(\frac{\underline{\iota}+\Gamma}{\delta}\right)^n}{\log\left(\frac{\delta}{\delta-\underline{\iota}-\Gamma}\right)}$$

and for high cost firms

$$\frac{M_n(\bar{\tau})}{M(\bar{\tau})} = \frac{\frac{1}{n}\left(\frac{\bar{\iota}}{\delta+\Lambda}\right)^n}{\log\left(\frac{\delta+\Lambda}{\delta-\bar{\iota}+\Lambda}\right)}$$

Note, the distribution is more skewed on higher n (leftward skewed) for the low cost firms than high cost firms since it is immediate that $\frac{\underline{\iota}+\Gamma}{\delta} > \frac{\bar{\iota}}{\delta+\Lambda}$.

4.9 Pinning down equilibrium M&A market tightness

Taking (w, δ) as fixed we show in this section that there exists a fixed point market tightness θ.

By definition,

$$\theta = \frac{\sum_{n=1}^{\infty} nM_n(\underline{\tau})\gamma}{\sum_{n=1}^{\infty} nM_n(\bar{\tau})\lambda} = \frac{\gamma}{\lambda}\frac{\delta+\Lambda-\bar{\iota}}{\delta-(\underline{\iota}+\Gamma)}\frac{\Upsilon(\underline{\tau})}{\Upsilon(\bar{\tau})} \tag{45}$$

where the second equality follows from directly computing $\sum_n nM_n(\tau)$.

Assume Assumption 2, $c_B(\gamma) = a_B\gamma^\alpha$ and $c_S(\lambda) = a_S\lambda^\alpha$, which from the optimal search intensities we have $\lambda = \max\{0, (c_S')^{-1}(\frac{\tilde{\Lambda}S}{w})\}, \gamma = \max\{0, (c_B')^{-1}(\frac{\tilde{\Gamma}S}{w})\}$.

Theorem 4.9 (Fixed point of θ exists)**.** *Suppose Assumption 2 holds. Given (w, δ) and provided $\bar{\iota} < \underline{\iota}$, $\Upsilon(\bar{\tau}) > 0$, there exists a fixed point θ solving (45).*

Proof. See proof B.4 in the appendix for details. □

4.10 Solving for the value of money given ΔR

I will for now restrict attention to size independent cash equilibria, so (37) holds provided $\theta \in [\underline{\theta}, \bar{\theta}]$ and so cash demand is positive, finite and given by

$$M_t^D = \sum_{n=1}^{\infty} M_n(\underline{\tau})p_0(1).$$

Note in the case where (37) does not hold then there will exist some threshold N such that all acquirers of size $n \geq N$ will accumulate cash and otherwise not.

Thus, equating supply and demand and solving for φ' we have

$$\varphi' = \frac{\sum_{n=1}^{\infty} M_n(\underline{\tau})}{M_t^S}(1+r)[\beta_1(\theta)\Sigma(\underline{\tau}) + (1-\beta_1(\theta))\Sigma(\bar{\tau})].$$

By definition $\varphi'(1+\phi) = \varphi$ where $1+\phi$ is the gross rate of money growth, we then have

$$\varphi = \frac{\sum_{n=1}^{\infty} M_n(\underline{\tau})}{M_t^S}(1+r)(1+\phi)[\beta_1(\theta)\Sigma(\underline{\tau}) + (1-\beta_1(\theta))\Sigma(\bar{\tau})]. \tag{46}$$

Using the definition of the nominal interest rate $i = (1+r)(1+\phi) - 1$ we have

$$\varphi = \frac{\sum_{n=1}^{\infty} M_n(\underline{\tau})}{M_t^S}(1+i)[\beta_1(\theta)\Sigma(\underline{\tau}) + (1-\beta_1(\theta))\Sigma(\bar{\tau})]. \tag{47}$$

In other words, conditional on (θ, w, δ) there is a unique equilibrium value of money. This is in line with Galenianos and Kircher (2008), but contrasts with the mulitiplicity in other monetary models with bargaining. Here higher inflation leads to higher nominal interest rates, but real money demand is not affected except (as we will see in the later subsections) through the general equilibrium effects from the lower value of incumbents depressing entry.

4.11 Market-clearing conditions

Recall that the free-entry condition

$$\sum_{\tau} V_1(\tau, 0)\Upsilon(\tau) = \frac{w}{h}.$$

Since $V_1(\tau,0) = \Sigma(\tau) + r^{-1}R_0(\tau)$, and $R_0(\tau) = -\varphi\underline{m} + \frac{\varphi'}{1+r}\underline{m} = -\frac{\varphi'\underline{m}}{1+r}i$ we have

$$\Upsilon(\bar{\tau})[\Sigma(\bar{\tau})] + (1-\Upsilon(\bar{\tau}))[\Sigma(\underline{\tau}) - \frac{\varphi'\underline{m}}{1+r}\frac{i}{r}] = \frac{w}{h} \tag{48}$$

and rate of creative destruction (analogous computation as LM (see lines after eq 17 in LM)):

$$\delta = \eta + \sum_{\tau}\sum_{n}\iota(\tau)nM_n(\tau) = \eta\sum_{\tau}\Upsilon(\tau)\left(\frac{\delta+\Lambda(\tau)-\Gamma(\tau)}{\delta+\Lambda(\tau)-\Gamma(\tau)-\iota(\tau)}\right). \tag{49}$$

Now assuming a unit measure of products, then following essentially the same arguments as above we get

$$1 = \eta\sum_{\tau}\frac{\Upsilon(\tau)}{\delta+\Lambda(\tau)-\Gamma(\tau)-\iota(\tau)}. \tag{50}$$

With linear production, number of workers employed in the making of intermediate goods per product of quality q is $x = \frac{1}{wq} = \frac{(1-\pi)}{w}$, total labour for production demanded of a firm of size n is $L_x(n,\tau) = n\frac{(1-\pi)}{w}$. Noting that having assumed a unit measure of products, $\sum_n\sum_\tau nM_n(\tau) = 1$ we then have

$$L = \sum_{\tau}\sum_{n}[L_x(1,\tau) + L_R(1,x) + L_B(1,x) + L_S(1,x)]nM_n(\tau) + L_E$$

$$\begin{aligned}
L &= \frac{(1-\pi)}{w} + \sum_{n=1}[c(\iota(\underline{\tau})) + c_B(\gamma)]nM_n((\underline{\tau})) + \sum_{n}[c(\iota(\bar{\tau})) + c_S(\lambda)]nM_n(\bar{(\tau)}) + \frac{\eta}{h} \\
&= \frac{(1-\pi)}{w} + \big[c(\iota(\underline{\tau})) + c_B(\gamma)\big]\frac{\Upsilon(\underline{\tau})\eta}{\delta-(\Gamma+\underline{\iota})} + \big[c(\iota(\bar{\tau})) + c_S(\lambda)\big]\frac{\Upsilon(\bar{\tau})\eta}{\delta+\Lambda-\bar{\iota}} + \frac{\eta}{h} \\
&= \eta\left(\frac{(1-\pi)}{w} + \big[c(\iota(\underline{\tau})) + c_B(\gamma)\big]\frac{\Upsilon(\underline{\tau})}{\delta-(\Gamma+\underline{\iota})} + \big[c(\iota(\bar{\tau})) + c_S(\lambda)\big]\frac{\Upsilon(\bar{\tau})}{\delta+\Lambda-\bar{\iota}} + \frac{1}{h}\right).
\end{aligned} \tag{51}$$

or equivalently,

$$= \eta\left([\frac{1-\pi}{w} + c(\iota(\underline{\tau})) + c_B(\gamma)]\big]\frac{\Upsilon(\underline{\tau})}{\delta-(\Gamma+\underline{\iota})} + \big[\frac{1-\pi}{w} + c(\iota(\bar{\tau})) + c_S(\lambda)]\big]\frac{\Upsilon(\bar{\tau})}{\delta+\Lambda-\bar{\iota}} + \frac{1}{h}\right). \tag{52}$$

It will be convenient in the next section to derive the value-added income identity for this economy which is stated in the next lemma.

