

Learning News Bias: Misspecifications and Consequences

Lin Hu* Matthew Kovach† Anqi Li‡

Abstract

We study how a decision maker (DM) learns about the bias of unfamiliar news sources. Absent any frictions, a rational DM uses known sources as a yardstick to discern the true bias of a source. If a DM has misspecified beliefs, this process fails. We derive long-run beliefs, behavior, welfare, and corresponding comparative statics, when the DM has dogmatic, incorrect beliefs about the bias of known sources. The distortion due to misspecified learning is succinctly captured by a single-dimensional metric we introduce. Our model generates the hostile media effect and false polarization, and has implications for fact-checking and misperception recalibration.

*Research School of Finance, Actuarial Studies and Statistics, Australian National University, lin.hu@anu.edu.au.

†Department of Economics, Purdue University, mlkovach@purdue.edu.

‡Department of Economics, University of Waterloo, angellianqi@gmail.com.

1 Introduction

News bias is pervasive. Failure to understand the biases of news sources could have dire consequences for our collective well-being, as showcased by the anti-vaccine debates instigated by opportunity seekers, climate misinformation backed by interest groups aiming to confuse the public and block evidence-based policy changes, and the recent incident of innocent citizens falling prey to the anti-liberal campaign propagated by Macedonian teens (Bursztyn et al., 2023; Farrell et al., 2019; Hughes and Waismel-Manor, 2021). Today’s digital news sphere is full of new, unfamiliar sources. Some of these are hosted by people who seldom meet in person, while others are operated by bots that drive a significant volume of the digital traffic (Wojcik et al., 2018). Some disseminate genuine, neutral, information; others spread biases and falsehood. As these sources become an integral part of our news diet (Matsa and Lu, 2016), cultivating a good understanding of their biases is crucial for the functioning of democratic society (Stocking and Sumida, 2018).

In an ideal world, news bias would be ineffective, at least in the long run. Consider a Republican voter (hereinafter, the DM) who wishes to stay informed on an evolving issue. He consults a number of news sources, some with established brands and reputations (e.g., CNN, Fox News), while others are lesser-known (e.g., unfamiliar accounts on social media). As the DM observes the reporting from lesser-known sources, he updates his belief about their biases. Even better, he may use established sources as yardsticks, placing a new source as far-right (resp. far-left) if it is consistently more conservative (resp. liberal) than Fox News (resp. CNN). Over time the DM’s posterior belief becomes increasingly precise, and, under fairly general conditions, converges to the truth.

Reality, however, is not frictionless. In this paper, we focus on a particular kind of learning friction whereby the DM holds dogmatic, wrong, beliefs about the biases of established sources. Such dogmatism can arise from several well-known cognitive biases, and so is potentially quite common. Recall our DM. As someone who grew up watching Fox News, the DM believes that this source is more neutral than it actually is due to familiarity bias (Pennycook and Rand, 2021). He underestimates the bias associated with his own opinions and judgments, partly because of the bias blind spot and third-person effect (Pronin et al., 2002; Davison, 1983),¹ and partly

¹Bias blind spot refers to the tendency for people to recognize biases in human judgment —

because he is working towards self-enhancement and would like to view himself as middle-of-the-road (Dennis, 1988). Finally, he believes in a liberal media bias, like many other Republicans, and displays excessive hostility towards CNN (Hassell et al., 2020). These cognitive biases are known to enhance with age, be resistant to corrective information, or even pass down through generations (Pronin et al., 2002; Jennings et al., 2009). We take them as given and examine their consequences for the DM’s learning about the unfamiliar sources.

We follow the paradigm of misspecified Bayesian learning to model the aforementioned situation. The DM in our model consults a number of news sources on an evolving state of the world. The signal generated by a source in a given period equals the horizontal bias of the source, plus the true state of the world in that period and a Gaussian error. Errors are independent of the true state and of each other in the baseline model, but can take more general forms in the online appendix. Initially, the DM holds a prior belief about the biases of the various sources. For some sources (e.g., CNN, Fox News, DM’s private sources) the prior is misspecified; it assigns probability one to an event in which the bias perceived by the DM differs from the truth. For others (e.g., lesser-known sources on social media), the prior is nondegenerate everywhere and, crucially, contains the truth in its support. In every subsequent period, the DM observes signal realizations and updates his belief about news biases in a Bayesian manner. We examine how misspecified Bayesian learning breeds persistent misperception about the unfamiliar sources and fuels distortions of the DM’s long run behavior and expected utility.

Our main result captures the distortionary effects of misspecified learning using a simple metric. In the baseline model, the metric equals the total misspecification across the various sources, weighted by their relative precision. It is negative in our leading example, where the DM begins by underperceiving the conservative biases of Fox News and his own private sources, while overperceiving the liberal bias of CNN. When learning about the biases of unfamiliar sources, the DM uses sources with misspecified biases as yardsticks. The leftward distortion of his worldview is then mirrored in his beliefs about the unfamiliar sources through misspecified learning. In the long run, the DM underestimates the bias of any conservative source and

except when that bias is their own. The third-person effect predicts that people perceive mass media messages to have a greater effect on others than on themselves. Both phenomena have proven to entail important consequences for democratic society.

overestimates the bias of any liberal source. Our model thus formalizes a channel through which the aforementioned cognitive biases, however small, may breed persistent misperceptions about new, unfamiliar, sources, generating patterns such as hostile media effect and false polarization in the digital news sphere. This misperception is shown to have a polarizing effect on the DM’s behavior — a prediction that traces the misspecified-learning origin of behavioral polarization. Details are in Sections 3.2 and 3.3.

An important feature of our metric is its independence from the lesser-known sources about which the DM is learning. While these sources clearly matter for the long run outcomes under the true model, they do not affect the differential outcomes between the misspecified model and true model. Consequently, improving the accuracy of these sources through, e.g., fact-checking, always enhances the DM’s welfare, for standard reasons. The same cannot be said about sources with misspecified biases, as the welfare loss from misspecified learning, as captured by our metric, is non-monotonic in their precision. We discuss the intuition behind the comparative statics in Section 3.2, as well as their policy implications in Section 3.3.

We examine a number of model variations in Section 4. Among others, we find that if the precision of news signals is also subject to misspecified learning, then the DM will act more moderately than in the benchmark scenario where the true covariance matrix of news signals is known from the outset. The reason is that misspecified learning about news biases leads the DM to perceive the environment as noisier than it actually is, in a sense that we formalize in Section 4.1. Consequently, the DM moderates his behavior and ends up being better off in the long run. This finding speaks to the literature on misperception recalibration (e.g., provision of corrective information, cross-cutting exposure). It suggests that even if recalibration efforts are effective, their behavioral and welfare consequences may remain ambiguous, due to the compound learning effect discussed above.

Methodologically, our analysis leverages existing mathematical results on low-rank updates of invertible matrices. We demonstrate the usefulness of our tool in Section 3.4 where we sketch the proof of our main result, and in the online appendix where we present robustness checks of the baseline model.

The current paper adds to the following strands of the economics literature.

Misspecified Bayesian learning. The literature on misspecified Bayesian learning has grown rapidly in recent years. We add to the literature on passive mis-

specified learning, whereby the signals that guide the learning process are generated exogenously rather than endogenously by the DM’s actions (as in models of active misspecified learning). Our analysis builds on the seminal work of Berk (1966), who shows that the long-run outcome of misspecified Bayesian learning minimizes the Kullback-Leibler divergence from the true fundamental. Heidhues et al.’s (2019) analysis of overconfidence and prejudice characterizes the KL minimizer in a Gaussian environment like we do, but restricts the DM to misspecifying the mean of a single random variable: his calibre. We obtain new insights through entertaining the possibility of multifaceted misspecifications, at the cost of imposing restrictions on the covariance matrix that HKS allow to be fully general.

The theory of active misspecified Bayesian learning was recently advanced by Esponda and Pouzo (2016) among others.² Developments on the theoretical frontier have fueled the study of political economy models with misspecifications. Notably, Levy et al. (2022) study the recurrence of populism among voters with differing degrees of model complexity. Eliaz and Spiegler (2020) and Eliaz et al. (2022) examine the consequences of causal misspecifications on politicians’ narratives and platforms. Frick et al. (2022) study dispersed investment decisions in a society with assortativity neglect. And Bowen et al. (2023) examine the divergence of beliefs when news-sharing friends hold misperceptions about each other’s access to first-hand information.³

Our analysis differs from the aforementioned in its focus on a new form of misspecification and passive learning. Naturally, we adopt Berk(-Nash) as the solution concept and focus on characterizing the unique steady state. See, however, Eliaz et al. (2022) and Frick et al. (2022) for strengthened solution concepts, and Levy et al. (2022) for convergence properties of the equilibrium.

Media bias. The economic literature on media bias is vast but is devoted to understanding actual media biases (Anderson et al., eds, 2016).⁴ A notable exception

²Esponda and Pouzo (2016) propose Berk-Nash equilibrium as a steady-state notion for misspecified learning dynamics. Esponda et al. (2021), Fudenberg et al. (2021), Bohren and Hauser (2021), and Frick et al. (2023) examine convergence of Berk-Nash equilibrium in single- and multi-agent environments, while Ba (2021) and Lanzani (2022) entertain the possibility of regime shifts.

³Earlier, non-steady-state models are surveyed by Levy and Razin (2019) and Spiegler (2020).

⁴Theories of media bias — as synthesized in Chapters 14 and 16 of Anderson et al., eds (2016) — fall broadly into two categories: demand-driven bias and supply-driven bias. The former arises from consumers’ preference for confirming information or their attention bottleneck, whereas the latter from partisan media’s intent to distort voters’ behavior.

(Mis)perceived media bias is an important topic in political science, communication, and related fields (Feldman, 2014). These strands of literature are discussed in Section 3.3.

is by Gentzkow et al. (2018) (GWZ), whose model features a payoff-irrelevant, independent, confounder, along with a payoff-relevant state. A DM observes the signal generated by his personal source and chooses one of the many external sources to attend to. A source is biased if it exhibits a nonzero correlation with the confounder. The DM misperceives his personal source as unbiased and uses this belief to guide his learning about the other sources.

An important finding of GWZ is that the DM ends up underperceiving the biases of like-minded sources. We provide precise conditions when this is true in our model, and notice other crucial differences between the models' constructs and predictions. First and foremost, the nature of GWZ's news bias (as the correlation with a zero-mean confounder) implies that the distortion of the DM's long run behavior has zero unconditional mean. We, instead, study horizontal news bias and predict nonzero, unconditional, distortions of the DM's behavior.

A central focus of GWZ is on the endogenous trust in news sources, defined as the latter's perceived correlation with the true state. Trust is exogenous in our baseline model. When endogenized (as in Section 4.1 and Online Appendix O.2), we find — unlike GWZ — no effect on perceived news bias but a moderation of the DM's unconditional behavior. Overall, we view our approaches as complementary, as they address different aspects of reality and require different methods to analyze.

Misperception. There is an abundant literature documenting a wide variety of misperceptions. A DM may have misperceptions about himself, e.g., ability, or about others, e.g., behaviors and political opinions (Bursztyn and Yang, 2022; Haaland et al., 2023).

Misperceptions are often explained by learning frictions. Commonly utilized frictions include non-Bayesian learning or errors in reasoning (Benjamin, 2019). Motivated reasoning, confirmation bias, or other judgment errors may contribute to misperceptions. According to Nyhan (2020), these are the key drivers of political misinformation and its resistance to corrections. In economics, Alesina et al. (2020) propose a conceptual framework involving endogenous, perception-dependent responses to information while highlighting recent evidence on polarization.

In contrast, our DM has a misspecified prior but is otherwise a standard Bayesian. We view the two approaches as complementary. Their connection is explored by Bohren and Hauser (2023), who provide conditions when a non-Bayesian updating rule has a misspecified-learning representation.

