Econometrica, Vol. 63, No. 3 (May, 1995), 545–565

PERFECT EQUILIBRIA IN A NEGOTIATION MODEL

BY LUTZ-ALEXANDER BUSCH AND QUAN WEN[1]

Rubinstein's alternating-offers bargaining model is enriched by assuming that players' payoffs in disagreement periods are determined by a normal form game. It is shown that such a model can have multiple perfect equilibria, including inefficient ones, provided that players are sufficiently patient. Delay is possible even though there is perfect information and the players are fully rational. The length of delay depends only on the payoff structure of the disagreement game and not on the discount factor. Not all feasible and individually rational payoffs of the disagreement game can be supported as average disagreement payoffs. Indeed, some negotiation games have a unique perfect equilibrium with immediate agreement.

KEYWORDS: Bargaining, negotiation, repeated games, delay.

1. INTRODUCTION

STÅHL (1972) AND RUBINSTEIN (1982) were among the first to investigate formally the dynamic and strategic aspects of bargaining situations. As in the earlier cooperative treatment of bargaining by Nash (1950), their models abstract from any possible relationship the bargaining partners may have apart from the bargain over a jointly owned surplus. The focus is solely on the bargaining process itself, modeled as a sequence of offers and responses. Payoffs are determined only by the agreement which is reached, not by the history of play. In contrast, the repeated game literature has focused on a repeated strategic relationship with periodic payoffs (e.g., Benoit and Krishna (1985) or Fudenberg and Maskin (1986)). In a repeated game, the players' actions in any one period directly determine their payoffs in that period, and therefore the payoffs from the game depend on the entire history of play.

The present paper provides a model which combines the structure of a repeated game with that of an alternating-offers bargaining game. The model describes a situation in which two players bargain over the allocation of a constant stream of periodic surpluses of known and fixed value via a (possibly infinite) sequence of offers and counter-offers. As in bargaining, an accepted offer constitutes a binding agreement on an allocation. In contrast to most bargaining models, however, after any rejection of an offer the players play a one-shot game to determine their payoffs for the "disagreement period." This "disagreement game" summarizes any strategic relationship between the players apart from the bargaining process itself. As in repeated games, contracting on the moves cannot be done in the disagreement games. Although this model can equally well be viewed as a repeated game with a valuable exit option, and thus

[1] We wish to thank Ignatius Horstmann and Philip Reny for their comments, patience, and encouragement. Valuable comments were also received from Mike Peters, Chantalle LaCasse, Arthur Robson, Abhijit Sengupta, Alan Slivinski, and various seminar audiences. Extensive comments by a co-editor and three anonymous referees greatly improved the paper.

an endogenous time horizon, it will be presented in the bargaining interpretation outlined above.

The economic situation we have in mind is that of two parties involved in a repeated relationship which can be transformed by mutual agreement. This situation arises, for example, during the horizontal or vertical integration of two firms, or during contract renewals. The traditional bargaining and repeated game models do not capture this type of situation well, since bargaining models ignore the strategic disagreement payoffs, while repeated game models do not take the possibility of a binding agreement into account.

The question of how the disagreement payoff of a bargaining game is determined has been modeled before. Nash (1953) analyzed the question of endogenous disagreement payoffs within the cooperative framework. He extended his original analysis of the bargaining problem (Nash (1950)) by letting players' disagreement payoffs be determined by a normal form game, played if disagreement occurs. More recently, papers by Fernandez and Glazer (1991) and Haller and Holden (1990) examine a noncooperative model of firm-union wage bargaining in which the union's decision to reject the firm's wage offer—delay—is distinct from the union's decision to forego the old wage for one period—strike. They conclude that this game has multiple subgame perfect equilibria, some of which feature delay and strike before an agreement is reached.