Lemma 4.10 (The Income Identity)**.** *The equilibrium income identity equating the aggregate value-added in the economy (which is unity by the chosen numeraire) with the total wage bill plus the economy wide return on the* *incumbent firms (including the last term below which captures the holding costs of cash).*

$$1 = wL + \frac{\sum_\tau \left\{(r+\Gamma(\tau)-\Lambda(\tau))\Sigma_\tau + \tau + (\widehat{\Gamma}_\tau + \widehat{\Lambda}_\tau)S\right\}\frac{\Upsilon(\tau)}{\delta-\Gamma(\tau)-\iota(\tau)+\Lambda(\tau)}}{\sum_\tau \frac{\Upsilon(\tau)}{\delta-\Gamma(\tau)-\iota(\tau)+\Lambda(\tau)}} - \frac{\frac{i}{r}[\beta_1\underline{\Sigma} + (1-\beta_1)\bar{\Sigma}]}{\sum_\tau \frac{\Upsilon(\tau)}{\delta-\Gamma(\tau)-\iota(\tau)+\Lambda(\tau)}} \tag{53}$$

Proof. Multiplying w times (52) after substituting in the free-entry condition (48) we have

$$\frac{wL}{\eta} = \sum_\tau \left(\frac{1-\pi+w[c(\iota(\tau))+c_B(\gamma(\tau))+c_S(\lambda(\tau))]}{\delta-\Gamma(\tau)-\iota(\tau)+\Lambda(\tau)} + \Sigma(\tau) + r^{-1}R_0(\tau)\right)\Upsilon(\tau).$$

Using the equilibrium entry rate condition (50) to substitute out η, we have

$$wL = \frac{\sum_\tau \left(\frac{1-\pi+w[c(\iota(\tau))+c_B(\gamma(\tau))+c_S(\lambda(\tau))]}{\delta-\Gamma(\tau)-\iota(\tau)+\Lambda(\tau)} + \Sigma(\tau) + r^{-1}R_0(\tau)\right)\Upsilon(\tau)}{\sum_\tau \frac{\Upsilon(\tau)}{\delta+\Lambda(\tau)-\Gamma(\tau)-\iota(\tau)}} \tag{54}$$

Recalling that $R_0(\bar{\tau}) = 0$ and $R_0(\underline{\tau}) = -i\frac{\varphi'}{1+r}p_0(1) = -i[(1-\beta_1)\Sigma(\bar{\tau}) + \beta\Sigma(\underline{\tau})]$ we then have that (54) is independent of φ and is fully characterized by (δ, θ). Some algebra and re-arrangement yields the result.

□

4.12 Equilibrium existence

We have from the above that θ, φ and all other objects determined by (w, δ). To pin down the equilibrium values of (w, δ) it reduces to finding an intersection of the labour market clearing condition and free-entry with a consistent θ, or equivalently from the value-added income identity (53) and the free-entry condition (5). we establish the existence of an equilibrium in the next theorem.

The proof is given in Appendix B.5. First observe that if we shutdown the M&A market (e.g. add large fixed costs to searching in M&A) then the model is a special case of Lentz and Mortensen (2005) which by their Theorem 4.4, for L sufficiently large, a steady state equilibrium exists and is unique if $\bar{\tau} \to \underline{\tau}$.

Now for the case with M&A, the proof will follows the following steps. First, we define a boundary on the admissible set of (δ, w) such that the firm mass is finite. we then provide a sufficient condition so that the high-cost firm mass is non-zero (which then assures that θ fixed point exists from

Theorem Eq Market Tightness fixed point exists). we then move on to step (3) characterize the set of candidate (w, δ) in equilibrium for a given θ, and step 4 characterize the super-set which contains the set found in step 3 for any $\theta \in [0, \infty)$. Finally, in step 5 we define a continuous function mapping the super-set into itself and appeal to an appropriate fixed point theorem to establish the result.

Theorem 4.11 (Equilibrium existence)**.** *For sufficiently large L, high rate of entry innovation h and sufficiently small nominal interest rate, i (as well as low difference of $\chi_0 - \chi_1$ and cost functions sufficiently steep, ie $c''(0), c''_B(0), c''_S(0)$ sufficiently large), an equilibrium with value functions satisfying the conjecture exist and feature positive cash demand exists.*

Proof. See Appendix B.5. ☐

The restriction to low holding costs is needed to ensure that all size low cost firms find it optimal to accumulate cash. The relaxation of this restriction implies that there will be some interior size $\hat{n}$ such that all acquirers with $n \geq \hat{n}$ will accumulate cash and those smaller will not. This implies an adjustment to the value function of acquirers depend on the distance from $\hat{n}$ which in turn leads to size dependent surplus for acquirers. Thus for sufficiently large firms, they will stockpile cash which lowers their per product line value by the holding cost, inducing to a first order approximation lower innovation rates, but higher acquisition rates (driven by higher share of expected surplus gained in the M&A market). Future work will either attempt to solve this case analytically or solve computationally.

5 Calibrating to the 1990 US economy

As a benchmark, we parameterize the model to the US economy in 1990. We take $r = 0.05$ as in Lentz and Mortensen (2008) and Acemoglu et al. (2018). Due to the potential multiplicity in equilibria, we follow Lentz and Mortensen (2008) and fix the level of η and wages w, and solve the remaining parts of the model (although we ensure they will be consistent with market clearing in equilibrium). Since the model here nests Lentz and Mortensen (2005) we take their estimated wage level $w = 190.29$ which (with the functional form of the cost functions assumed here) is without loss of generality, since all other parameters are flexible. The parameters to calibrate are given in Table 3.

We take the job creation rate (births) in the BDS survey as the estimate for η, which in 1990 was $\eta_{1990} = 6.4\%$ while in 2015 is $\eta_{2015} = 4.6\%$. Similarly, the inflation rate from FRED (series CPIAUCSL) was 6.1% while in 2015 this rate fell to .12% (with even some months staying negative) and the GDP growth rate (implicit price deflated) was 3.7% in 1990 vs 0.93% in 2015. Estimates on markups are taken from De Loecker and Eeckhout (2017) who find the average markup in 1990 was 1.31 while in 2015 was 1.61. For the sample of non-financial firms in Compustat, the average cash to asset ratio in 1990 was 11%, M&A cash share was 62%, and the coefficient of variation of R&D to assets was .34.

We map the probability of an auction as the auction share which for 1990 is taken from Boone and Mulherin (2007). The share of firms acquiring in a given year is estimated by David (2021) to be 3.9% of public firms. The proxy for the buyer-seller ratio (θ) in the M&A market is taken from Liu and Mulherin (2018) who studying raw SEC merger documents find that on average for the 1990s there were 1.81 formal indications of interest per target firm in their sample. This average increased to 2.75 in the 2000s (from 2000-2014) which is in contrast to the number of publicly reported bidders which has been relatively flat over time.

Moving to the medium of payment contingent M&A parameters, Fee and Thomas (2004) find that US antitrust authorities (e.g. Department of Justice or Federal Trade Commission) intervened in 39/554 cases, implying a 7% exogenous rate of breakdown. Since the median transaction is not a 100% cash transaction, we apply this 7% to the breakdown of external financing, χ_0. To necessitate a positive demand for cash $\chi_0 \geq \chi_1$. Absent any granular data on the exogenous breakdown probability of 100% cash transactions, we thus for simplicity set the cash breakdown probability to zero, and fix χ_R to be 1 p.p. higher than the externally financed level. For the speed differential between cash/fully financed offers vs externally financed, we compute the average duration to deal completion for tender offers between a public and private bidder, with no revisions (across the sample period 1990-2015), and compare it against the average duration for non-tender offers between public and private bidder, and find that non-tender offers took about 15% longer on average which pins down $\frac{T_0}{T_1}$.

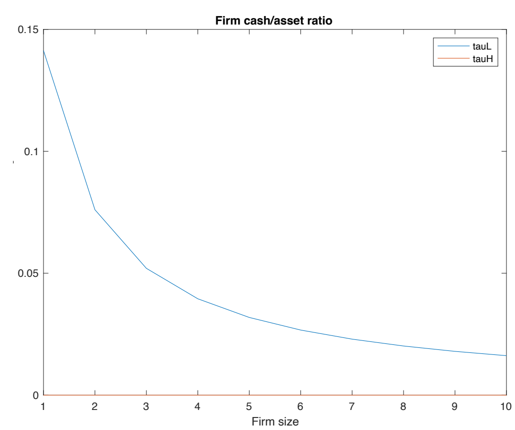
The median merger premium (measured as is standard in the literature as target valuation after merger value over ex-ante target value) is taken from Andrade et al. (2001) of 34.5% (measured from 1990-1998). Using local stock market reactions around patent grants to the patenting firm, Kogan et al. (2017) (see their eq. (10) for the analytical expression) they compute the average firm patent innovation output to be on average 3.1% of assets (see their Table III).