2 Model

Setup. Time is discrete and infinite. In each period $t = 1, 2, \dots$, a random state ω_t is drawn independently from the standard normal distribution. A decision maker (DM) — informed by a finite set $\mathcal{S} := \{1, \dots, N\}$ of news sources — takes an action $a_t \in \mathbb{R}$ and earns a utility $-(a_t - \omega_t)^2$. The signal generated by source $i \in \mathcal{S}$ in period t is $X_{i,t} = \omega_t + b_i + \varepsilon_{i,t}$, where $b_i \in \mathbb{R}$ is the horizontal bias of the source, and $\varepsilon_{i,t}$ is a Gaussian error with mean zero and variance $v_i > 0$ that is independent of ω_t and of each other. Let $X_t := [X_{1,t} \ \dots \ X_{N,t}]^\top$ denote the vector of period- t signals. Suppose that X_t s are independent over time.

Initially, the DM holds a prior belief P_0 about the biases of news sources. In every subsequent period, he updates his belief in a Bayesian manner, based on the prior and realizations of news signals. It is well known that if the true values of news biases, which we denote by b_i^* s, lie in the support of the prior, then the DM's posterior belief will eventually converge to the truth. His long run action is the best linear predictor of the state given news signals, under the (correct) belief that news biases have value $b^* := [b_1^* \ \dots \ b_N^*]^\top$.

We focus on the case where the DM's prior is partly misspecified. We examine a particular kind of misspecification, whereby the DM holds dogmatic, wrong, beliefs about the biases of some sources and uses them to guide his learning about other sources. We examine how such misspecifications distort the DM's learning and, ultimately, his long run belief, behavior, and expected utility.

Formally, we say that the DM *misspecifies the bias of source i* if he initially assigns probability one to an event in which $b_i = \tilde{b}_i$, for some $\tilde{b}_i \neq b_i^*$. Let $\mathcal{M} := \{1, \dots, M\}$ denote the set of sources with misspecified biases. Suppose that $1 \leq M < N$, so that the DM misspecifies the biases of some but not all sources. For those sources in $\mathcal{S} - \mathcal{M}$, we assume that the DM's prior is nondegenerate on \mathbb{R}^{N-M} . The support of his prior is thus $\{b \in \mathbb{R}^N : b_i = \tilde{b}_i \ \forall i \in \mathcal{M}\}$. Outcomes of Bayesian learning based on the misspecified prior are the subjects of the current study.

Two remarks are in order. First, we assume, for now, that signals equal biases plus the true state and independent errors, and that the latter's variances are known to the DM from the outset. In Section 4, we entertain more general forms of signals whose covariance matrix is also a subject of misspecified learning. Second, it will become clear that adding sources with correctly specified biases to the analysis would

not affect our predictions in any meaningful way.

Leading example. The situation described in the introduction can be succinctly captured by letting $\mathcal{M} = \{\text{CNN, Fox News, DM's private sources}\}$ and $\tilde{b}_i - b_i^* < 0 \forall i \in \mathcal{M}$. This leading example serves as a basis for illustrating our framework and results.

3 Analysis

3.1 Preliminaries

We first formalize the problem faced by the DM. Let $\mathcal{N}(b, \Sigma)$ denote an arbitrary N -dimensional normal distribution with mean b and covariance matrix Σ , and $\mathcal{N}(b^*, \Sigma^*)$ denote the true distribution of news signals. The Kullback-Leibler divergence from $\mathcal{N}(b, \Sigma)$ to $\mathcal{N}(b^*, \Sigma^*)$ is

$$D_{KL}(b^*, \Sigma^* || b, \Sigma) = \frac{1}{2} \left(\text{tr}(\Sigma^{-1}\Sigma^*) - N + (b - b^*)^\top \Sigma^{-1}(b - b^*) + \log \frac{\det \Sigma}{\det \Sigma^*} \right). \quad (1)$$

From the seminal result of Berk (1966), we know that the DM's long run belief concentrates on the set of distributions that minimizes the Kullback-Leibler divergence from the true distribution (hereinafter, the *KL minimizers*). That is, for every open cover of the set of KL minimizers, the DM assigns probability one to it in the limit, as $t \rightarrow +\infty$. Let $\mathcal{N}(\hat{b}, \hat{\Sigma})$ denote a typical KL minimizer. Since Σ^* is known to the DM from the outset, $\hat{\Sigma} = \Sigma^*$ must hold. Simplifying (1) accordingly turns the DM's problem into

$$\min_{b \in \text{supp} P_0} (b - b^*)^\top \Sigma^{*-1}(b - b^*). \quad (2)$$

In the appendix, we show that the solution to (2) is unique and is fully captured by $\Delta := \hat{b} - b^*$. Call Δ *the DM's misperception about news biases in the long run*, and note that $\Delta_i = \tilde{b}_i - b_i^* \forall i \in \mathcal{M}$. Let $\text{BLP}^*(X)$ and $\widehat{\text{BLP}}(X)$ denote the best linear predictors of the state given news signals, under the beliefs that news biases have values b^* and \hat{b} , respectively. These are the long run actions taken by the DM under the correct and misspecified models, respectively, and their difference captures the distortionary effect of misspecified learning on the DM's long run behavior. The expected utilities generated by these actions, evaluated at the true distribution of

news signals, are denoted by EU^* and $\widehat{\text{EU}}$, respectively. Their difference represents the welfare loss from misspecified learning.

We develop a single-dimensional metric for the DM's overall misspecification. To this end, write ν_i for the signal-to-noise ratio $1/v_i$ of source i . For each $i \in \mathcal{M}$, define

$$\gamma_i := \frac{\nu_i}{1 + \sum_{i=1}^M \nu_i}$$

as the *relative precision* of the signal generated by source i , compared to the other sources in \mathcal{M} . The term

$$\overline{\Delta} := \sum_{i \in \mathcal{M}} \gamma_i (\tilde{b}_i - b_i^*)$$

captures the total misspecification associated with the sources in \mathcal{M} , weighted by their relative precision. As we worsen the misspecification $|\tilde{b}_i - b_i^*|$ associated with source i or raise its relative precision γ_i , the influence of the source on $\overline{\Delta}$ increases.

3.2 Results

We present two main results. The first characterizes the distortionary effects of misspecified learning on the DM's long run belief, behavior, and expected utility. The second investigates the comparative statics of distortionary effects.

Theorem 1. (i) $\Delta_i = \tilde{b}_i - b_i^*$ if $i \in \mathcal{M}$ and $\overline{\Delta}$ if $i \notin \mathcal{M}$; (ii) $\widehat{\text{BLP}}(X) - \text{BLP}^*(X) = -\overline{\Delta}$; (iii) $\widehat{\text{EU}} - \text{EU}^* = -\overline{\Delta}^2$.

Theorem 2. $\overline{\Delta}$ depends only on $(\Delta_i, \nu_i)_{i \in \mathcal{M}}$. For each $i \in \mathcal{M}$, $\partial \overline{\Delta} / \partial \Delta_i > 0$, whereas $\partial \overline{\Delta} / \partial \nu_i$ has an ambiguous sign in general.

Theorem 1 reduces the distortionary effects of misspecified learning to a single metric: $\overline{\Delta}$. The latter depends only on the characteristics of the sources with misspecified biases, but not on those with initially unknown biases. The irrelevance of sources of the second kind for the distortionary effect is interesting and surprising, and we postpone the discussion of its mathematical underpinning till Section 3.4. Here we focus on results that are more intuitive, i.e., those that relate the distortionary effect to sources of the first kind.

We illustrate Theorem 1 in the leading example, where the DM begins by underperceiving the conservative bias of Fox News and his own private sources, while

overperceiving the liberal bias of CNN, i.e., $\Delta_i = \tilde{b}_i - b_i^* < 0 \forall i \in \mathcal{M}$. The overall distortion of his worldview takes the form of a leftward shift of magnitude $|\overline{\Delta}|$. When learning about the biases of the unfamiliar sources, the DM uses sources that he holds dogmatic, wrong, beliefs about as yardsticks. The distortion of his worldview is then mirrored in his beliefs about the unfamiliar sources through misspecified learning. In the long run, the DM overestimates (resp. underestimates) the bias of any liberal (resp. conservative) source by $|\overline{\Delta}|$. He also behaves more conservatively than under the true model by $|\overline{\Delta}|$ and, as a result, experiences a utility loss of $|\overline{\Delta}|^2$.

Theorem 2 investigates the comparative statics of the distortionary effect. The result that $\partial\overline{\Delta}/\partial\Delta_i > 0$ is the most intuitive. In our leading example, this means that a slight increase in the misspecification associated with any established source will worsen the voter’s misperception about any unfamiliar source, resulting in more polarized behaviors and a lowered expected utility in the long run.

The comparative statics regarding source precision are more nuanced. As we raise the absolute precision of source i (as captured by ν_i), we also raise its relative precision (as measured by γ_i), while diminishing the relative precision γ_j of any other source $j \in \mathcal{M} - \{i\}$. As a consequence, source i is taken more seriously by the DM in the learning process, and misspecifications of its bias distort the DM’s long run belief and behavior more heavily than before. The opposite happens to source j . Depending on whether the DM misspecifies the bias of source i more significantly than that of source j , or the other way round, the outcome is either an exacerbation of the distortionary effect, or a correction thereof.⁵

3.3 Implications

Hostile media effect. The *hostile media effect* refers to the phenomenon that opposing partisans perceive news coverage to be more biased against their side than it actually is. This phenomenon, first discovered by Vallone et al. (1985), has been replicated by numerous studies in political science, communication, and psychology (Feldman, 2014). In a classical experiment by Arpan and Raney (2003), student subjects read a balanced story about their hometown college football team in one of three newspapers: the home-town, the rival-town, or an out-of-state neutral-town. Subjects viewed the coverage as favoring the rival university even if it was believed

⁵Note that the ambiguity arises only when $M \geq 2$ and is absent when $\mathcal{M} = \{1\}$ (as in HKS).

to be from the neutral source. More recently, Hassell et al. (2020) present journalists with balanced news stories that differ only in their ideological content (e.g., whether the candidate running for office is Democratic or Republican). Journalists exhibit no gatekeeping bias when selecting which news stories to cover, despite the fact that many of them are ideologically liberal. This finding debunks the “liberal media bias” widely perceived among Republicans.⁶

Early studies on the hostile media effect focus on how brand names and reputation serve as heuristics to influence people’s perceptions about mainstream media bias (Baum and Gussin, 2008). The issue of learning and belief formation about new online sources has recently caught the attention of a few scholars, with Peterson and Kagalwala (2021) stressing the role of stereotype — rather than content per se — in explaining partisans’ hostility toward unfamiliar, out-party news sources. Part (i) of Theorem 1 formalizes a mechanism of stereotype formation, whereby dogmatic, wrong, beliefs about established sources breed persistent misperceptions about the biases of unfamiliar sources.

False polarization. *False polarization* is defined as the difference between the perceived distance between the positions of two groups and the actual distance between the positions of these groups. The presence of false polarization in the American public was rigorously established by Levendusky and Malhotra (2016b). These authors analyze survey data gleaned from a nationally representative sample, where respondents were asked about their self-placements on an issue scale, as well as their placements of a typical Democratic and Republican voter on that scale. The data show that Americans significantly misperceive the public to be more divided along partisan lines than it is in reality and that misperceptions about opposing partisans are larger than those about their own party.

To the extent that individual human beings can now voice their opinions online at almost no cost, our model predicts false polarization in the digital news sphere. It also establishes a causal relation between misspecified news bias and false polarization — a finding that is consistent with the literature’s common attribution of false polarization to partisan media exposure (Ahler, 2014; Levendusky and Malhotra, 2016a).

⁶Suggestive evidence for hostile media effect abounds. According to a 2009 Pew Research Center study, half of regular CNN viewers see Fox News as mostly conservative, as do 45% of its own viewers; 50% of regular Fox News viewers say NBC News is mostly liberal, compared with only about third of regular viewers of CNN (35%) and MSNBC (31%).