Our results reinforce those of Fernandez and Glazer and of Haller and Holden, and provide further insight into the interplay of the forces active in a bargaining process with those deriving from the repeated game aspects of the model. Strategic interaction outside the bargaining process per se will, in general, generate multiple equilibria. Some of these equilibria are inefficient, even though there is perfect information.[2] Our model demonstrates that such inefficiency may arise even if the disagreement game has uniformly small payoffs relative to agreement. On the other hand, our results show that the set of disagreement game payoffs which can be supported in a subgame perfect equilibrium are markedly restricted vis-à-vis the folk theorem with the disagreement game as stage game. Indeed, for some nontrivial disagreement games only the Nash equilibrium payoff vector of the disagreement game can be supported in disagreement periods. This in turn implies that immediate agreement is the unique equilibrium outcome in the bargaining process, and thus for the negotiation game. This result can occur even though some payoffs in the disagreement game are on the bargaining frontier. Finally, we show that only the structure of the disagreement game, and not the gains from agreement, determines multiplicity and the range of payoffs that can be supported.

[2] Inefficiency in bargaining has previously been explained as a signaling device under imperfect information. See, for example, Admati and Perry (1987), Gul and Sonnenschein (1988), Chaterjee and Samuelson (1987), Ausubel and Deneckere (1989), or the survey by Wilson (1987).

What is the intuition behind these results? Multiple equilibria in standard repeated games are supported by history-dependent strategies. These allow for a player to be punished for deviations from proposed equilibrium actions, thereby deterring deviations. However, history-dependent strategies do not have the same power in Rubinstein bargaining, as is clearly demonstrated by the proof of Rubinstein's (1982) result by Shaked and Sutton (1984). The reason is that a player will always accept any offer which yields at least as much as rejecting it, since acceptance cannot be punished. The player making the offer therefore can capture all of the surplus from immediate agreement. Along the path which leads to a player's worst payoff, this player's first offer is thus accepted and ends the game. Together with the stationary structure of the bargaining game, and the fact that a player who deviates from his own punishment can only be punished by restarting the same punishment, a unique equilibrium results. As we show, this argument continues to hold for any given sequence of disagreement payoffs in a bargaining game. In a negotiation game, when disagreement payoffs are endogenous, this force towards uniqueness is counteracted by the presence of the repeated game element provided by the disagreement game. The disagreement game allows for a player's payoff in disagreement periods to vary in response to past actions. Only if it is possible to support multiple disagreement payoffs after rejections, can multiple equilibrium agreements in the bargaining process also be supported.

The question arises why the presence of the disagreement game does not lead to a result akin to the folk theorem? In particular, why cannot any feasible and individually rational payoff of the disagreement game be supported as equilibrium disagreement payoff? This restriction derives from the bargaining process. Since for extreme payoffs the bargaining process will end the game immediately, the disagreement game is at most played once, and any punishments and rewards for behavior in the disagreement game have to be applied in the immediately following offer. The bargaining process thus rules out a distant reward phase, as in Benoit and Krishna (1985). On the other hand, it is possible to make offers contingent on the outcome of the previous disagreement game, so that some payoffs which are not Nash can be supported. Since the bargaining frontier is negatively sloped, there is a one-to-one tradeoff between players' payoffs, however. Due to the fact that a player's punishment becomes more severe the higher his opponent's disagreement payoff from rejecting the punished player's offers, the important magnitude is the opponent's highest disagreement payoff after taking any compensation which may be necessary into account. This payoff is determined only by the structure of the disagreement game, not the position of the bargaining frontier.

The outline of the paper is as follows. Section 2 presents the model formally, while Section 3 presents two simple examples of negotiation games with unique and multiple equilibria. Section 4 contains the formal analysis and results. Section 5 concludes. An appendix provides proofs and formal descriptions of equilibrium strategies.

2. THE NEGOTIATION MODEL

Two players, 1 and 2, are bargaining over the allocation of a periodic surplus. Each player in turn proposes a partition, and his opponent may accept or reject this offer. The negotiation game ends if an agreement is reached with an accepted offer. The players then forever share the stream of surpluses according to the agreement. After a rejection, and before the rejecting player makes a counter-offer in the next period, the players play a one-shot game, called the disagreement game, to determine their payoffs in the current period. This process then repeats, possibly indefinitely.