The calibrated model moments are given in Table 2. Here we see that the model can reasonably simultaneously capture the targeted moments of both firm entry, innovation, cash demand and merger market microstructure for the 1990s. The buyer-seller ratio in the M&A market probability of an auction are quite well matched. The calibration is within about 1% points for the growth rate, firm return from innovation (E[innovation ROA]) and cash/asset ratio. The calibration undershoots the acquisition share of firms and median (target) merger premium slightly, and the scale-free dispersion in R&D of firms is 17% too low. Where the calibration falls substantially short is with capturing the cash share of transactions in the M&A market, with the model share at 29% while in the data the share is around twice that. While not perfect, given the broad scope of the model linking firm concentration, innovation and M&A activity with a two type distribution, the model does a reasonable job capturing the key features of the data.

The model provides rich predictions on the dynamics of firm cash holdings, R&D and acquisition intensity. In Figure 3 we depict how the cash/asset ratio varies by size in the model. Cash to asset ratios are only positive for firms that are prospective acquirers, and the cash/asset ratio is monotonically decreasing in the size of the acquirer as cash stockpiles are set to the expected purchase price of an acquisition target.

Since size conditional on survival and age are positively correlated in the model, the figure also qualitatively captures the cash/asset ratio evolution of firms consistent with Begenau and Palazzo (2017). That is, mapping entry to initial public offering (IPO), average firm cash asset ratios are highest at the time of IPO and then decline after entry to an apparent target ratio. As it has been well-documented in the literature (see Begenau and Palazzo (2017)), IPO tends to precede an active period of expansion including acquisitions. Thus, we see that the evolution of firm cash to asset ratios in the model is consistent with the empirical evidence.

Figure 3: Firm Cash Holding Dynamics



Model-implied cash-asset ratio across firm-size and by fixed cost type for the 1990 calibration. τ_L is the low cost prospective acquirers, τ_H is high cost non-acquirers.

6 Quantification & Counterfactuals

6.1 Secular cash-stockpile decomposition

In this subsection, we take the calibrated model from the previous section and examine how underlying structural shifts capture this change. In particular, we examine the quantitative importance of (i) declining entry rates, (ii) declining real interest rates, (iii) rising markups and (iv) increasing dispersion in profits / costs of firms to match key targeted moments in 2015. For each of these comparative static exercises, we allow the fixed cost to vary to match the observed buyer-seller ratio $\theta_{2015} = 2.75$, but otherwise we leave all parameters unchanged from the 1990 calibration.

The main results of this exercise are given in Table **??**. In the first column, we simply report the percentage deviations from the 1990 benchmark results when we re-solve the benchmark model with the lower entry rate $\eta = 4.6\%$ observed in 2015. This is tied to more competition in the M&A market (73% higher buyer-seller ratio - see Table 5) yielding a 35.53% higher share of the surplus for selling firms in an initial offer. This impels acquires to increase their cash holdings to an average of 24% of a fraction of assets leading to a 19% higher holding cost on the acquiring firms. We see a one to one decline in the growth rate of output g and creative destruction rate δ to the decline in entry. With the lower entry rate and rate of creative destruction, the total firm mass falls implying that products become increasingly concentrated amongst the existing firms and especially the low cost firms. This lowers average firm innovative productivity by 1.61%.

Unlike column 1, which keeps the entry cost h fixed, in the experiments presented in columns 2-7, we use h to match the M&A buyer-seller ratio estimated by Liu and Mulherin (2018) for 2000-2014 (in terms of the number of formal indications of interest per target), $\theta_{2015} = 2.75$. In column 2, the opportunity cost of entry wh^{-1} (where $1/h$ is the expected duration until a potential entrant discovers a new innovation) is reduced by 3.7% relative to the benchmark in column 1. The lower entry cost leads to lesser competition in the M&A market (33% higher buyer-seller ratio - see Table 5) as compared to 73% higher buyer-seller ratio in column 1, leading to a smaller increase (21%) in the share of the surplus for selling firms in an initial offer. This implies a smaller increase in the cash holdings to an average of 9% as a fraction of assets, leading to a 10% higher holding cost on the acquiring firms as compared to the 1990 benchmark. This further reduces the growth rate of output g and creative destruction rate δ relative to column 1 (-21.82% in column 2 vs -21.52% in column 1), and is associated with a fall in the total firm mass (-4.71% in column 2 vs -2.51% in column 1).

In the third experiment, rather than drop the entry rate to the level observed in 2015, we examine the effect that simply lowering holding costs would have in the absence of any other changes in the model (besides the implied entry cost by varying h). We find that the lower inflation rate actually boosts aggregate growth by .62% and M&A competition by 45% after adjusting for a 29% higher cost of entry. This decreases concentration and cash share in M&A. In column 4, both the entry rate and inflation rate of 2015 are applied which leads to a smaller increase in the entry cost compared with inflation alone. In general the effects from declining entry seem to outweigh the reduced holding costs of cash.

In columns 5 and 6 we consider the increases in average firm markups documented by De Loecker and Eeckhout (2017) who find that the average markup in 2015 was approximately 1.61. From our model, a rise in markups is equivalent to a rise in the quality of the good q. It is important to note that unfortunately, no equilibrium seems to exist with this higher markup on its own or jointly with a reduced entry rate η for any entry cost h that matches θ_{2015}. The presented results are for the closest obtained θ which is more than 60% lower than the benchmark θ. With that caveat aside, we find that consumer growth g increases by 80% with the higher markup. Firm concentration increases while nonetheless cash demand increases driven by a nearly 20 times higher standalone value of the high cost firm. Cash to asset ratio increases with the heightened cash demand but is depressed by the higher value of the acquiring firm's assets.

Finally, in column 7 we combine the three different fundamental observable changes in the environment η, inflation and markups q, and examine how lowering the operating costs of the low cost firm interacts with these observable changes. Reducing the fixed cost of the low cost type by nearly a factor of 6, we find an overall positive effect on growth of 46.37% driven by the increased markups. Despite the higher growth rate of consumption, the rate of innovation decreases by 17% with on net higher firm concentration and a larger market share of the low cost firms. This higher concentration amongst the high types induces a huge spike in the average cash/asset ratio of 132% and leads to a 120% increase in the cash share of transactions in the M&A market while also increasing the probability of an auction.

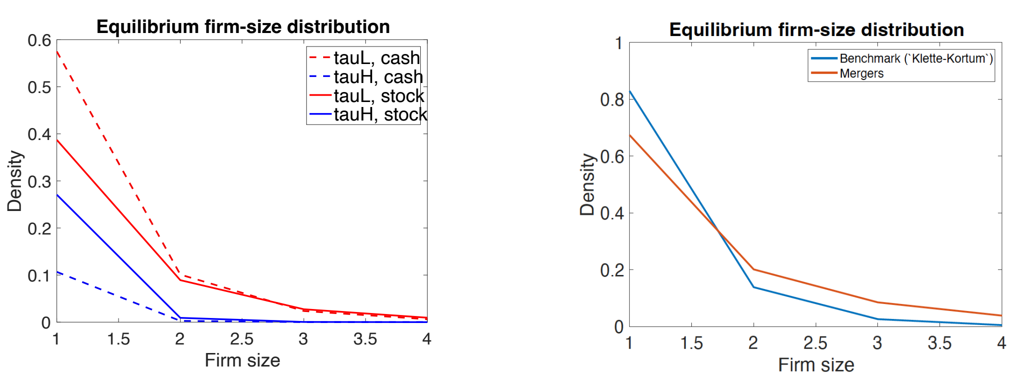
This last comparative static is what we take as the benchmark calibration for the US economy in 2015. The targeted moments in the data and the calibrated model are reported in Table **??**. Overall the model fit is if anything better in 2015 than in the 1990s. Of note, the cash share in M&A is over 60% which is still under-cutting the observed level, but by substantially less than in the benchmark calibration. The cash/asset ratio is almost exactly on target despite not having calibrated any additional parameters besides $\underline{\tau}$. Where this calibration does worse is with matching the growth rate of output. This suggests either some of markups are a function of increased market power not tied to higher quality improvements, i.e., there is some mis-measurement of quality improvements in output (Corrado et al. (2009)) or that dispersion in markups is substantively important (as examined by Lentz and Mortensen (2008) with three levels of markups/quality improvements). Altogether, the results above suggest that increasing productivity differences between firms is crucial in generating the increase in cash/asset ratios observed in the past 30 years.

6.2 Quantifying cash-use, M&A on growth

In this section, we examine the benchmark calibrated model but where we shut down cash mergers, or shutdown mergers completely. This is interesting first, to quantify the importance of this mechanism and second, as an extreme policy tool which could be utilized in an attempt to ameliorate the stagnation. The results from this exercise are presented in Table **??**. Here we find that preventing all mergers reduces growth by 4.5%, while banning cash mergers reduces growth by 2.3%. This decline is driven by the lower option value of high cost firms having the opportunity to sell in the M&A market.