Polarized behavior. There is clear evidence that (mis)perceived polarization and media bias increases affective polarization and undermines trust in media and government (Feldman, 2014). As for the impact on voters’ actual issue and policy platforms, the answers provided by the existing literature are mixed. While some authors, like Stanovich (2021), embrace the idea that myside bias in the news sphere fuels more polarized behaviors, others, including Levendusky and Malhotra (2016a), suggest that false polarization could lead voters to moderate their issue positions. Theorem 1 (ii) suggests that the finding of Levendusky and Malhotra (2016a) may be reversed as time goes by; the Republican voter in our leading example behaves more conservatively than under the true model in the long run.

Implication for fact-checking. Perhaps the most surprising finding of ours is that sources with initially unknown biases are irrelevant for the distortionary effect of misspecified learning. While these sources clearly matter for the long run outcomes under the true model, they do not affect the differential outcomes between the true model and misspecified model. As we raise the precision of these sources, the DM’s expected utility under the true model and, hence, the misspecified model, increases unambiguously (indeed, $EU^* = -(1 + \sum_{i \in \mathcal{S}} \nu_i)^{-1}$). The same cannot be said about sources with misspecified biases, as their precision has, in general, ambiguous effects on the welfare distortion $\overline{\Delta}^2$ from misspecified learning.

The last finding has important consequences for policy interventions that target misinformation and fake news. It suggests that interventions aimed at improving source accuracy (e.g., fact-checking) may entail unintended welfare consequences, especially when they operate through modulating sources that people already hold dogmatic misperceptions about. When it comes to lesser-known sources, however, fact-checking always improves welfare. This result is, to our best knowledge, new to the literature on (political) misinformation. The latter is surveyed by Nyhan (2020), who outlines the cognitive and psychological impediments to effective fact-checking. Misspecified learning is absent from the author’s list.

3.4 Proof sketch

The proof of Theorem 1 is instructive and worth sketching. For ease of notation, write v for $[v_1 \cdots v_N]^\top$, ν for $[\nu_1 \cdots \nu_N]^\top$, and $\mathbf{1}_k$ and $\mathbf{0}_k$ for the k -vector of ones and zeros, $k \in \mathbb{N}$.

We first solve for Δ . Rewrite (2) as

$$\min_{\Delta \in \mathbb{R}^N: \Delta_i = \tilde{b}_i - b_i^* \forall i=1, \dots, M} \Delta^\top \Sigma^{*-1} \Delta,$$

and note that any solution to this problem is fully determined by the following system of first-order conditions: $\forall j = M + 1, \dots, N$,

$$2 \sum_{i=1}^N \Sigma_{ji}^{*-1} \Delta_i = 0.$$

Simplifying the system of first-order conditions yields

$$\underbrace{\begin{bmatrix} \Sigma_{M+1M+1}^{*-1} & \cdots & \Sigma_{M+1N}^{*-1} \\ \vdots & \ddots & \vdots \\ \Sigma_{NM+1}^{*-1} & \cdots & \Sigma_{NN}^{*-1} \end{bmatrix}}_A \begin{bmatrix} \Delta_{M+1} \\ \vdots \\ \Delta_N \end{bmatrix} = - \underbrace{\begin{bmatrix} \Sigma_{M+11}^{*-1} & \cdots & \Sigma_{M+1M}^{*-1} \\ \vdots & \ddots & \vdots \\ \Sigma_{N1}^{*-1} & \cdots & \Sigma_{NM}^{*-1} \end{bmatrix}}_B \begin{bmatrix} \Delta_1 \\ \vdots \\ \Delta_M \end{bmatrix}.$$

Solving A^{-1} and B is challenging in general.⁷ A seemingly innocuous assumption turns out to be key, namely news signals, net of their biases, equal a common (standard-normal) state plus independent errors. Thus their covariance matrix Σ^* is the sum of an invertible matrix: $\text{diag}(v)$, and a matrix of rank one: $\mathbf{1}_N \mathbf{1}_N^\top$. The next lemma — proven by Sherman and Morrison (1950) — prescribes a formula for inverting rank-one updates of invertible matrices.

Lemma 1 (Sherman and Morrison 1950). *Suppose that $A \in \mathbb{R}^{n \times n}$ is an invertible matrix and that $u, v \in \mathbb{R}^n$ are column vectors. Then $A + uv^\top$ is invertible if and only if $1 + v^\top A^{-1}u \neq 0$, in which case*

$$(A + uv^\top)^{-1} = A^{-1} - \frac{A^{-1}uv^\top A^{-1}}{1 + v^\top A^{-1}u}.$$

Applying the Sherman-Morrison formula to Σ^* yields

$$\Sigma^{*-1} = \text{diag}(v) - \frac{v v^\top}{1 + \mathbf{1}_N^\top v}, \quad (3)$$

⁷HKS's approach works for general Σ^* when $M = 1$. Ours works for arbitrary M but imposes restrictions on Σ^* . We replicate the analysis of HKS in Online Appendix O.2.

from which we can read off the expressions for A and B . Furthermore, since Σ^{*-1} is itself a rank-one update of an invertible matrix, we can apply the Sherman-Morrison formula again to obtain A^{-1} and, in turn, Δ . Details are in the appendix.

We next demonstrate that

$$\Sigma^{*-1}\Delta = \left[\begin{array}{ccc|ccc} \Sigma_{11}^{*-1} & \cdots & \Sigma_{1M}^{*-1} & \Sigma_{1M+1}^{*-1} & \cdots & \Sigma_{1N}^{*-1} \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ \Sigma_{M1}^{*-1} & \cdots & \Sigma_{MM}^{*-1} & \Sigma_{MM+1}^{*-1} & \cdots & \Sigma_{MN}^{*-1} \\ \hline & & B & & & A \end{array} \right] \left[\begin{array}{c} \Delta_1 \\ \vdots \\ \Delta_M \\ \hline \overline{\Delta}\mathbf{1}_{N-M} \end{array} \right] = \left[\begin{array}{c} \nu_1(\Delta_1 - \overline{\Delta}) \\ \vdots \\ \nu_M(\Delta_M - \overline{\Delta}) \\ \hline \mathbf{0}_{N-M} \end{array} \right]. \quad (4)$$

The fact that $[\Sigma^{*-1}\Delta]_j = 0 \forall j = M+1, \dots, N$ is nothing but the first-order conditions. Meanwhile, notice, from (3), that

$$[\Sigma^{*-1}\Delta]_j = \nu_j \left(\Delta_j - \frac{\nu^\top \Delta}{1 + \mathbf{1}_N^\top \nu} \right) \quad \forall j = 1, \dots, N.$$

Substituting $\Delta_{M+1} = \overline{\Delta}$ and $[\Sigma^{*-1}\Delta]_{M+1} = 0$ into the above expression yields $\nu^\top \Delta / (1 + \mathbf{1}_N^\top \nu) = \overline{\Delta}$ and, in turn, $[\Sigma^{*-1}\Delta]_j = \nu_j(\Delta_j - \overline{\Delta}) \forall j = 1, \dots, M$, as desired.

Importantly, $\Sigma^{*-1}\Delta$ depends only on sources with misspecified biases but nothing else. As demonstrated in the appendix, this is the precise reason why sources with initially unknown biases do not enter the determination of distortionary effects.

4 Extensions

4.1 Unknown covariance matrix

So far we have assumed that the covariance matrix of news signals is known to the DM from the outset. In light of today's news landscape, perhaps a more natural assumption is that the covariance matrix is itself a subject of misspecified learning. We entertain this possibility in this section, with a focus on whether the additional source of misspecified learning leads to a polarization of the DM's long run belief and behavior, or a moderation thereof.

Formally, suppose that the DM holds a prior belief P_0 about the biases of the news sources and their covariance matrix at the outset. The prior misspecifies the biases of some but not all news sources, and is nondegenerate on the set of $N \times N$ covariance matrices. Its support equals $\text{supp}P_0 = \{(b, \Sigma) \in \mathbb{R}^N \times \mathbb{R}^{N \times N} : b_i =$

$\tilde{b}_i \forall i \in \mathcal{M}$ and Σ is positive definite}. In every subsequent period, the DM observes realizations of news signals and updates his belief in a Bayesian manner. In the long run, his belief concentrates on the KL minimizer solving

$$\min_{(b, \Sigma) \in \text{supp} P_0} \quad (1).$$

Let $(\hat{b}, \hat{\Sigma})$ denote the solution to the above problem, and (b^*, Σ^*) denote the truth. Define the DM's misperception of news biases, as well as the distortion of his long run action and expected utility, the same way as before. The next theorem pinpoints the difference between these quantities and their analogs in the baseline model.

Theorem 3. *Let everything be as above. Then (i) Δ is the same as in Theorem 1, whereas $\hat{\Sigma} = \Sigma^* + \Delta\Delta^\top$ and satisfies $\det(\hat{\Sigma}) = \det(\Sigma^*)/\delta > \det(\Sigma^*)$, where $\delta := (1 + \Delta^\top \Sigma^{*-1} \Delta)^{-1} \in (0, 1)$. (ii) $\mathbb{E}_{(b^*, \Sigma^*)}[\widehat{\text{BLP}}(X) - \text{BLP}^*(X)] = -\delta\bar{\Delta} \in (-\bar{\Delta}, 0)$. (iii) $\widehat{\text{EU}} - \text{EU}^* = -\delta\bar{\Delta}^2 \in (-\bar{\Delta}^2, 0)$.*

Part (i) of Theorem 3 follows from Theorem 1 of Heidhues et al. (2019), whereas Parts (ii) and (iii) are new. On the one hand, misspecified learning of the covariance matrix does not interfere with that of news biases and leaves the DM's long run misperception about news biases unaffected. On the other hand, misspecified learning of news biases makes the environment look more noisy to the DM than it actually is, i.e., $\det(\hat{\Sigma}) > \det(\Sigma^*)$.⁸ The resulting adjustment in the DM's behavior takes the form of a moderation and entails a welfare gain, compared to the scenario where the true covariance matrix is known to the DM from the outset.

To understand the moderation effect, we decompose the distortion of the DM's long run behavior as follows:

$$\begin{aligned} \widehat{\text{BLP}}(X) - \text{BLP}^*(X) &= \mathbf{1}_N^\top \hat{\Sigma}^{-1} (X - \hat{b}) - \mathbf{1}_N^\top \Sigma^{*-1} (X - b^*) \\ &= \underbrace{-\mathbf{1}_N^\top \Sigma^{*-1} \Delta}_{\text{(I)}} + \underbrace{\mathbf{1}_N^\top (\hat{\Sigma}^{-1} - \Sigma^{*-1}) (X - b^*)}_{\text{(II)}} + \underbrace{-\mathbf{1}_N^\top (\hat{\Sigma}^{-1} - \Sigma^{*-1}) \Delta}_{\text{(III)}}. \end{aligned}$$

Effect (I) in the above expression stems from misspecified learning of news biases and is shown to equal $-\bar{\Delta}$ in Theorem 1(ii). Effect (II) stems from misspecified learning of the covariance matrix; it has zero mean and undermines the DM's long run welfare.

⁸The generalized variance of a random vector is the determinant of the covariance matrix.

Effect (III) captures the compound effect of misspecified learning. It equals

$$(III) = -\mathbf{1}_N^\top \left(-\frac{\Sigma^{*-1} \Delta \Delta^\top \Sigma^{*-1}}{1 + \Delta^\top \Sigma^{*-1} \Delta} \right) \Delta = \bar{\Delta} \left(1 - \frac{1}{1 + \Delta^\top \Sigma^{*-1} \Delta} \right),$$

where the first equality uses the Sherman-Morrison formula, and the second equality the fact that $\mathbf{1}_N^\top \Sigma^{*-1} \Delta = \bar{\Delta}$. The combined effect of (I) and (III) is $-\bar{\Delta}/(1 + \Delta^\top \Sigma^{*-1} \Delta) := -\delta \bar{\Delta}$, which is less negative than $-\bar{\Delta}$ because Σ^{*-1} is positive definite.

Our last theorem investigates the comparative statics of compound misspecified learning.

Theorem 4. $\bar{\Delta}$ and δ depend only on $(\Delta_i, \nu_i)_{i \in \mathcal{M}}$. For each $i \in \mathcal{M}$, $\partial \bar{\Delta} / \partial \Delta_i$ and $\partial \bar{\Delta} / \partial \nu_i$ are the same as in Theorem 2, whereas $\partial \delta \bar{\Delta} / \partial \Delta_i$, $\partial \delta \bar{\Delta} / \partial \nu_i$, $\partial \delta \bar{\Delta}^2 / \partial \Delta_i$, and $\partial \delta \bar{\Delta}^2 / \partial \nu_i$ have ambiguous signs in general.

In light of Theorem 2, it is unsurprising that changes in source accuracy have, in general, ambiguous effects on outcomes of misspecified learning. What is new here is the ambiguity associated with correcting misspecifications; the latter unambiguously moderates the DM’s behavior and improves his welfare in the baseline model. The ambiguity stems from compound misspecified learning: by raising the misspecification associated with an established source, we make the environment look more noisy to the DM than before, i.e., $1 + \Delta^\top \Sigma^{*-1} \Delta$ increases. This amplifies the moderation effect discussed above, i.e., δ decreases, and in some cases could moderate the DM’s behavior and improve his welfare.

There is a large empirical literature studying the treatment effects of misperception recalibration (Bursztyn and Yang, 2022). Early research indicated the possibility of a “backfire effect,” whereby recalibration efforts (e.g., provision of corrective information, cross-cutting exposure) end up reinforcing rather than reducing subjects’ misperceptions (see, e.g., Nyhan and Reifler 2010). More recently, an emerging research consensus finds that the backfire effect could be more tenuous than previously thought and that more attention should be paid to alternative issues, such as the durability and scalability of the interventions (Nyhan, 2021; Hartman et al., 2022). We take no stand on this debate and instead caution that even if misspecifications can partly be corrected (bringing Δ_i , $i \in \mathcal{M}$, closer to zero in our leading example), their behavioral and welfare consequences may remain ambiguous, due to the compound learning effect studied in Theorem 4.

4.2 General signal structure

The news signals in the baseline model equal biases plus the true state and independent errors. In the online appendix, we examine a variant of the baseline model where the signal generated by source i in period t equals

$$X_{i,t} = \underbrace{b_i}_{\text{bias}} + \alpha_i \underbrace{\omega_t}_{\text{true state}} + \overbrace{\sum_{k=1}^K \beta_{i,k} \underbrace{\theta_{k,t}}_{\text{common confounder}}}_{\text{total error}} + \underbrace{\epsilon_{i,t}}_{\text{idiosyncratic error}} .$$

The common confounder $\theta_{k,t}$ represents, e.g., shocks to the editorial sentiment at the various sources. It is independent of the true state ω_t , as well as the source-specific, idiosyncratic error $\epsilon_{i,t}$. The baseline model assumes that $\alpha_i \equiv 1$ and $\beta_{i,k} \equiv 0$. By allowing these coefficients to take any real values, we can entertain arbitrary correlations between signals and the true state, as well as correlated total errors through common confounders.

We examine the robustness of our results to the above model variation. Methodologically, we note that the covariance matrix of the above signals is a rank- $(K + 1)$ update of an invertible matrix, so its inverse can be obtained from applying the Sherman-Morrison formula iteratively. Prediction-wise, while some baseline results — such as the use of a simple metric to capture distortions of the DM’s worldview and behavior — remain unchanged, others — such as the DM’s misperceptions about the biases of individual sources — become more nuanced. Details are in the online appendix.

5 Discussions

Other biases. Our leading example focused on a DM who underperceives conservative bias and overperceives liberal bias. That is, the DM perceives a left shift: $\tilde{b}_i - b_i^* < 0 \forall i \in \mathcal{M}$. Other biases can be studied within our framework by adjusting the pattern of (multifaceted) misspecification. For example, the widely observed *false-consensus* effect (Ross et al., 1977) shows that many people overestimate the extent to which others share their own beliefs, while those who disagree are viewed as extreme. Let $\mathcal{N} := \{i \in \mathcal{M} \mid |b_{DM}^* - b_i^*| < r\}$ denote the “nearby” sources about which the DM is misspecified. False-consensus is captured when $|\tilde{b}_i| < |b_i^*|$ for $i \in \mathcal{N}$

and $|b_j^*| < |\tilde{b}_j|$ for $j \in \mathcal{M} - \mathcal{N}$.

In-group/out-group effects can be captured when sources are labelled as left and right: $\mathcal{L} := \{i \in \mathcal{M} : b_i^* < 0\}$ and $\mathcal{R} := \{i \in \mathcal{M} : b_i^* > 0\}$. *Group asymmetry* is exhibited when a Republican voter (mis)perceives more bias among liberal sources than conservative sources: $\forall i \in \mathcal{L}$ and $j \in \mathcal{R}$: $\Delta_i < 0$, $\Delta_j > 0$, and $|\Delta_i| > \Delta_j$. Such misperceptions were documented by Levendusky and Malhotra (2016b). Similarly, Stroud et al. (2014) find that politically dissimilar media are seen as having a more uniform partisan bias. A Republican voter (mis)perceiving excessive out-group uniformity if \tilde{b}_i s form a contraction of b_i^* s among $i \in \mathcal{L}$: $\exists r < 0$ such that $\tilde{b}_i > b_i^*$ if $b_i^* < r$ and $\tilde{b}_i < b_i^*$ if $b_i^* > r$.

Multidimensional positions. Online Appendix O.3 entertains the possibility that the payoff-relevant state is a multidimensional vector. Each dimension represents a distinct issue such as climate change, gun policies, etc. The reporting from a news source is potentially biased along every dimension. We provide closed-form solutions to the distortionary effects of misspecified learning using the Woodbury formula for block matrix inversion (Woodbury, 1950). We discuss how correlations between different dimensions can potentially complicate the comparative statics and leave a detailed investigation for future research.

A Proofs

Proof of Theorem 1. Part (i): In Section 3.4, we already demonstrated that

$$\underbrace{\begin{bmatrix} \Sigma_{M+1M+1}^{*-1} & \cdots & \Sigma_{M+1N}^{*-1} \\ \vdots & \ddots & \vdots \\ \Sigma_{NM+1}^{*-1} & \cdots & \Sigma_{NN}^{*-1} \end{bmatrix}}_A \begin{bmatrix} \Delta_{M+1} \\ \vdots \\ \Delta_N \end{bmatrix} = - \underbrace{\begin{bmatrix} \Sigma_{M+11}^{*-1} & \cdots & \Sigma_{M+1M}^{*-1} \\ \vdots & \ddots & \vdots \\ \Sigma_{N1}^{*-1} & \cdots & \Sigma_{NM}^{*-1} \end{bmatrix}}_B \begin{bmatrix} \Delta_1 \\ \vdots \\ \Delta_M \end{bmatrix}$$

where

$$A = \text{diag}(\nu_{M+1} \cdots \nu_N) - \frac{1}{1 + \sum_{i=1}^N \nu_i} \begin{bmatrix} \nu_{M+1} \\ \vdots \\ \nu_N \end{bmatrix} \begin{bmatrix} \nu_{M+1} & \cdots & \nu_N \end{bmatrix},$$

and

$$B = \frac{-1}{1 + \sum_{i=1}^N \nu_i} \begin{bmatrix} \nu_{M+1} \\ \vdots \\ \nu_N \end{bmatrix} \begin{bmatrix} \nu_1 & \cdots & \nu_M \end{bmatrix}.$$

Since A is itself a rank-one update of an invertible matrix, applying the Sherman-Morrison formula to it shows that

$$\begin{aligned} A^{-1} &= \text{diag}^{-1}(\nu_{M+1} \cdots \nu_N) \\ &+ \frac{1}{1 + \sum_{i=1}^N \nu_i} \frac{\text{diag}^{-1}(\nu_{M+1} \cdots \nu_N) \begin{bmatrix} \nu_{M+1} \\ \vdots \\ \nu_N \end{bmatrix} \begin{bmatrix} \nu_{M+1} \cdots \nu_N \end{bmatrix} \text{diag}^{-1}(\nu_{M+1} \cdots \nu_N)}{1 - \begin{bmatrix} \nu_{M+1} \cdots \nu_N \end{bmatrix} \text{diag}^{-1}(\nu_{M+1} \cdots \nu_N) \begin{bmatrix} \nu_{M+1} \\ \vdots \\ \nu_N \end{bmatrix}} / (1 + \sum_{i=1}^N \nu_i) \\ &= \text{diag}(\nu_{M+1} \cdots \nu_N) + \frac{1}{1 + \sum_{i=1}^M \nu_i} \mathbf{1}_{N-M} \mathbf{1}_{N-M}^\top \end{aligned}$$

and, in turn, that

$$A^{-1}B = \frac{-1}{1 + \sum_{i=1}^N \nu_i} \left(1 + \frac{\sum_{i=M+1}^N \nu_i}{1 + \sum_{i=1}^M \nu_i} \right) \mathbf{1}_{N-M} \begin{bmatrix} \nu_1 \cdots \nu_M \end{bmatrix} = \frac{-1}{1 + \sum_{i=1}^M \nu_i} \mathbf{1}_{N-M} \begin{bmatrix} \nu_1 \cdots \nu_M \end{bmatrix}$$

and that

$$\begin{bmatrix} \Delta_{M+1} \\ \vdots \\ \Delta_N \end{bmatrix} = -A^{-1}B \begin{bmatrix} \Delta_1 \\ \vdots \\ \Delta_M \end{bmatrix} = \frac{\sum_{i=1}^M \nu_i \Delta_i}{1 + \sum_{i=1}^M \nu_i} \mathbf{1}_{N-M} = \bar{\Delta} \mathbf{1}_{N-M}.$$

Parts (ii) and (iii): The best linear predictor of the state given news signals is $\text{BLP}^*(X) = \mathbf{1}_N^\top \Sigma^{*-1} (X - b^*)$ under the true model, and $\widehat{\text{BLP}}(X) = \mathbf{1}_N^\top \Sigma^{*-1} (X - \hat{b})$ under the misspecified model.

$$\widehat{\text{BLP}}(X) - \text{BLP}^*(X) = -\mathbf{1}_N^\top \Sigma^{*-1} \Delta = -\sum_{i=1}^M \nu_i (\Delta_i - \bar{\Delta}) = -\bar{\Delta}, \quad (5)$$

irrespective of X , where the second equality uses (4), and the last equality uses the identity that $\sum_{i=1}^M \nu_i \Delta_i = \bar{\Delta}(1 + \sum_{i=1}^M \nu_i)$. As a consequence.

$$\begin{aligned}
\widehat{\text{E}}\text{U} - \text{E}\text{U}^* &= \mathbb{E}_{(b^*, \Sigma^*)}[-(\widehat{\text{BLP}} - \omega)^2 + (\text{BLP}^* - \omega)^2] \\
&= \mathbb{E}_{(b^*, \Sigma^*)}[-(-\bar{\Delta} + \text{BLP}^* - \omega)^2 + (\text{BLP}^* - \omega)^2] \\
&= -\bar{\Delta}^2 + 2\bar{\Delta}\mathbb{E}_{(b^*, \Sigma^*)}[\text{BLP}^* - \omega] \\
&= -\bar{\Delta}^2. \quad \square
\end{aligned}$$

Proof of Theorem 3. Part (i): The result that Δ is the same as before whereas $\widehat{\Sigma} = \Sigma^* + \Delta\Delta^\top$ is established by Theorem 1 of Heidhues et al. (2019). Since $\widehat{\Sigma}$ is a rank-one update of Σ^* , $\det(\widehat{\Sigma}) = \det(\Sigma^*)(1 + \Delta^\top \Sigma^{*-1} \Delta)$ by Lemma 1.1 of Ding and Zhou (2007). The last term is greater than $\det(\Sigma^*)$ because Σ^* is positive definite, and so $\det(\Sigma^*) > 0$ and Σ^{*-1} is also positive definite.