Another way to see the aggregate effects of mergers and cash is to consider the firm size distribution implied by the different policies depicted in Figure 4a. Here we see that cash based acquisitions re-allocate output from high cost producers to low cost producers, skewing right the mass of low cost firms and left the mass of high cost firms. In addition, Figure 4b highlights the change in the firm-size distribution with the possibility of mergers in the benchmark Klette-Kortum framework. The firm-size tail is thickened by low-cost firms (superstars) via acquisitions and cash-use.

Figure 4: Firm-Size Distribution Counterfactuals



(a) Cash vs No Cash Mergers (b) Mergers vs No Mergers

Comparing equilibrium firm-size distribution from 2015 calibration with cash mergers & all mergers banned in left & right panel respectively. Left panel conditions on firm cost type τ, right panel pools across firm cost types.

6.3 Real Effects of Monetary Policy

In-progress

7 Conclusion

We presented in this paper a novel model linking innovative firm concentration to firm cash holdings and growth. As a result the model links monetary policy and the level of long-term interest rates to growth. Despite the richness of the model, analytical results were obtained yielding a cross-sectional distribution of firm productivity, size and cash holdings. A current limitation of the model is that the model is only analytically tractable when cash policies are size invariant which occurs only for low interest rates. Future work should attempt to extend this analytically or examine some variant numerically to capture additional possible size distortions with the cash advantage.

To our knowledge, the paper provides a new, analytically tractable general equilibrium theory which links market concentration and M&A market conditions to a firm's demand for liquidity and their incentives to innovate. Despite the richness of firm heterogeneity in the model, the majority of the equilibrium objects can be characterized in closed form (conditional on the wage and buyer/seller ratio in the M&A market which in general must be solved numerically). Key to the tractability is a built-in size invariance of policies coming from Klette and Kortum (2004), however, this can be violated for firms optimal choice of stockpiled liquidity if holding costs (i.e. interest rates) are sufficiently high to preclude small, but high efficiency firms from accumulating liquidity. Nonetheless, restricting attention to estimating the model over the past three decades this issue seems to be only a theoretical concern. The size invariance of the cash policy find broad empirical support amongst US public firms since it gives rise to a declining cash/asset ratio observed in Compustat data when sorted by size. The value of money is endogenous, and the growth rate of money has a non-neutral, and quantitatively significant effect on the distribution of innovative activity, firm-size and aggregate growth through the M&A market. It also provides a theory of cash-demand over the lifecycle consistent with the findings by Begenau and Palazzo (2017) and Gao et al. (2013) in which private firms tend to hold little cash, while around the time of IPO firms cash asset ratios spike and steadily decline over the following years.

It is also among the first to provide real-linkages between monetary policy, firm dynamics and aggregate growth. While at the firm level, R&D and M&A activity will be positively correlated, the equilibrium impacts of monetary policy can cause aggregate substitution between external and internal growth. Counterfactual exercises from our 2015 calibrated economy suggest that the congestion externality and costly firm cash demand can entirely unwind the dynamic gains from reallocating to more efficient producers in M&A.

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A Main tables

Table 1: Logistic Regression - Predicting Controlling Acquisitions

	Dependent variable:
	Acquisition$_t$
$\Delta cash_{t-1}/size_{t-1}$	0.417***
	(0.089)
high_tech$_t$	0.138***
	(0.037)
% rivals acquiring$_{t-1}$	0.357***
	(0.050)
10th rival similarity$_{t-1}$	1.978***
	(0.331)
Controls	
$profitability_{t-1}$	8.349***
	(0.328)
$tobinsQ_{t-1}$	0.029***
	(0.007)
$capx_{t-1}$	-0.0001^{***}
	(0.00004)
Other controls	(omitted)
Observations	55,089
Log Likelihood	$-20{,}828.880$
Akaike Inf. Crit.	41,689.750
Note:	*p<0.1; **p<0.05; ***p<0.01

Combines Compustat and US M&A data obtained from Thompson Reuters SDC Platinum over the sample period: 1990 to 2016 inclusive, as well as Hoberg and Phillips (2016) product similarity data. We restrict the sample of acquisitions to those which were completed, were for controlling shares (over 50% ownership ex-post) and involved US firms as targets yielding a sample of 69,790 transactions. We remove all firms from Compustat not of US origin and with assets less than $10 million. We define % rivals acquiring as the 10 closest rivals in the product market and compute the percentage which acquired in the previous year. We take the 10-th rival similarity score as the distance in product similarity of their 10th closest rival, providing a measure of how competitive they are within a product market space. High tech is an indicator based on Ritter's classification of SIC codes. Other controls include one year lags of: book assets and sales, as well as average of rival characteristics including assets, cash equivalents, min distance of rival, total acquisitions divested of rivals.

Table 2: Benchmark calibration to US 1990 Economy

Moments	Data	Model
entry rate (eta)	0.064	0.064
markups (q)	1.310	1.310
inflation	0.061	0.061
GDP growth rate	0.037	0.022
E[cash/assets]	0.110	0.090
cv(R&D/assets)	0.340	0.281
M&A competition (# interest)	1.810	1.985
M&A cash share	0.620	0.290
Auction prob	0.471	0.423
Acquisition rate	0.039	0.011
E[innovation ROA]	0.031	0.012
Median Merger Premium	0.345	0.257

Table 3: Benchmark Parameters

Parameters	Values
η	0.064
π	0.237
$\bar{\tau}$	0.228
$\underline{\tau}$	0.055
h	1900.000
a_ι	2000.000
α_ι	5.000
a_γ	30.000
α_γ	4.600
a_λ	1.000
α_λ	6.000
ψ	1.300
T_1	0.250
T_0	0.287
T_R	0.360
χ_1	0.000
χ_0	0.070
χ_R	0.071

Table 4: Calibration of the model to the US economy in 2015

Moments	Data	Model
entry rate (eta)	0.046	0.046
markups (q)	1.610	1.610
inflation	0.001	0.001
GDP growth rate	0.04	0.032
E[cash/assets]	0.225	0.210
cv(R&D/assets)	0.418	0.301
M&A competition (# interest)	2.750	2.779
M&A cash share	0.750	0.639
Auction prob	0.471	0.538
Acquisition rate	0.039	0.009
E[innovation ROA]	0.031	0.011
Median Merger Premium	0.345	0.183

A.1 Additional Tables

	η_{2015} (h fixed)	η_{2015}	i_{2015}	η_{2015}, i_{2015}	q_{2015}	η_{2015}, q_{2015}	$\eta_{2015}, i_{2015}, q_{2015}, \underline{\tau}_{2015}$
entry rate (η)	-28.12	-28.12	0.00	-28.12	0.00	-28.12	-28.12
entry cost (h)	0.00	3.68	-28.95	-24.21	-21.05	0.00	-48.53
markups (q)	0.00	0.00	0.00	0.00	60.11	60.11	60.11
inflation	0.00	0.00	-98.36	-98.36	0.00	0.00	-98.36
M&A competition (θ)	72.67	33.31	45.36	46.96	-79.27	-62.33	40.02
g	-21.52	-21.82	0.62	-21.70	79.78	42.24	46.37
net entry share	-8.41	-8.06	-0.61	-8.21	-1.90	-10.88	-13.40
δ	-21.52	-21.82	0.62	-21.70	1.93	-19.35	-17.01
$\Upsilon(\underline{\tau})$	14.74	5.11	14.20	9.03	-73.13	-58.49	6.80
total firm mass	-2.51	-4.71	2.49	-3.84	-1.56	-14.12	-7.32
low cost mass share	8.86	5.18	7.11	6.76	-67.51	-43.79	5.81
low cost sales share	7.42	4.64	5.69	5.82	-59.35	-32.18	5.14
cv r&d / assets	-5.24	7.47	-17.98	2.43	-13.78	35.81	7.22
E[new innovation value/assets]	-1.61	-4.19	4.28	-2.40	10.09	-7.09	-4.68
E[cash/assets]	23.92	8.71	20.13	9.48	-24.78	10.13	131.83
cash share in M&A	-10.13	-4.74	-6.43	-6.55	210.95	191.05	119.90
auction prob	44.90	22.82	30.09	31.02	-74.52	-55.75	26.93
share acquiring firms/year	-44.68	-21.93	-34.08	-31.05	7.69	63.56	-19.41
median merger premium	108.84	43.78	70.32	57.49	-83.11	-73.81	-28.60
avg initial offer premium	87.08	36.71	59.17	46.84	-46.04	-16.48	109.34
avg premium	109.10	43.79	70.62	57.58	-83.71	-74.72	-29.56
β	35.53	20.58	26.28	26.97	-74.87	-55.99	23.88
$\Sigma(\underline{\tau})$	-1.45	-1.64	0.41	-1.56	72.11	71.03	99.56
$\Sigma(\bar{\tau})$	-19.79	-10.51	-12.43	-14.00	1850.83	1813.09	1788.35
S	-0.63	-1.24	0.99	-1.00	-7.77	-7.21	23.71
$a(\underline{\tau})$	18.83	10.47	-48.12	-48.91	484.05	485.59	194.57
$\iota(\underline{\tau})$	-0.37	-0.41	0.10	-0.39	14.54	14.36	18.85
$\iota(\bar{\tau})$	-5.36	-2.74	-3.27	-3.70	110.16	109.14	108.46
φ	26.20	10.77	20.08	10.57	85.73	182.07	509.53
$\Sigma(\bar{\tau}) + \beta S$	18.83	10.47	15.90	14.14	484.05	485.59	558.10