Part (ii): The best linear predictors of the state under the correct and misspecified models are now $\text{BLP}^*(X) = \mathbf{1}_N^\top \Sigma^{*-1}(X - b^*)$ and $\widehat{\text{BLP}}(X) = \mathbf{1}_N^\top \widehat{\Sigma}^{-1}(X - \hat{b})$, respectively. Their difference equals

$$\begin{aligned}
&\widehat{\text{BLP}}(X) - \text{BLP}^*(X) \\
&= \mathbf{1}_N^\top \widehat{\Sigma}^{-1}(X - \hat{b}) - \mathbf{1}_N^\top \Sigma^{*-1}(X - b^*) \\
&= \mathbf{1}_N^\top \left(\Sigma^{*-1} - \frac{\Sigma^{*-1} \Delta \Delta^\top \Sigma^{*-1}}{1 + \Delta^\top \Sigma^{*-1} \Delta} \right) (X - b^* - \Delta) - \mathbf{1}_N^\top \Sigma^{*-1}(X - b^*) \\
&= \frac{-\bar{\Delta}}{1 + \Delta^\top \Sigma^{*-1} \Delta} (1 + \Delta^\top \Sigma^{*-1}(X - b^*)), \tag{6}
\end{aligned}$$

where the second equality uses the Sherman-Morrison formula, and the third equality uses (5). Since the second term in the last line has zero expectation under the true model, $\mathbb{E}_{(b^*, \Sigma^*)}[\widehat{\text{BLP}} - \text{BLP}^*] = -\bar{\Delta}(1 + \Delta^\top \Sigma^{*-1} \Delta)^{-1}$ as desired.

Part (iii): Expanding $\widehat{\text{E}}\text{U}$ and EU^* yields

$$\begin{aligned}
\widehat{\text{E}}\text{U} - \text{E}\text{U}^* &= -\mathbb{E}_{(b^*, \Sigma^*)}[(\omega - \widehat{\text{BLP}})^2] + \mathbb{E}_{(b^*, \Sigma^*)}[(\omega - \text{BLP}^*)^2] \\
&= -\mathbb{E}_{(b^*, \Sigma^*)}[(\omega - \text{BLP}^* - (\widehat{\text{BLP}} - \text{BLP}^*))^2] + \mathbb{E}_{(b^*, \Sigma^*)}[(\omega - \text{BLP}^*)^2] \\
&= -\underbrace{\mathbb{E}_{(b^*, \Sigma^*)}[(\widehat{\text{BLP}} - \text{BLP}^*)^2]}_C + 2 \underbrace{\mathbb{E}_{(b^*, \Sigma^*)}[(\omega - \text{BLP}^*)(\widehat{\text{BLP}} - \text{BLP}^*)]}_D.
\end{aligned}$$

The remainder of the proof verifies in two steps that $C = \overline{\Delta}^2(1 + \Delta^\top \Sigma^{*-1} \Delta)^{-1}$ and that $D = 0$.

Step 1. Expanding C yields

$$C = \left(\frac{\overline{\Delta}}{1 + \Delta^\top \Sigma^{*-1} \Delta} \right)^2 \mathbb{E}_{(b^*, \Sigma^*)} [(\Delta^\top \Sigma^{*-1} (X - b^*))^2 + 1 + 2\Delta^\top \Sigma^{*-1} (X - b^*)],$$

where the third term in the expectation operator has zero mean. Meanwhile,

$$\begin{aligned} \mathbb{E}_{(b^*, \Sigma^*)} (\Delta^\top \Sigma^{*-1} (X - b^*))^2 &= \mathbb{E}_{(b^*, \Sigma^*)} \left(\sum_{i=1}^M \nu_i (\Delta_i - \overline{\Delta}) (\omega + \varepsilon_i) \right)^2 \\ &= \left(\sum_{i=1}^M \nu_i (\Delta_i - \overline{\Delta}) \right)^2 \mathbb{E}[\omega^2] + \sum_{i=1}^M \nu_i^2 (\Delta_i - \overline{\Delta})^2 \mathbb{E}[\varepsilon_i^2] \\ &= \sum_{i=1}^M \nu_i \Delta_i^2 - (\overline{\Delta})^2 (1 + \sum_{i=1}^M \nu_i), \end{aligned}$$

where the first equality uses (4), the second equality the fact that ω, ε_i are independent with zero mean, and the third equality the fact that $\mathbb{E}[\omega^2] = 1$, $\nu_i = 1/\mathbb{E}[\varepsilon_i^2]$, and $\sum_{i=1}^M \nu_i \Delta_i = \overline{\Delta}(1 + \sum_{i=1}^M \nu_i)$. The last line equals $\Delta^\top \Sigma^{*-1} \Delta$ because

$$\Delta^\top \Sigma^{*-1} \Delta = \sum_{i=1}^M \nu_i (\Delta_i - \overline{\Delta}) \Delta_i = \sum_{i=1}^M \nu_i \Delta_i^2 - (\overline{\Delta})^2 (1 + \sum_{i=1}^M \nu_i).$$

Taken together, we conclude that $C = \overline{\Delta}^2(1 + \Delta^\top \Sigma^{*-1} \Delta)^{-1}$ as desired.

Step 2. Straightforward algebra shows that

$$\begin{aligned} \omega - \text{BLP}^*(X) &= \omega - \mathbf{1}_N^\top \Sigma^{*-1} (X - b^*) \\ &= \omega - \mathbf{1}_N^\top \left(\text{diag}(\nu) - \frac{\nu \nu^\top}{1 + \mathbf{1}_N^\top \nu} \right) (\omega \mathbf{1}_N + \varepsilon) \\ &= \frac{\omega - \sum_{i=1}^N \nu_i \varepsilon_i}{1 + \mathbf{1}_N^\top \nu}, \end{aligned}$$

and that

$$\begin{aligned}
\Delta^\top \Sigma^{*-1}(X - b^*) &= [\nu_1(\Delta_1 - \bar{\Delta}) \cdots \nu_M(\Delta_M - \bar{\Delta}), 0 \cdots 0](\omega \mathbf{1}_N + \varepsilon) \\
&= \omega \sum_{i=1}^M \nu_i(\Delta_i - \bar{\Delta}) + \sum_{i=1}^M \nu_i(\Delta_i - \bar{\Delta})\varepsilon_i \\
&= \bar{\Delta}\omega + \sum_{i=1}^M \nu_i(\Delta_i - \bar{\Delta})\varepsilon_i.
\end{aligned}$$

Simplifying D accordingly yields

$$\begin{aligned}
D &= \mathbb{E}_{(b^*, \Sigma^*)}[(\omega - \text{BLP}^*)(\widehat{\text{BLP}} - \text{BLP}^*)] \\
&= \frac{-\bar{\Delta}}{(1 + \mathbf{1}_N^\top \nu)(1 + \Delta^\top \Sigma^{*-1} \Delta)} \mathbb{E}_{(b^*, \Sigma^*)} \left[\left(\omega - \sum_{i=1}^M \nu_i \varepsilon_i \right) \left(1 + \bar{\Delta}\omega + \sum_{i=1}^M \nu_i(\Delta_i - \bar{\Delta})\varepsilon_i \right) \right] \\
&= \frac{-\bar{\Delta}}{(1 + \mathbf{1}_N^\top \nu)(1 + \Delta^\top \Sigma^{*-1} \Delta)} \left(\bar{\Delta} \mathbb{E}[\omega^2] - \sum_{i=1}^M \nu_i^2 (\Delta_i - \bar{\Delta}) \mathbb{E}[\varepsilon_i^2] \right) \\
&= \frac{-\bar{\Delta}}{(1 + \mathbf{1}_N^\top \nu)(1 + \Delta^\top \Sigma^{*-1} \Delta)} \left(\bar{\Delta} - \sum_{i=1}^M \nu_i (\Delta_i - \bar{\Delta}) \right) \\
&= 0,
\end{aligned}$$

where the second equality uses (6), the third equality the fact that ω, ε_i s are independent with mean zero, the fourth equality the fact that $\mathbb{E}[\omega^2] = 1$ and $\nu_i = 1/\mathbb{E}[\varepsilon_i^2]$, and the last equality the identity that $\sum_{i=1}^M \nu_i \Delta_i = \bar{\Delta}(1 + \sum_{i=1}^M \nu_i)$. \square

Proofs of Theorems 2 and 4. Results follow from straightforward algebra that are omitted for brevity. \square

Online Appendix for
“Learning News Bias: Misspecifications
and Consequences”
by Lin Hu, Matthew Kovach, and Anqi Li

O.1 General covariance between signals and true state

This appendix examines a variant of the baseline model where the signal generated by source i in period t is $X_{i,t} = b_i + \alpha_i \omega_t + \varepsilon_{i,t}$. The coefficient α_i captures the covariance of the signal and true state and is set equal to one in the baseline model. By allowing this coefficient to take arbitrary values, we can entertain new possibilities such as signals that are negatively correlated with the true state,⁹ and differing α_i s across sources. The generality does not come for free, however, as we can no longer capture the precision of a news source by a single parameter.¹⁰ Now both α_i and ν_i affect the precision of source i , and the covariance matrix of news signals is $\text{diag}(v) + \alpha\alpha^\top$, where $\alpha := [\alpha_1 \ \cdots \ \alpha_N]^\top$.

The learning environment is as in the baseline model. Initially, the DM holds a prior belief about the true fundamental, which we denote by (b^*, Σ^*) . The prior correctly specifies the covariance matrix but misspecifies the biases of the sources in \mathcal{M} . Its support equals $\{(b, \Sigma) \in \mathbb{R}^N \times \mathbb{R}^{N \times N} : b_i = \tilde{b}_i \ \forall i \in \mathcal{M} \text{ and } \Sigma = \Sigma^*\}$. In every subsequent period, the DM observes signal realizations and updates his belief about the fundamental in a Bayesian manner. We examine outcomes of misspecified learning in this new setting.

For starters, let $\Delta_i = \tilde{b}_i - b_i^* \ \forall i \in \mathcal{M}$, and redefine

$$\bar{\Delta} := \frac{\sum_{i \in \mathcal{M}} \alpha_i^* \nu_i^* \Delta_i}{1 + \sum_{i \in \mathcal{M}} \alpha_i^{*2} \nu_i^*}$$

as the total misspecification across the sources in \mathcal{M} , weighted by their relative precision that now depends on α^* . The next proposition establishes the analog of Theorem 1 in the current setting.

⁹The case of positive correlation still seems the most plausible. Under normal circumstances, reporting of the true state should become more conservative (resp. liberal) as the latter leans more towards the right (resp. left).

¹⁰It will soon become clear that the lack of a unique identification has little impact on the analysis if the true covariance matrix is known to the DM from the outset. When the covariance matrix is also a subject of misspecified learning, Part (i) of Theorem 3 that characterizes the DM's long run beliefs continues to hold. Parts (ii) and (iii) of the theorem will be affected, because distortions of the DM's behavior and utility depend on the misperceived values of α_i s and ν_i s. These can be backed out from the DM's beliefs, but the analysis is purely algebraic and isn't pursued here.

Proposition O.1. *Let everything be as above. Then (i) $\Delta_i = \alpha_i^* \bar{\Delta} \forall i \notin \mathcal{M}$; (ii) $\widehat{\text{BLP}}(X) - \text{BLP}^*(X) = -\bar{\Delta}$; and (iii) $\widehat{\text{EU}} - \text{EU}^* = -\bar{\Delta}^2$.*

Compared to the baseline scenario, two key features remain unchanged. First of all, a single metric $\bar{\Delta}$ still captures distortions of the DM’s long run belief, behavior, and welfare. Secondly, $\bar{\Delta}$ is still independence from sources with initially unknown biases. What become more nuanced are the DM’s long run misperceptions about the latter’s biases. For one thing, the sign of the misperception now differs, depending on whether the correlation between the source and true state is positive or negative. For another, the magnitude of the misperception can now vary across sources.