Table 5: Decomposing the cash build-up

Decomposing the structural change between 1990 and 2015. Each column presents the changes (in percent deviation from the 1990 benchmark) from targeting the 2015 level rather than 1990. Column 1 shows the result of entry rates η falling to the 2015 level, $\eta_{2015} = 4.6\%$, holding all else fixed (including entry cost h). In the remainder of the columns, I allow the entry cost to vary (with h) in order to target the 2015 M&A bidder market tightness level, $\theta_{2015} = 2.75$, in addition to the parameter listed in the column title. Column 2 presents the entry rate fall combined with rising M&A competition. Column 3 presents the fall in inflation. Column 4 jointly considers the fall in entry rates and inflation. Column 5 considers an increase in markups to 1.61, column 6 decline in entry rates and rise in markups. Column 6 considers all of these changes as well as allows the low cost fixed cost $\underline{\tau}$ to fall in order to calibrate moments to 2015 economy (see Table **??**).

B Appendix - Additional Proofs

B.1 M&A matching primitive derivations

Assume k buyers are matched to ℓ sellers situated on a line so N is the number of potential bidders for a given seller,

$$N \sim Bin(k, p)$$

where $p = \frac{1}{\ell}$. If $N = 0$ then the seller cannot sell this period. One of these N bidders is randomly selected as the initial bidder who (unaware of the number N) of other matched bidders with the seller selects the medium of exchange d which in turn determines the bidding window duration. $d = 1$ indicates a cash bid, while $d = 0$ is externally financed.

With urn-ball matching of buyers to sellers, each buyer who entered into the M&A market is paired with a seller with probability 1. Let N_i denote the number of competitors who are matched with the same seller as bidder i. Then

$$N_i \sim Bin(k, p)$$

and the probability of bidder i being the initial bidder given N_i other bidders is $\frac{1}{N_i+1}$. Because bidder i does not observe how many other bidders are initially matched with the seller, their perceived probability of being the initial bidder integrates over the possible number of initial matched competitors, that is the probability of i being the initial bidder for the seller they are matched with is

$$Pr(\text{i is initial bidder}) = E[\frac{1}{N_i + 1}].$$

WLOG assume that i is the initial bidder. Each of the N_i other buyers matched with the seller draw an inter-arrival time $\tilde{t}$ with per-instant arrival intensity ψ following an exponential distribution independently. The bidding window d specifies a terminal horizon point $\widehat{T_\omega}$ so that buyers with arrival times $\tilde{t} \leq T_d$ have the opportunity to make a bid to the seller while those with $\tilde{t} > T_d$ arrive too late and are excluded from making a bid.

Consequently, the number of realized competitor bidders to the initial bidder matched with a given seller is

$$C|N_i, d \sim Bin(N_i, z_d)$$

where $z_d = Pr(\tilde{t} \leq T_d) = 1 - \exp(-\psi T_d)$. Using the fact that a binomial conditional on a binomial is also binomial (see conditional binomials), we have

$$C|d \sim Bin(k, pz_d).$$

Taking the number of buyers $k \to \infty$ while keeping the buyer-seller ratio seller fixed $\theta = \frac{k}{\ell}$ we get

$$C|d \rightarrow Poisson(\theta z_d).$$

Let $\hat{d}$ denote the seller's assessed probability of a cash window being selected by their initial bidder. Then the unconditional number of realized bidders, $b+1$ (b competitors) for a given seller is a weighted average of two Poisson's:

$$P_b^T \equiv Pr(B = b+1) = [\hat{d}\frac{(z_1\theta)^b}{b!}e^{-z_1\theta} + (1-\hat{d})\frac{(z_0\theta)^b}{b!}e^{-z_0\theta}](1-e^{-\theta})$$

where since $N \rightarrow Poisson(\theta)$ with $k \rightarrow \infty$, it follows the probability of the seller receiving no bidders is $P_0^T = e^{-\theta}$.

Straightforward calculations gives that the probability of being the initial bidder is $\nu = E[\frac{1}{N+1}] = \frac{1-e^{-\theta}}{\theta}$,[26] and that the probability of b competitors for bidder i matched with the seller is P_b^A with $P_0^A = e^{-\theta_d}$, and

$$P_b^A = \nu[P_{b,1}d + (1-d)P_{b,0}] + (1-\nu)[P_{b,1}\hat{d} + (1-\hat{d})P_{b,0}]$$

where $\hat{d}$ is the anticipated choice of bidding window by a rival initial bidder.

B.2 Investment-savings explicit continuous time formulation

$$\begin{aligned} rV_n^\tau(z) = \max_{a\geq -z,\iota,\gamma,\lambda\geq 0} & n[\pi-\tau] - a + \frac{\partial V}{\partial z}\dot{z} \\ & +n\big(\iota E\big[V_{n+1}^\tau(z) - V^\tau(z)\big] - c(\iota)w\big) \\ & +n\big(\gamma[W_n^A(z) - V_n^\tau(z)] - c_A(\gamma)w\big) \\ & +n\big(\lambda[W_n^T(z) - V_n^\tau(z)] - c_T(\lambda)w\big) \\ & +n\delta\big[V_{n-1}^\tau(z) - V_n^\tau(z)\big] \end{aligned} \tag{55}$$

subject to

$$\dot{z} = \frac{\dot{\varphi}}{\varphi}z + a,$$ [27]

if $a > -z$, and $\dot{z} = -z$ else.

[26] Here we implicitly take the event of a given acquirer themselves being selected as an outside the match bidder is a zero measure event.

[27] Observe that we have $z = \varphi m$ so $\dot{z} = \dot{\varphi}m + \varphi\dot{m}$ and $\dot{m} = \frac{y}{\varphi}$.

B.3 Solving equilibrium surplus's details

Having solved for the equilibrium prices, we now move to characterizing the value of innovation ΔR for the high and low cost firms.

Starting with the low cost firm, we have from the body that $\Delta R(\underline{\tau})$ is given implicitly by

$$(r+\delta)\Delta R(\underline{\tau}) = \max_{\iota} \iota\Sigma(\underline{\tau}) - wc(\iota) + \gamma\tilde{\Gamma}_{d^*}(1-\beta_d)S - wc_B(\gamma_{d^*}) \tag{56}$$

where

$$\tilde{\Gamma}_d \equiv \nu(\theta)\omega_d(\theta) = (1-\chi_d)\frac{1-e^{-\theta}}{\theta}e^{-\theta_d} \tag{57}$$

and d^* is 1 if $\hat{m} = p_0(1)$ and 0 otherwise. Notice that the total acquisition probability for a given acquirer, given the uniform probability of being the winner in an auction with tied offers amongst buyers is

$$\Gamma_d = \gamma_d(1-\chi_d)\nu \tag{58}$$

while the total probability of a target selling an innovation is

$$\Lambda = \lambda(1-\chi_1)(1-e^{-\theta}).$$

Now applying similar logic for the high cost firm, we have

$$(r+\delta)\Delta R(\bar{\tau}) = \max_{\iota} \iota\Sigma(\bar{\tau}) - wc(\iota) + \max_{\lambda} \lambda\widetilde{\Lambda}_d\tilde{\beta}_d S - wc_S(\lambda)$$

where $\widetilde{\Lambda}_d = (1-\chi_d)(1-e^{-\theta})$ and $\tilde{\beta} = [e^{-\theta_d}\beta_d + (1-e^{-\theta_d})]$. To reduce on clutter also define $\hat{\Lambda}_d$ as the expected surplus share received in the M&A market as a seller

$$\hat{\Lambda}_d \equiv \lambda(1-\chi_d)(1-e^{-\theta})(1-e^{-\theta_d}(1-\beta_d)) \tag{59}$$

and so

$$(r+\delta)\Delta R(\bar{\tau}) = \bar{\iota}\Sigma(\bar{\tau}) - w(c(\bar{\iota}) + c_\lambda(\bar{\lambda})) + \hat{\Lambda}_d S. \tag{60}$$