In the leading example considered in the main text, the DM begins by underperceiving the conservative biases of Fox News and his own private source, while overperceiving the liberal bias of CNN, i.e., $\Delta_i = \tilde{b}_i - b_i^* < 0 \forall i \in \mathcal{M}$. If, in addition, $\alpha_i^* > 0 \forall i \in \mathcal{S}$, then $\bar{\Delta} < 0$, and by Proposition O.1 (ii) the DM’s long run behavior is more conservative than under the true model. To the extent that behavior reflects the distortion of his worldview, we conclude that the DM misperceives a liberal media bias overall.

As for individual sources that the DM is learning about, Part (i) of Proposition O.1 predicts that the DM ends up underestimating the bias of any conservative source and overestimating the bias of any liberal source. Raising α_i^* , $i \notin \mathcal{M}$, makes source i more informative and, hence, more susceptible to the distortionary effect of misspecified learning. In the long run, this worsens the DM’s misperception about source i compared to any other source $j \notin \mathcal{M} - \{i\}$.

O.2 Correlated errors

So far the errors in news signals are assumed to be independent. While this assumption greatly simplifies the analysis, in reality errors could be correlated through, e.g., common/conflicting editorial sentiments. In this appendix, we examine a variant of the baseline model where the signal generated by source i in period t is again $X_{i,t} = b_i + \omega_t + \varepsilon_{i,t}$. The vector $[\varepsilon_{1,t} \cdots \varepsilon_{N,t}]^\top$ of errors is independent of the true state ω_t and over time as before, but can now follow any N -dimensional Gaussian distribution with mean zero and covariance matrix Ω . The covariance matrix of news signals is $\Omega + \mathbf{1}_N \mathbf{1}_N^\top$.

Initially, the DM misspecifies biases of the sources in \mathcal{M} , and he may or may not know the true covariance matrix. The support of his prior is $\{(b, \Sigma) : b_i = \tilde{b}_i \forall i \in \mathcal{M} \text{ and } \Sigma = \Sigma^*\}$ in the first case (hereinafter, Case I), and $\{(b, \Sigma) : b_i = \tilde{b}_i \forall i \in \mathcal{M} \text{ and } \Sigma \text{ is positive definite}\}$ in the second case (hereinafter, Case II). Everything else is as in the main text.

The remainder of this appendix examines two special cases.

- $\mathcal{M} = \{1\}$; **arbitrary** Ω This case was first studied by Heidhues et al. (2019). To adapt these authors' findings to the current setting, redefine

$$\bar{\Delta} := \frac{\Delta_1}{\Sigma_{11}^*},$$

where $\Delta_1 = \tilde{b}_1 - b_1^*$. The next proposition establishes analogs of Theorems 1 and 3 in the current setting.

Proposition O.2. *Let everything be as above. In Case I, (i) $\Delta_i = \Sigma_{i1}^* \bar{\Delta} \forall i \in \mathcal{S}$; (ii) $\widehat{\text{BLP}}(X) - \text{BLP}^*(X) = -\bar{\Delta}$; and (iii) $\widehat{\text{EU}} - \text{EU}^* = -\bar{\Delta}^2$. In Case II, everything is as in Theorem 3.*

Compared to the baseline scenario, what remain unchanged are: (i) the use of a single metric $\bar{\Delta}$ to capture distortions of the DM's long run belief, behavior, and welfare; (ii) the independence of $\bar{\Delta}$ from sources with initially unknown biases; and (iii) the moderation effect of compound misspecified learning.

Part (i) of Proposition O.2. is new. To illustrate, suppose that the DM in our leading example begins by underperceiving the conservative bias of Fox News only, i.e., $\mathcal{M} = \{\text{Fox News}\}$ and $\Delta_1 < 0$. Proposition O.2 predicts, then, that any $\Delta_i, i \neq 1$, is negative if $\Sigma_{i1}^* = 1 + \text{Cov}(\varepsilon_1, \varepsilon_i) > 0$ and is positive otherwise. In the case where $\Sigma_{i1}^* > 0 \forall i \neq 1$, the baseline message remains valid: the DM ends up overestimating the bias of any liberal source and underestimating the bias of any conservative source. The message is reversed, however, if the sentiment of the (liberal) reporter at source i goes strongly against that of Fox News such that $\Sigma_{i1}^* < 0$.

Regardless of which situation we end up with, Part (ii) of Proposition O.2 predicts that the DM always behaves more conservatively than under the true model. To the extent that behavior reflects the distortion of his worldview, we conclude that the DM misperceives a liberal bias overall.

• **Correlation through common confounders** Suppose now that

$$\underbrace{\epsilon_{i,t}}_{\text{total error}} = \sum_{k=1}^K \beta_{i,k} \underbrace{\theta_{k,t}}_{\text{common confounder} \sim \mathcal{N}(0,1)} + \underbrace{\epsilon_{i,t}}_{\text{idiosyncratic error} \sim \mathcal{N}(0, v_i)},$$

where $\theta_{k,t}$, $k = 1, \dots, K$, are state-independent, confounding variables that are independent of each other and the source-specific, idiosyncratic error $\epsilon_{i,t}$. These confounders serve to produce correlations between total errors and can be interpreted as shocks to the editorial sentiment at the various sources.¹¹ Define $\beta_k := [\beta_{1,k} \cdots \beta_{N,k}]^\top$ for $k = 1, \dots, K$. The covariance of news signals is $\text{diag}(v) + \mathbf{1}_N \mathbf{1}_N^\top + \sum_{k=1}^K \beta_k \beta_k^\top$.

At first sight, it is nonobvious whether the above model variation is amenable to analysis. The Sherman-Morrison formula turns out again to be the key. A central step of our analysis is to obtain the inverse of the covariance matrix (recall the proof sketch presented in Section 3.4). In the current setting, the covariance matrix is a rank- $(K + 1)$ update of an invertible matrix, so its inverse can be obtained from applying the Sherman-Morrison formula iteratively to $\Sigma_0, \dots, \Sigma_{K+1}$, where $\Sigma_0 := \text{diag}(v)$, $\Sigma_1 := \text{diag}(v) + \mathbf{1}_N \mathbf{1}_N^\top$, and $\Sigma_{k+1} := \Sigma_k + \beta_k \beta_k^\top$, $k = 1, \dots, K$.

The algebra is, however, brutal in general. To illustrate the usefulness of the proposed approach, consider the case where $K = 1$, and drop the subscript “ k ” from the notations. Suppose the truth satisfies $\sum_{i=1}^N \beta_i^* \nu_i^* = \sum_{i=1}^M \beta_i^* \nu_i^* = 0$ — a condition that holds in a symmetric environment where opposing liberal and conservative sources differ in the signs of their β_i^* s, but their $\beta_i^* \nu_i^*$ s have the same magnitude.¹² The next proposition establishes the analog of Theorem 1 in this simple setting.

Proposition O.3. *Let everything be as above. Define*

$$\overline{\Delta}_\omega := \frac{\sum_{i=1}^M \nu_i^* \Delta_i}{1 + \sum_{i=1}^M \nu_i^*} \quad \text{and} \quad \overline{\Delta}_\beta := \frac{\sum_{i=1}^M \beta_i^* \nu_i^* \Delta_i}{1 + \sum_{i=1}^M \beta_i^{*2} \nu_i^*},$$

where $\Delta_i = \tilde{b}_i - b_i^* \forall i \in \mathcal{M}$. The following are true in Case I: (i) $\Delta_i = \overline{\Delta}_\omega + \beta_i^* \overline{\Delta}_\beta$

¹¹ Gentzkow et al. (2018) define the bias of a news source as its correlation with a single confounding variable (think of it as the beta coefficient), while setting the horizontal biases of all news sources equal to zero. Their DM misspecifies the beta coefficient of his personal source, and uses this to guide his learning about other parts of the covariance matrix (think of them as the alphas in Online Appendix O.1 and the betas). In contrast, our DM never misspecifies any part of the covariance matrix; the latter always lies in the support of his prior.

¹²The last assumption is only interesting when $M \geq 2$: when $\mathcal{M} = \{1\}$, this assumption holds only if $\beta_1^* = 0$. But then $\overline{\Delta}_\beta = 0$, and we are essentially back to the baseline model.

$\forall i \notin \mathcal{M}$; (ii) $\widehat{\text{BLP}}(X) - \text{BLP}^*(X) = -\bar{\Delta}_\omega$; and (iii) $\widehat{\text{EU}} - \text{EU}^* = -\bar{\Delta}_\omega^2$.

Proposition [O.3](#) combines insights of the baseline model with that of the extension explored in Online Appendix [O.1](#). The DM’s misperception about the bias of any source $i \notin \mathcal{M}$ equals $\bar{\Delta}_\omega$ in the first case and $\beta_i \bar{\Delta}_\beta$ in the second case. The main difference lies in the interpretation: now $\bar{\Delta}_\beta$ represents the total misspecification across the sources in \mathcal{M} , weighted by their editorial sentiments. If a source $i \notin \mathcal{M}$ shares a majority of these sentiments, i.e., $\text{sgn}(\beta_i^*) = \text{sgn}(\beta_j^*)$ for many $j \in \mathcal{M}$, then $\text{sgn}(\beta_i^* \bar{\Delta}_\beta) = \text{sgn}(\Delta_j)$ in case the latter is constant across $j \in \mathcal{M}$. In our leading example, this means that the DM ends up underestimating the bias of source i if it is conservative and overestimating its bias if it is liberal. However, the message is reversed when strong opposing sentiments prevail between source i and those in \mathcal{M} .

Meanwhile, distortions of the DM’s long run behavior and expected utility depend only on $\bar{\Delta}_\omega$ but not on $\bar{\Delta}_\beta$. This result exploits the symmetry of the environment; its proof is relegated to the section after the next.

O.3 Multidimensional positions

This appendix extends the baseline model to encompass multidimensional positions. In each period $t = 1, 2, \dots$, Nature draws the true state ω_t from a K -dimensional Gaussian distribution with mean $\mathbf{0}_K$ and covariance matrix V . Each dimension represents a distinct issue such as climate change, gun policy, gay marriage, illegal immigration, etc. The DM consults news sources before taking a K -dimensional action $a_t \in \mathbb{R}^K$ and earns a utility $-\|a_t - \omega_t\|_2$. The signal generated by source i in period t is $X_{i,t} = b_i + \omega_t + \varepsilon_{i,t}$, where $b_i \in \mathbb{R}^K$ is a K -dimensional bias, and $\varepsilon_{i,t}$ is a source-specific, idiosyncratic error with mean $\mathbf{0}_K$ and covariance matrix Ω_i .

Initially, the DM knows the true covariance matrix Σ^* of news signals but misspecifies the biases of some sources. The support of his prior is $\{(b, \Sigma) \in \mathbb{R}^{NK} \times \mathbb{R}^{NK \times NK} : b_i = \tilde{b}_i \forall i \in \mathcal{M} \text{ and } \Sigma = \Sigma^*\}$. Everything else is as in the main text, except that all variables of our interest are now K -dimensional vectors. These include the DM’s long run misperception about the bias of source i : $\Delta_i := \hat{b}_i - b_i^*$, as well as his long run behaviors under the misspecified model and true model: $\widehat{\text{BLP}}(X)$ and $\text{BLP}^*(X)$.

The next proposition establishes the analog of Theorem 1 in the current, multidimensional, setting.