Using $\Sigma = \Delta R(\tau) + \frac{\pi-\tau}{r+\delta}$ we can re-write in terms of the surplus for the low cost firm as

$$(r+\delta)\Sigma(\underline{\tau}) = \pi - \underline{\tau} + \iota(\underline{\tau})\Sigma(\underline{\tau}) - wc(\underline{\tau}) + \widehat{\Gamma}S - wc_B(\gamma) \tag{61}$$

and for the high cost firm as

$$(r+\delta)\Sigma(\bar{\tau}) = \pi - \bar{\tau} + \iota(\bar{\tau})\Sigma(\bar{\tau}) - wc(\bar{\tau}) + \widehat{\Lambda}S - wc_S(\lambda) \tag{62}$$

Subtracting (62) from (61), using the FOC $\Sigma(\tau) = c'(\iota(\tau))w$ and the assumption on $c(\cdot)$ so that $c'(\iota) = \frac{\alpha}{a}\frac{c(\iota)}{\iota}$, after a little algebra we have

$$S = \frac{\bar{\tau} - \underline{\tau} - w\left[\left(c(\underline{\tau}) - c(\bar{\tau})\right)(1 - \frac{\alpha}{a}) + c_B(\gamma) - c_S(\lambda)\right]}{r + \delta - \widehat{\Gamma} + \widehat{\Lambda}}. \tag{63}$$

Consequently, re-arranging $\Sigma(\underline{\tau})$ above we have

$$\Sigma(\underline{\tau}) = \frac{\pi - \underline{\tau} - w[c(\underline{\tau}) + c_B(\gamma)] + \widehat{\Gamma}S}{r + \delta - \underline{\iota}} \tag{64}$$

and similarly,

$$\Sigma(\bar{\tau}) = \frac{\pi - \bar{\tau} - w[c(\bar{\tau}) + c_S(\lambda)] + \widehat{\Lambda}S}{r + \delta - \bar{\iota}}. \tag{65}$$

QED

B.4 Proof of fixed point market tightness given

A simple but useful lemma is below.

Lemma B.1. *Given c_B, c_S have the form specified above, for any $\theta \leq \bar{\theta}_\beta$ we have that $\frac{\gamma}{\lambda}$ is strictly decreasing in θ.*

Proof. By their definition we have

$$\left(\frac{\tilde{\Gamma}}{\tilde{\Lambda}}\right) = \frac{e^{-\theta_d}(1 - \beta_d)}{\theta[1 - e^{-\theta_d}(1 - \beta_d)]}.$$

From the section on β we have β monotonically increasing for $\theta \leq \bar{\theta}_\beta$. Thus for $\theta \in [0, \bar{\theta}_\beta]$ it is simple to verify that $e^{-\theta_d}(1 - \beta_d)$ is monotonically decreasing. Given this, it follows immediately that $\frac{1}{1 - e^{-\theta_d}(1-\beta_d)}$ is also monotonically decreasing. Finally, $\frac{1}{\theta}$ is also monotonically decreasing, hence the product of decreasing functions is decreasing and we are done.

□

Proof of Theorem (4.9). With the functional form assumption above

$$\text{RHS (45)} = \frac{a_S}{a_B}\left(\frac{\tilde{\Gamma}}{\tilde{\Lambda}}\right)^{\frac{1}{\alpha}} \frac{\delta + \Lambda - \bar{\iota}}{\delta - (\underline{\iota} + \Gamma)} \frac{\Upsilon(\underline{\tau})}{\Upsilon(\bar{\tau})}.$$

As this is a composite of continuous functions it is also continuous. Using the solution of $\tilde{\Gamma} = (1 - \chi_d)\nu(\theta)e^{-\theta_d}(1 - \beta_d)$ and $\tilde{\Lambda} = (1 - \chi_d)(1 - e^{-\theta})(1 - e^{-\theta_d}(1 - \beta_d))$ and $\Gamma = \gamma(1 - \chi_d)\nu(\theta)$, $\Lambda = \lambda(1 - \chi_d)(1 - e^{-\theta})$ we have

$$\left(\frac{\tilde{\Gamma}}{\tilde{\Lambda}}\right) = \frac{e^{-\theta_d}(1 - \beta_d)}{\theta[1 - e^{-\theta_d}(1 - \beta_d)]}.$$

and

$$\frac{\delta+\Lambda-\bar{\iota}}{\delta-(\underline{\iota}+\Gamma)}=\frac{\left[\frac{\delta-\bar{\iota}}{(1-e^{-\theta})(1-\chi_d)}+\lambda\right](1-\chi_d)(1-e^{-\theta})}{\left[\frac{\delta-\underline{\iota}}{(1-e^{-\theta})(1-\chi_d)}+\frac{\gamma}{\theta}\right](1-\chi_d)(1-e^{-\theta})}=\frac{\frac{\delta-\bar{\iota}}{(1-e^{-\theta})(1-\chi_d)}+\lambda}{\frac{\delta-\underline{\iota}}{(1-e^{-\theta})(1-\chi_d)}+\frac{\gamma}{\theta}}.$$

Multiply both sides of (45) by $\theta^{\frac{1}{\alpha}}$. Thus the modified RHS (45) is

$$R\tilde{H}S(45)=\frac{a_S}{a_B}\left(\frac{e^{-\theta_d}(1-\beta_d)}{\theta[1-e^{-\theta_d}(1-\beta_d)]}\right)^{\frac{1}{\alpha}}\frac{\frac{\delta-\bar{\iota}}{(1-e^{-\theta})(1-\chi_d)}+\lambda}{\frac{\delta-\underline{\iota}}{(1-e^{-\theta})(1-\chi_d)}+\frac{\gamma}{\theta}}\frac{\Upsilon(\underline{\tau})}{\Upsilon(\bar{\tau})}.$$

Now since $\lambda\geq 0$ and $\bar{\iota}<\underline{\iota}$,

$$R\tilde{H}S(45)\geq\frac{a_S}{a_B}\left(\frac{e^{-\theta_d}(1-\beta_d)}{\theta[1-e^{-\theta_d}(1-\beta_d)]}\right)^{\frac{1}{\alpha}}\frac{\frac{\delta-\underline{\iota}}{(1-e^{-\theta})(1-\chi_d)}}{\frac{\delta-\underline{\iota}}{(1-e^{-\theta})(1-\chi_d)}+\frac{\gamma}{\theta}}\frac{\Upsilon(\underline{\tau})}{\Upsilon(\bar{\tau})}.$$

Using the solution for γ, $\lim_{\theta\to 0}\beta_d(\theta)=0$ and L'Hopitals rule to get $\lim_{\theta\to 0}\frac{\gamma(1-e^{-\theta})}{\theta}=(1-\chi_d)$ it follows that

$$\lim_{\theta\to 0}\left(\frac{e^{-\theta_d}(1-\beta_d)}{1-e^{-\theta_d}(1-\beta_d)}\right)^{\frac{1}{\alpha}}=\frac{1}{1-1}=\infty$$

and since

$$\lim_{\theta\to 0}\frac{\frac{\delta-\underline{\iota}}{(1-e^{-\theta})(1-\chi_d)}}{\frac{\delta-\underline{\iota}}{(1-e^{-\theta})(1-\chi_d)}+\frac{\gamma}{\theta}}>0$$

it follows that $\lim_{\theta\to 0}R\tilde{H}S(45)\geq\infty$.
On the other hand, since $\gamma\geq 0$, we have

$$\lim_{\theta\to\infty}R\tilde{H}S(45)\leq\lim_{\theta\to\infty}\left(\frac{e^{-\theta_d}(1-\beta_d)}{1-e^{-\theta_d}(1-\beta_d)}\right)^{\frac{1}{\alpha}}\frac{\frac{\delta-\bar{\iota}}{(1-e^{-\theta})(1-\chi_d)}+\lambda}{\frac{\delta-\underline{\iota}}{(1-e^{-\theta})(1-\chi_d)}}\frac{\Upsilon(\underline{\tau})}{\Upsilon(\bar{\tau})}$$

$$=\lim_{\theta\to\infty}\left(\frac{e^{-\theta_d}(1-\beta_d)}{1-e^{-\theta_d}(1-\beta_d)}\right)^{\frac{1}{\alpha}}\left(\frac{\delta-\bar{\iota}}{\delta-\underline{\iota}}+\frac{(1-e^{-\theta})(1-\chi_d)\lambda}{\delta-\underline{\iota}}\right)=0,$$

where the last equality follows from

$$\lim_{\theta\to\infty}\theta^{\frac{1}{\alpha}}\frac{\gamma}{\lambda}=\lim_{\theta\to\infty}\left(\frac{e^{-\theta_d}(1-\beta_d)}{1-e^{-\theta_d}(1-\beta_d)}\right)^{\frac{1}{\alpha}}=0.$$

Finally the LHS (45) multiplied by $\theta^{\frac{1}{\alpha}}$ has $\lim_{\theta\to 0}=0$ and $\lim_{\theta\to\infty}=\infty$ and is continuous / monotonic, thus we have that at least one fixed point exists.

□

B.5 Equilibrium existence proof

The proof follows the following steps. First, we define a boundary on the admissible set of (δ, w) such that the firm mass is finite. we then provide a sufficient condition so that the high-cost firm mass is non-zero (which then assures that θ fixed point exists from Theorem Eq Market Tightness fixed point exists). we then move on to step (3) characterize the set of candidate (w, δ) in equilibrium for a given θ, and step 4 characterize the super-set which contains the set found in step 3 for any $\theta \in [0, \infty)$. Finally, in step 5 we define a continuous function mapping the super-set into itself and appeal to an appropriate fixed point theorem to establish the result.

Proof. Step 1: (boundary on the admissible $((w, \delta)$

First we will define a boundary on the admissible set of (δ, w) such that $\delta > \Gamma - \Lambda + \iota$ to ensure a finite firm mass. Combining the FOC of $\iota(\underline{\tau})$ and (41) and solving for w we have

$$w = \frac{\pi - \underline{\tau} + \frac{\widehat{\Gamma}}{r+\delta-\widehat{\Gamma}+\widehat{\Lambda}}[\bar{\tau} - \underline{\tau}]}{C} \tag{66}$$

where

$$C \equiv (r+\delta-\underline{\iota})c'(\underline{\iota})+c(\underline{\iota})+c_B(\underline{\tau})+\frac{\widehat{\Gamma}}{r+\delta-\widehat{\Gamma}+\widehat{\Lambda}}\left[(c(\underline{\tau})-c(\bar{\tau}))(1-\frac{\alpha}{a})+c_B(\underline{\tau})-c_S(\bar{\tau})\right].$$

Now from the FOCs, $c', c'_B, c'_S > 0 = c(0) = c_S(0) = c_B(0)$ and given $\underline{\Sigma} \geq \bar{\Sigma}$ it is immediate that (i) $\underline{\iota} \geq \bar{\iota}$, (ii) $\gamma\tilde{\Gamma}(1-\beta) \leq \gamma \leq \underline{\iota}$ (since $\tilde{\Gamma}, \beta \in (0,1)$) and $\Gamma(\bar{\tau}) = \Lambda(\underline{\tau}) = 0$. Thus, we have $\delta \geq \Gamma(\underline{\tau}) + \iota(\underline{\tau}) > \iota(\bar{\tau}) - \Lambda(\bar{\tau})$ and with the above $2\underline{\iota} > \Gamma(\underline{\tau}) + \underline{\iota}$. Thus a more stringent sufficient condition is $\delta \geq 2\underline{\iota}$ to ensure finite firm mass.

Setting $\gamma\tilde{\Gamma}(1-\beta) = \underline{\iota} = \bar{\iota} = \frac{\delta}{2}$ and $\Lambda = 0$ in (66)

$$w \equiv B(\delta) = \frac{\pi - \underline{\tau} + \frac{\delta}{2r+\delta}[\bar{\tau} - \underline{\tau}]}{(r+\frac{\delta}{2})c'(\frac{\delta}{2}) + c(\frac{\delta}{2}) + c_B(\frac{\delta}{2}) + \frac{\delta}{2r+\delta}(c_B(\frac{\delta}{2}) - c_S(0))} \tag{67}$$

where $c_S(0) = 0$. It is immediate that $B(\delta) > 0$ (since $\bar{\tau} > \underline{\tau}$) and tends to infinity as $\delta \to 0$ (since $c'(0) = c'_B(0) = c(0) = c_B(0) = 0$) while tends to 0 as $\delta \to \infty$.

Step 2: (ensuring positive high-cost firm mass)

To ensure that $\Upsilon(\bar{\tau}) > 0$, so that $\theta < \infty$, note from the free-entry condition that

$$\Upsilon(\bar{\tau}) = \frac{[\Sigma(\underline{\tau}) - \frac{i}{r}\frac{\varphi' p_0(1)}{1+r}] - \frac{w}{h}}{S}.$$

Thus for $i \to 0$ and h sufficiently large we have that $\Upsilon(\bar{\tau}) > 0$.

Step 3: characterizing the set of candidate (w, δ), $\Xi(\theta)$

I now move to characterizing the pair of equations pinning down (w, δ).

Taking $\Upsilon(\underline{\tau}) \to 1$, we then have w implicitly defined in the free-entry condition (48) by $w = \underline{E}(\delta, \theta)$ and given by

$$\Sigma(\underline{\tau}) - \frac{i}{r}\left[\beta\Sigma(\underline{\tau}) + (1-\beta)\Sigma(\bar{\tau})\right] = \frac{w}{h} \tag{68}$$

while for the high cost firm we have $w = \bar{E}(\delta, \theta)$ given by

$$\Sigma(\bar{\tau}) = \frac{w}{h}. \tag{69}$$

Taking $i \to 0$, $\underline{E}(\delta, \theta) \geq \bar{E}(\delta, \theta)$ since $\bar{\tau} > \underline{\tau}$. Straightforward differentiation (and applying the envelope theorem) yields $\frac{\partial \underline{E}(\delta,\theta)}{\partial \delta}, \frac{\partial \bar{E}(\delta,\theta)}{\partial \delta} < 0$.

Now moving to the national income identity (53) (the modified labour market clearing condition), and again, taking $\Upsilon(\underline{\tau}) \to 1$, while holding θ fixed, we then have w implicitly defined by $w = \underline{L}(\delta; \theta)$ with

$$wL = 1 - (r + \Gamma)\underline{\Sigma} + \underline{\tau} + \widehat{\Gamma}S \tag{70}$$

while when $\Upsilon(\bar{\tau}) \to 1$ for the high cost firm we have implicitly $w = \bar{L}(\delta, \theta)$

$$wL = 1 - (r - \Lambda)\bar{\Sigma} + \bar{\tau} + \widehat{\Lambda}S.^{28} \tag{71}$$

Now, since this model with M&A nests Lentz and Mortensen (2005), shutting down the M&A market yields the simplified equilibrium conditions given by free-entry (LM eq. 20), and labour-market clearing (LM eq. 21) in their paper from (48) and (52) here. In this case, the solution to these lies within the compact set depicted in Figure 3 of Lentz and Mortensen (2005), where $\bar{L}$ evaluates the labour market clearing condition with all the weight set on the high profit firm and similarly, $\underline{E}$ sets the entry probability of the low profit firm to 1 in evaluating the free-entry condition. In the graph, $\bar{L}$ is roughly equivalent to $\underline{L}(\theta; \delta)$ but with $\gamma = \lambda = 0$. Similarly, $\underline{E}$ in the figure corresponds to $\bar{E}(\theta; \delta)$ in the paper, where the flip comes because high type in paper is the low profit firm. we will refer to this depicted set as Ξ_0.

In the next lemma, we show that for $\theta \geq 1$ we have that $\underline{L} \geq \bar{L}$ while for θ sufficiently small we have the reverse.

Lemma B.2. *If $D > 0$ and $\theta \geq 1$ then $\underline{L} \geq \bar{L}$, while for θ sufficiently close to 0 $\underline{L} \leq \bar{L}$.*[29]

Proof.

$$RHS(\bar{L}) - RHS(\underline{L}) = \bar{\tau} - \underline{\tau} - S(r - \widehat{\Gamma} + \widehat{\Lambda}) + \Lambda\bar{\Sigma} - \Gamma\underline{\Sigma}$$

[28] Of course if θ were to adjust then $\Lambda = 0 = \widehat{\Lambda}$ and $\Gamma = 0$ since no mass of positive surplus to trade with.

[29] This result hinges on the fixed cost not affecting labour.

if $\Lambda \geq \Gamma$, (which is true when $\theta \geq 1$)

$$\geq \bar{\tau} - \underline{\tau} - S(r - \widehat{\Gamma} + \widehat{\Lambda} + \Gamma) \geq wD > 0.$$

$\square$

Regardless, of the configuration, a compact, convex set can be defined by the convex hull of $\underline{L}, \bar{L}, \underline{E}, \bar{E}$ defined as $\Xi(\theta)$. Observe that from LM Ξ_0 is non-empty and by construction $\Xi_0 \subseteq \{\Xi(\theta) : \theta \geq 0\}$.