Proposition O.4. *Let everything be as above. Define*

$$\bar{\Delta} := \left[\mathbb{I}_{K \times K} + (V^{*-1} + \sum_{i=1}^M \Omega_i^{*-1})^{-1} \sum_{i=M+1}^N \Omega_i^{*-1} \right] \left(V^{-1} + \sum_{i=1}^N \Omega_i^{*-1} \right)^{-1} \sum_{i=1}^M \Omega_i^{*-1} \Delta_i,$$

where $\Delta_i = \tilde{b}_i - b_i^* \forall i \in \mathcal{M}$. Then (i) $\Delta_i = \bar{\Delta} \forall i \notin \mathcal{M}$; (ii) $\widehat{\text{BLP}}(X) - \text{BLP}^*(X) = -V^* \sum_{i=1}^M \Omega_i^{*-1} (\Delta_i - \bar{\Delta})$.

As in the baseline scenario, we can use a simple metric $\bar{\Delta}$ to capture the distortionary effect of misspecified learning. But unlike its baseline counterpart, $\bar{\Delta}$ can now depend on the characteristics of sources with initially unknown biases, i.e., those in $\mathcal{S} - \mathcal{M}$. When the various dimensions of the issue position are independent so that V and Ω_i s become diagonal matrices, Proposition O.4 reduces to Theorem 1. In general, this equivalence breaks down. When that happens, the comparative statics of the distortionary effect become more nuanced and await further investigation.

O.4 Proofs

Throughout this appendix, we drop the superscript “*” from the notations of true fundamentals whenever convenient. Since the proofs closely parallel that of Theorems 1 and 3, we omit most algebra and outline only key steps of the derivation.

Proof of Proposition O.1. The first-order conditions associated KL minimization stipulate that

$$\begin{bmatrix} \Delta_{M+1} \\ \vdots \\ \Delta_N \end{bmatrix} = -A^{-1}B \begin{bmatrix} \Delta_1 \\ \vdots \\ \Delta_M \end{bmatrix},$$

where A and B were defined in the proof of Theorem 1. Since $\Sigma^* = \text{diag}(\nu) + \alpha\alpha^\top$ is a rank-one update of an invertible matrix,

$$\Sigma^{*-1} = \text{diag}(\nu) - \frac{1}{1 + \sum_{i=1}^N \alpha_i^2 \nu_i} \begin{bmatrix} \alpha_1 \nu_1 \\ \vdots \\ \alpha_N \nu_N \end{bmatrix} \begin{bmatrix} \alpha_1 \nu_1 & \cdots & \alpha_N \nu_N \end{bmatrix}$$

by the Sherman-Morrison formula. Substituting this result into the definitions of A and B yields

$$A = \text{diag}(\nu_{M+1} \cdots \nu_M) - \frac{1}{1 + \sum_{i=1}^N \alpha_i^2 \nu_i} \begin{bmatrix} \alpha_{M+1} \nu_{M+1} \\ \vdots \\ \alpha_N \nu_N \end{bmatrix} \begin{bmatrix} \alpha_{M+1} \nu_{M+1} & \cdots & \alpha_N \nu_N \end{bmatrix}$$

and

$$B = -\frac{1}{1 + \sum_{i=1}^N \alpha_i^2 \nu_i} \begin{bmatrix} \alpha_{M+1} \nu_{M+1} \\ \vdots \\ \alpha_N \nu_N \end{bmatrix} \begin{bmatrix} \alpha_1 \nu_1 & \cdots & \alpha_M \nu_M \end{bmatrix}.$$

Since A is itself a rank-one update of an invertible matrix, applying the Sherman-Morrison formula to it shows that

$$A^{-1} = \text{diag}(\nu_{M+1}, \dots, \nu_N) + \frac{1}{1 + \sum_{i=1}^M \alpha_i^2 \nu_i} \begin{bmatrix} \alpha_{M+1} \\ \vdots \\ \alpha_N \end{bmatrix} \begin{bmatrix} \alpha_{M+1} & \cdots & \alpha_N \end{bmatrix}$$

and, in turn, that

$$A^{-1}B = -\frac{1}{1 + \sum_{i=1}^M \alpha_i^2 \nu_i} \begin{bmatrix} \alpha_{M+1} \\ \vdots \\ \alpha_N \end{bmatrix} \begin{bmatrix} \alpha_1 \nu_1 & \cdots & \alpha_M \nu_M \end{bmatrix}.$$

Simplifying the first-order conditions accordingly yields

$$\begin{bmatrix} \Delta_{M+1} \\ \vdots \\ \Delta_N \end{bmatrix} = \bar{\Delta} \begin{bmatrix} \alpha_{M+1} \\ \vdots \\ \alpha_N \end{bmatrix} \text{ and } \Sigma^{*-1} \Delta = \begin{bmatrix} \nu_1(\Delta_1 - \alpha_1 \bar{\Delta}) \\ \vdots \\ \nu_M(\Delta_M - \alpha_M \bar{\Delta}) \\ \mathbf{0}_{N-M} \end{bmatrix}.$$

The best linear predictors of the state under the true model and misspecified model are $\text{BLP}^*(X) = \alpha^\top \Sigma^{*-1}(X - b^*)$ and $\widehat{\text{BLP}}(X) = \alpha^\top \Sigma^{*-1}(X - \hat{b})$, respectively.

Their difference equals

$$\widehat{\text{BLP}}(X) - \text{BLP}^*(X) = -\alpha^\top \Sigma^{*-1} \Delta = -\overline{\Delta},$$

irrespective of X , from which $\widehat{\text{E}}\text{U} - \text{E}U^* = -\overline{\Delta}^2$ follows. \square

Proof of Proposition O.2. Case I: setting the term M in Theorem 1 of Heidhues et al. (2019) equal to the identity matrix yields $\Delta_i = \Sigma_{i1}^* \overline{\Delta} \forall i$, that is Part (i). Multiplying Δ by Σ^{*-1} yields

$$\Sigma^{*-1} \Delta = \Sigma^{*-1} \begin{bmatrix} \Sigma_{11}^* & \Sigma_{21}^* & \cdots & \Sigma_{N1}^* \end{bmatrix}^\top \overline{\Delta} = \begin{bmatrix} 1 & 0 & \cdots & 0 \end{bmatrix}^\top \overline{\Delta}, \quad (7)$$

where the second equality uses the fact that $\Sigma^{*-1} \Sigma^*$ equals the $N \times N$ identity matrix. Thus

$$\widehat{\text{BLP}}(X) - \text{BLP}^*(X) = \mathbf{1}_N^\top \Sigma^{*-1} (X - \hat{b}) - \mathbf{1}_N^\top \Sigma^{*-1} (X - b^*) = -\mathbf{1}_N^\top \Sigma^{*-1} \Delta = -\overline{\Delta},$$

that is Part (ii). Part (iii) that $\widehat{\text{E}}\text{U} - \text{E}U^* = -\overline{\Delta}^2$ is an immediate consequence of Part (ii).

Case II: setting the term M in Theorem 1 of Heidhues et al. (2019) equal to the identity matrix shows that Δ is the same as in Case I, whereas $\widehat{\Sigma} = \Sigma^* + \Delta \Delta^\top$. Further duplicating the proof of Theorem 3(ii) line by line shows that $\mathbb{E}_{(b^*, \Sigma^*)}[\widehat{\text{BLP}}(X) - \text{BLP}^*(X)] = -\overline{\Delta} (1 + \Delta^\top \Sigma^{*-1} \Delta)^{-1}$. It remains to show that $\widehat{\text{E}}\text{U} - \text{E}U^* = -\overline{\Delta}^2 (1 + \Delta^\top \Sigma^{*-1} \Delta)^{-1}$, as in Theorem 3(iii). The reason is that

$$\begin{aligned} \widehat{\text{E}}\text{U} - \text{E}U^* &= -\mathbb{E}_{(b^*, \Sigma^*)}[(\omega - \widehat{\text{BLP}})^2] + \mathbb{E}_{(b^*, \Sigma^*)}[(\omega - \text{BLP}^*)^2] \\ &= -\underbrace{\mathbb{E}_{(b^*, \Sigma^*)}[(\widehat{\text{BLP}} - \text{BLP}^*)^2]}_C + 2 \underbrace{\mathbb{E}_{(b^*, \Sigma^*)}[(\omega - \text{BLP}^*)(\widehat{\text{BLP}} - \text{BLP}^*)]}_D, \end{aligned}$$

where $C = \overline{\Delta}^2 (1 + \Delta^\top \Sigma^{*-1} \Delta)^{-1}$ and $D = 0$. To verify the last statements, note that

$$\begin{aligned} \widehat{\text{BLP}}(X) - \text{BLP}^*(X) &= -\frac{\overline{\Delta}}{1 + \Delta^\top \Sigma^{*-1} \Delta} (1 + \Delta^\top \Sigma^{*-1} (X - b^*)) \\ &= -\frac{\overline{\Delta}}{1 + \Delta^\top \Sigma^{*-1} \Delta} (1 + \overline{\Delta}(\omega + \varepsilon_1)), \end{aligned}$$

where the first equality was established in the proof of Theorem 3, and the second

equality uses (7). Thus

$$C = \left(\frac{\bar{\Delta}}{1 + \Delta^\top \Sigma^{*-1} \Delta} \right)^2 (1 + \Sigma_{11}^* \bar{\Delta}^2) = \frac{\bar{\Delta}^2}{1 + \Delta^\top \Sigma^{*-1} \Delta}$$

as desired, where the first equality uses the fact that ω and ε_1 have zero mean and are independent of each other, and the second equality uses again (7) and the fact that $\bar{\Delta} := \Delta_1 / \Sigma_{11}^*$. Meanwhile,

$$D \propto \mathbb{E}_{(b^*, \Sigma^*)} [(\omega - \text{BLP}^*)(1 + \bar{\Delta}(\omega + \varepsilon_1))] \propto \mathbb{E}_{(b^*, \Sigma^*)} [(\omega - \mathbf{1}_N^\top \Sigma^{*-1} (\omega \mathbf{1}_N + \varepsilon))(\omega + \varepsilon_1)],$$

where the second step uses the definition of BLP^* and the fact that $\mathbb{E}_{(b^*, \Sigma^*)} [\omega - \text{BLP}^*] = 0$. Expanding the last term yields

$$\begin{aligned} & \mathbb{E}_{(b^*, \Sigma^*)} [(\omega - \mathbf{1}_N^\top \Sigma^{*-1} (\omega \mathbf{1}_N + \varepsilon))(\omega + \varepsilon_1)] \\ &= \mathbb{E}_{(b^*, \Sigma^*)} [\omega^2] + \mathbb{E}_{(b^*, \Sigma^*)} [\omega \varepsilon_1] - \mathbb{E}_{(b^*, \Sigma^*)} [\mathbf{1}_N^\top \Sigma^{*-1} \begin{bmatrix} \omega + \varepsilon_1 & \cdots & \omega + \varepsilon_N \end{bmatrix}^\top (\omega + \varepsilon_1)] \\ &= 1 - 0 - \mathbf{1}_N^\top \Sigma^{*-1} \begin{bmatrix} \Sigma_{11}^* & \cdots & \Sigma_{N1}^* \end{bmatrix}^\top = 0, \end{aligned}$$

where the second equality uses the fact that ω has unit variance and is independent of ε_1 , and the third equality the fact that $\Sigma^{*-1} \Sigma^*$ equals the $N \times N$ identity matrix. \square

Proof of Proposition O.3. Applying the Sherman-Morrison formula twice to $\Sigma^* = \text{diag}(\nu) + \mathbf{1}_N \mathbf{1}_N^\top + \beta \beta^\top$ shows that

$$\Sigma^{*-1} = \text{diag}(\nu) - \frac{1}{1 + \sum_{i=1}^N \nu_i} \nu \nu^\top - \frac{1}{1 + \sum_{i=1}^N \beta_i^2 \nu_i - (\sum_{i=1}^N \beta_i \nu_i)^2 / (1 + \sum_{i=1}^N \nu_i)} Z Z^\top,$$

where

$$Z = \begin{bmatrix} \beta_1 \nu_1 \\ \vdots \\ \beta_N \nu_N \end{bmatrix} - \frac{\sum_{i=1}^N \beta_i \nu_i}{1 + \sum_{i=1}^N \nu_i} \begin{bmatrix} \nu_1 \\ \vdots \\ \nu_N \end{bmatrix}.$$