Step 4: Establishing the containing super-set Ξ_∞

In this step we define a super-set Ξ_∞ such that $\{\Xi(\theta) : \theta \geq 0\} \subseteq \Xi_\infty$.

Lemma B.3 ($\exists$ income equality upper bound)**.** *There exists a function $L^{UB}(\delta)$ s.t. $L_\tau(\delta;\theta) \leq L^{UB}(\delta), \forall \theta \geq 0, \delta$ s.t. (w,δ) above $B(\delta)$.*

Proof. First, taking $\Upsilon(\bar{\tau}) \to 1$, we then have $\theta \to 0$, $\Lambda \to 0$, so that $\bar{L}(0;\delta) = \bar{L}_0(\delta)$ is given by

$$wL = 1 - r\bar{\Sigma} - \bar{\tau}.$$

Now,

$$(r - \Lambda)\bar{\Sigma} + \bar{\tau} + \widehat{\Lambda} S \geq \inf_\theta (r - \Lambda)\bar{\Sigma} - \bar{\tau} \geq (r - \lambda)\bar{\Sigma} + \bar{\tau}.$$

Then if $r \geq \delta$, then $r > \lambda$ and so this is still a positive quantity. Outside of a constant $\bar{\tau}$ and scaling, $\bar{L}^{UB}(\delta)$ given by

$$wL = 1 - (r - (1 - \chi_1))\bar{\Sigma} + \bar{\tau} \tag{72}$$

lies strictly above $\bar{L}(\delta;\theta)$.

On the other hand, for the low cost firm,

$$r\underline{\Sigma} + \underline{\tau} \leq \inf_\theta (r + \Gamma)\underline{\Sigma} + \underline{\tau} - \widehat{\Gamma} S$$

where the last inequality follows since $\widehat{\Gamma} \leq \Gamma$ and $S \leq \underline{\Sigma}$.

Thus, define $\underline{L}^{UB}$ as

$$wL = 1 - r\underline{\Sigma} + \underline{\tau}. \tag{73}$$

Finally, take $L^{UB}(\delta) = \max\{\bar{L}^{UB}(\delta), \underline{L}^{UB}(\delta)\}$ then by construction the result follows.[30]

$\square$

Lemma B.4 ($\exists$ lower bound on income-identity)**.** *There exists a function $L^{LB}(\delta)$ s.t. $L_\tau(\delta;\theta) \geq L^{LB}(\delta), \forall \theta$.*

[30] Note that if $\bar{\tau} - \underline{\tau}$ difference sufficiently small so that $\bar{\tau} - \underline{\tau} \leq r(\underline{\Sigma} - \bar{\Sigma}) + (1 - \chi_1)\bar{\Sigma}$, we have that $\bar{L}^{UB} > \underline{L}^{UB}$.

Proof. Define

$$\underline{\Sigma}^* = \frac{\pi - \underline{\tau} + \gamma^* \bar{S} - w[c(\iota) + c_B(\gamma^*)]}{r + \delta - \underline{\iota}}, \bar{S} = \frac{\bar{\tau} - \underline{\tau}}{r + \delta - \gamma^*}$$

where γ^* solves FOC $\max_\gamma \gamma \bar{S} - w c_B(\gamma)$.
Clearly, $\bar{S} \geq S$, and so $\underline{\Sigma}^* \geq \underline{\Sigma}$. Further,

$$(r + \Gamma)\underline{\Sigma} - \underline{\tau} - \widehat{\Gamma} S \leq (r + \gamma)\underline{\Sigma} - \underline{\tau} - \widehat{\Gamma} S \leq (r + \gamma)\underline{\Sigma} - \underline{\tau}$$

Thus, define $w = \underline{L}^{LB}$ by

$$wL = 1 - (r + \gamma^{LB})\underline{\Sigma} + \underline{\tau} \tag{74}$$

which by construction we have $\underline{L}^{LB}(\delta) \leq \underline{L}(\delta; \theta) \forall \theta \geq 0$.
By similar logic, defining $w = \bar{L}^{LB}$ by

$$wL = 1 - r\bar{\Sigma} + \bar{\tau} \tag{75}$$

with $\bar{L}^{LB}(\delta) \leq \bar{L}(\delta; \theta) \forall \theta \geq 0$.

□

Finally, we will show that L^{LB} is upper-ward sloping, (same logic can be applied to L^{UB}).

Lemma B.5 (Monotonicity of boundary constraints)**.** *The pure cost free-entry / labour-market clearing conditions for free-entry, $E^{UB}_\tau(\delta)$ and labour market clearing, $L^{UB}_\tau(\delta)$ are monotonic in δ for (w, δ) above, $i \to 0$ and $h \to \infty$, (and total costs of low cost firms $\geq$ total costs of high cost firms)*

$$\frac{\partial E^{UB}(\delta)}{\partial \delta} < 0 < \frac{\partial L^{UB}(\delta)}{\partial \delta}$$

Proof. Total differentiating (73) and re-arranging we have

$$\frac{dw}{d\delta} = \frac{\frac{\partial RHS(73)}{\partial \delta}}{L - \frac{\partial RHS(73)}{\partial w}}.$$

First, by isolating the terms with respect to w, we have $L > (r+\gamma)[c(\underline{\iota}) + c_B(\gamma)]$. Second, using the FOCs we have $\frac{\partial \underline{\Sigma}^*}{\partial \underline{\iota}} = 0$, and $\frac{\partial \underline{\Sigma}^*}{\partial \underline{\gamma}} = \frac{\bar{S} - w c'_B(\gamma)}{r + \delta - \underline{\iota}} + \frac{\gamma \bar{S}}{(r+\delta-\gamma)(r+\delta-\underline{\iota})} = \frac{\gamma \bar{S}}{(r+\delta-\gamma)(r+\delta-\underline{\iota})}$.

Third, total differentiating the FOC of γ we have $\frac{d\gamma}{dw} < 0$ and so

$$\begin{aligned} L - \frac{\partial RHS(73)}{\partial w} &= L - (r + \gamma)[c(\underline{\iota}) + c_B(\gamma)] \\ &+ \left[\frac{\gamma \bar{S}}{r + \delta - \underline{\iota}} \left(\frac{2\gamma - \delta}{r + \delta - \gamma} \right) - \left(\frac{\pi - \underline{\tau} - w[c(\underline{\iota}) + c_B(\gamma)]}{r + \delta - \underline{\iota}} \right) \right]. \end{aligned} \tag{76}$$

Restricting to the region that (w, δ) is above $B(\delta)$ implies $\delta > 2\gamma$ and further restricting $\pi, \underline{\tau}, c(\cdot), c_B()$ so that $\pi - \underline{\tau} - w[c(\underline{\iota}) + c_B(\gamma)] > 0$ for (w, δ) falling below $\underline{E}^{UB}(\delta)$ then yields the result that $L - \frac{\partial RHS(73)}{\partial w} > 0$.[31] Finally, differentiating the RHS with respect to δ we get immediately that $\frac{\partial RHS}{\partial \delta} > 0$.

The proof for the lower bounds L^{LB} and U^{LB} follow symmetric logic.

□

With this, defining Ξ_∞ as the convex hull of $L^{UB}, L^{UB}, E^{UB}, E^{LB}$ we have from the arguments above that $\Xi(\theta) \subseteq \Xi_\infty$ for any $\theta \in [0, \infty)$.

Step 5: Establishing existence of a fixed-point of (w, δ)

Define $\Theta((\delta, w))$ to be the mapping of θ given by (45), $\Theta : \Xi_\infty \to [0, \infty)$. Equipped with Ξ_∞, let Ψ denote the mapping of (w, δ) to (w', δ'), where (w', δ') satisfies the equilibrium conditions (54), and (48) within the set $\Xi(\theta(w, \delta))$. In other words, $\Psi : \Xi_\infty \to \Xi_\infty$. From the previous steps we have that Ξ_∞ is non-empty, compact and convex and that Ψ is a composite of continuous functions. Further, starting at any point along the boundary of Ξ_∞ yields a strictly interior convex, compact set $\Xi(\theta)$ and thus interior point (w', δ') satisfying the equilibrium conditions and hence Brouwer's fixed point theorem yields at least one solution (w, δ). Since this set Ξ_∞ is in the upper-contour set of $B(\delta)$, this solution yields a finite firm mass. QED

□

[31] $\underline{E}^{UB}(\delta)$ is $E(\delta; \theta)$ but with $\underline{\Sigma}^*$ rather than $\underline{\Sigma}$.