Simplifying the above expressions using $\sum_{i=1}^N \beta_i \nu_i = 0$ yields

$$\Sigma^{*-1} = \text{diag}(\nu) - \frac{1}{1 + \sum_{i=1}^N \nu_i} \nu \nu^\top - \frac{1}{1 + \sum_{i=1}^N \beta_i^2 \nu_i} \begin{bmatrix} \beta_1 \nu_1 \\ \vdots \\ \beta_N \nu_N \end{bmatrix} \begin{bmatrix} \beta_1 \nu_1 & \cdots & \beta_N \nu_N \end{bmatrix}$$

and, in turn,

$$A = \text{diag}(\nu_{M+1} \cdots \nu_N) - \frac{1}{1 + \sum_{i=1}^N \nu_i} \begin{bmatrix} \nu_{M+1} \\ \vdots \\ \nu_N \end{bmatrix} \begin{bmatrix} \nu_{M+1} & \cdots & \nu_N \end{bmatrix} \\ - \frac{1}{1 + \sum_{i=1}^N \beta_i^2 \nu_i} \begin{bmatrix} \beta_{M+1} \nu_{M+1} \\ \vdots \\ \beta_N \nu_N \end{bmatrix} \begin{bmatrix} \beta_{M+1} \nu_{M+1} & \cdots & \beta_N \nu_N \end{bmatrix},$$

and

$$-B \begin{bmatrix} \Delta_1 \\ \vdots \\ \Delta_M \end{bmatrix} = \frac{\sum_{i=1}^M \nu_i \Delta_i}{1 + \sum_{i=1}^N \nu_i} \begin{bmatrix} \nu_{M+1} \\ \vdots \\ \nu_N \end{bmatrix} + \frac{\sum_{i=1}^M \beta_i \nu_i \Delta_i}{1 + \sum_{i=1}^N \beta_i^2 \nu_i} \begin{bmatrix} \beta_{M+1} \nu_{M+1} \\ \vdots \\ \beta_N \nu_N \end{bmatrix}.$$

Since A is itself a rank-two update of an invertible matrix, applying the Sherman-Morrison formula to it twice shows that

$$A^{-1} = \text{diag}(\nu_{M+1} \cdots \nu_N) + \frac{1}{1 + \sum_{i=1}^M \nu_i} \mathbf{1}_{N-M} \mathbf{1}_{N-M}^\top \\ + \frac{1}{1 + \sum_{i=1}^M \beta_i^2 \nu_i} \begin{bmatrix} \beta_{M+1} \\ \vdots \\ \beta_N \end{bmatrix} \begin{bmatrix} \beta_{M+1} \cdots \beta_N \end{bmatrix}$$

and, in turn, that

$$-A^{-1}B \begin{bmatrix} \Delta_1 \\ \vdots \\ \Delta_M \end{bmatrix} = \frac{\sum_{i=1}^M \nu_i \Delta_i}{1 + \sum_{i=1}^N \nu_i} \mathbf{1}_{N-M} + \frac{\sum_{i=1}^M \beta_i \nu_i \Delta_i}{1 + \sum_{i=1}^N \beta_i^2 \nu_i} \begin{bmatrix} \beta_{M+1} \\ \vdots \\ \beta_N \end{bmatrix} \\ + \frac{1}{1 + \sum_{i=1}^M \nu_i} \left(\frac{(\sum_{i=1}^M \nu_i \Delta_i)(\sum_{i=M+1}^N \nu_i)}{1 + \sum_{i=1}^N \nu_i} + \frac{(\sum_{i=1}^M \beta_i \nu_i \Delta_i)(\sum_{i=M+1}^N \beta_i \nu_i)}{1 + \sum_{i=1}^N \beta_i^2 \nu_i} \right) \mathbf{1}_{N-M} \\ + \frac{1}{1 + \sum_{i=1}^M \beta_i^2 \nu_i} \left(\frac{(\sum_{i=1}^M \nu_i \Delta_i)(\sum_{i=M+1}^N \beta_i \nu_i)}{1 + \sum_{i=1}^N \nu_i} + \frac{(\sum_{i=1}^M \beta_i \nu_i \Delta_i)(\sum_{i=M+1}^N \beta_i^2 \nu_i)}{1 + \sum_{i=1}^N \beta_i^2 \nu_i} \right) \begin{bmatrix} \beta_{M+1} \\ \vdots \\ \beta_N \end{bmatrix}.$$

Simplifying the last two line using $\sum_{i=M+1}^N \beta_i \nu_i = \sum_{i=1}^N \beta_i \nu_i - \sum_{i=1}^M \beta_i \nu_i = 0$ yields

$$\begin{bmatrix} \Delta_{M+1} \\ \vdots \\ \Delta_N \end{bmatrix} = -A^{-1}B \begin{bmatrix} \Delta_1 \\ \vdots \\ \Delta_M \end{bmatrix} = \bar{\Delta}_\omega \mathbf{1}_{N-M} + \bar{\Delta}_\beta \begin{bmatrix} \beta_{M+1} \\ \vdots \\ \beta_N \end{bmatrix},$$

as desired.

The difference between the best linear predictors of the state under the misspecified and true models is $-\mathbf{1}_N^\top \Sigma^{*-1} \Delta$. Straightforward algebra shows that

$$\begin{aligned} \mathbf{1}_N^\top \Sigma^{*-1} \Delta &= \mathbf{1}_N^\top \begin{bmatrix} \nu_1(\Delta_1 - \bar{\Delta}_\omega - \beta_1 \bar{\Delta}_\beta) \\ \vdots \\ \nu_M(\Delta_M - \bar{\Delta}_\omega - \beta_M \bar{\Delta}_\beta) \\ \hline \mathbf{0}_{N-M} \end{bmatrix} \\ &= \sum_{i=1}^M \nu_i (\Delta_i - \bar{\Delta}_\omega - \beta_i \bar{\Delta}_\beta) \\ &= \sum_{i=1}^M \nu_i \Delta_i - \bar{\Delta}_\omega \sum_{i=1}^M \nu_i - \bar{\Delta}_\beta \sum_{i=1}^M \beta_i \nu_i \\ &= \bar{\Delta}_\omega + 0, \end{aligned}$$

where the first equality uses results concerning Σ^{*-1} and Δ , and the last line uses $\sum_{i=1}^M \nu_i \Delta_i = (1 + \sum_{i=1}^M \nu_i) \bar{\Delta}_\omega$ and $\sum_{i=1}^M \beta_i \nu_i = 0$. \square

Proof of Proposition O.4. With multidimensional positions, the true covariance matrix of news signals becomes

$$\Sigma^* = \underbrace{\begin{bmatrix} \Omega_1 & & \\ & \ddots & \\ & & \Omega_N \end{bmatrix}}_{\text{invertible}} + \underbrace{\begin{bmatrix} \mathbb{I}_{K \times K} \\ \vdots \\ \mathbb{I}_{K \times K} \end{bmatrix}}_{NK \text{ rows}} \overbrace{V \begin{bmatrix} \mathbb{I}_{K \times K} & \cdots & \mathbb{I}_{K \times K} \end{bmatrix}}^{\text{rank-}K \text{ update}} \underbrace{\quad}_{NK \text{ columns}}.$$

Since Σ^* is a rank- K update of an invertible matrix, applying the Woodbury formula (Woodbury, 1950) to it shows that

$$\Sigma^{*-1} = \begin{bmatrix} \Omega_1^{-1} & & \\ & \ddots & \\ & & \Omega_N^{-1} \end{bmatrix} - \begin{bmatrix} \Omega_1^{-1} \\ \vdots \\ \Omega_N^{-1} \end{bmatrix} \left(V^{-1} + \sum_{i=1}^N \Omega_i^{-1} \right)^{-1} \begin{bmatrix} \Omega_1^{-1} & \cdots & \Omega_N^{-1} \end{bmatrix},$$

and, in turn, that

$$A = \begin{bmatrix} \Omega_{M+1}^{-1} & & \\ & \ddots & \\ & & \Omega_N^{-1} \end{bmatrix} - \begin{bmatrix} \Omega_{M+1}^{-1} \\ \vdots \\ \Omega_N^{-1} \end{bmatrix} \left(V^{-1} + \sum_{i=1}^N \Omega_i^{-1} \right)^{-1} \begin{bmatrix} \Omega_{M+1}^{-1} & \cdots & \Omega_N^{-1} \end{bmatrix}$$

and

$$B = - \begin{bmatrix} \Omega_{M+1}^{-1} \\ \vdots \\ \Omega_N^{-1} \end{bmatrix} \left(V^{-1} + \sum_{i=1}^N \Omega_i^{-1} \right)^{-1} \begin{bmatrix} \Omega_1^{-1} & \cdots & \Omega_M^{-1} \end{bmatrix}.$$

Since A is itself a rank- K update of an invertible matrix, applying the Woodbury formula to it and doing lots of algebra yields

$$A^{-1} = \begin{bmatrix} \Omega_{M+1} & & \\ & \ddots & \\ & & \Omega_N \end{bmatrix} + \underbrace{\begin{bmatrix} \mathbb{I}_{K \times K} \\ \vdots \\ \mathbb{I}_{K \times K} \end{bmatrix}}_{(N-M)K \text{ rows}} \left(V^{-1} + \sum_{i=1}^M \Omega_i^{-1} \right)^{-1} \underbrace{\begin{bmatrix} \mathbb{I}_{K \times K} & \cdots & \mathbb{I}_{K \times K} \end{bmatrix}}_{(N-M)K \text{ columns}}.$$

Taken together, we obtain that

$$\begin{bmatrix} \Delta_{M+1} \\ \vdots \\ \Delta_N \end{bmatrix} = -A^{-1}B \begin{bmatrix} \Delta_1 \\ \vdots \\ \Delta_M \end{bmatrix} = \begin{bmatrix} \mathbb{I}_{K \times K} \\ \vdots \\ \mathbb{I}_{K \times K} \end{bmatrix} \bar{\Delta},$$

where

$$\bar{\Delta} = \left[\mathbb{I}_{K \times K} + \left(V^{-1} + \sum_{i=1}^M \Omega_i^{-1} \right)^{-1} \sum_{i=M+1}^N \Omega_i^{-1} \right] \left(V^{-1} + \sum_{i=1}^N \Omega_i^{-1} \right)^{-1} \sum_{i=1}^M \Omega_i^{-1} \Delta_i,$$

and that

$$\Sigma^{*-1}\Delta = \begin{bmatrix} \Omega_1^{-1}(\Delta_1 - \bar{\Delta}) \\ \vdots \\ \Omega_M^{-1}(\Delta_M - \bar{\Delta}) \\ \mathbf{0}_{(N-M)K \times K} \end{bmatrix}.$$

The best linear predictors of the state under the true model and misspecified model are

$$\text{BLP}^*(X) = V \begin{bmatrix} \mathbb{I}_{K \times K} & \cdots & \mathbb{I}_{K \times K} \end{bmatrix} \Sigma^{*-1}(X - b^*)$$

and

$$\widehat{\text{BLP}}(X) = V \begin{bmatrix} \mathbb{I}_{K \times K} & \cdots & \mathbb{I}_{K \times K} \end{bmatrix} \Sigma^{*-1}(X - \hat{b}),$$

respectively. Simplifying their difference using the last result yields

$$\widehat{\text{BLP}}(X) - \text{BLP}^*(X) = -V\Sigma^{*-1}\Delta = -V \sum_{i=1}^M \Omega_i^{-1}(\Delta_i - \bar{\Delta}),$$

as desired. □

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