Formally, let the value of the surplus equal one in every period. As in Rubinstein (1982), a partition of the surplus is identified by a number in the unit interval, $b \in [0,1]$, such that b is player 1's share and $(1-b)$ is player 2's share of the surplus. Acceptance and rejection of an offer are denoted by "Y" and "N," respectively. The disagreement game is modeled as a two-player game in normal form, $G = \{A_1, A_2, u_1(\cdot), u_2(\cdot)\}$, where A_i is the set of player i's actions in the disagreement game, and $u_i(\cdot)$: $A = A_1 \times A_2 \to R$ is player i's payoff function. The set A is also interpreted as the set of outcomes of the disagreement game. In this paper, we assume:

(A-1) *A_1 and A_2 are nonempty and compact*;

(A-2) *$u_1(\cdot)$ and $u_2(\cdot)$ are continuous, and $u(A)$ is convex*;

(A-3) *the disagreement game has at least one Nash equilibrium*;

(A-4) *each player's minimax payoff is zero*;

(A-5) *$u_1(a) + u_2(a) \leqslant 1$ for all outcomes $a \in A$ of the disagreement game.*

The disagreement game G will satisfy assumptions (A-1), (A-2), and (A-3), for example, if we assume that A_1 and A_2 consist of probability mixtures over finite pure action sets, that players can choose their actions contingent on the outcomes of a public randomizing device, and that the payoff functions are the corresponding expected payoff functions. Assumption (A-2) implies that any feasible payoff vector of the disagreement game, in terms of expected value, can result from a one-shot play. Assumption (A-5) formalizes the notion that agreement is in both players' interest, that is, agreement weakly dominates disagreement. Finally, we adopt the convention that player 1 makes offers in odd periods while player 2 makes offers in even periods. A schematic representation of the negotiation game is given in Figure 1.

An outcome path of the negotiation game consists of all disagreement game outcomes before agreement and the partition in the agreement. An outcome path is denoted by $\pi(T) = (a^1, a^2, \ldots, a^{T-1}, b^T, \{Y\})$ where a^t is the disagreement game outcome in period t for $1 \leqslant t < T$, and b^T is the partition agreed on in period T. By convention, T is set to infinity in an outcome path in which the players never reach an agreement. In the negotiation game, a player receives a payoff in every period, with or without agreement. The payoff derives from the disagreement game before an agreement is reached, and from the agreement

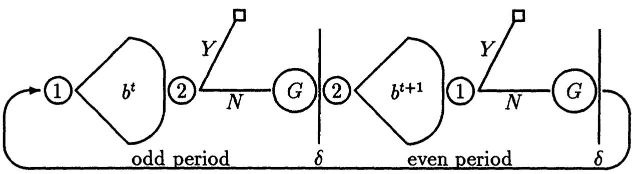

FIGURE 1.—Schematic of the negotiation game.

itself thereafter. Players' payoffs from the negotiation game are therefore the sum of their payoffs in all periods, discounted by a (common) discount factor $\delta \in (0,1)$. Player 1's and 2's average (discounted) payoffs from an outcome path $\pi(T)$ for $T < \infty$ are given by, respectively,

$$\left((1-\delta)\sum_{t=1}^{T-1}\delta^{t-1}u_1(a^t)\right)+\delta^{T-1}b^T$$

and

$$\left((1-\delta)\sum_{t=1}^{T-1}\delta^{t-1}u_2(a^t)\right)+\delta^{T-1}(1-b^T).$$

The players' payoffs from an outcome path without agreement, denoted by $\pi(\infty)$, are given by $(1-\delta)\sum_{t=1}^{\infty}\delta^{t-1}u_i(a^t)$, for $i = 1,2$.

In what follows, we will assume that each player can observe all past actions, including those in the disagreement game. This assumption is restrictive if the actions in the disagreement game involve probability mixtures. In contrast to the repeated game model of Fudenberg and Maskin (1986), the observability of probability mixtures over actions is necessary to our results as presented, however. We will discuss this issue further in Section 5. Under this assumption, we have the following specification of histories and strategies. There are three types of history in the negotiation game. The first type is the history at the beginning of a period, which consists of all rejected offers and disagreement game outcomes to date.[3] The set of histories of the first type in period t is $H_1(t) = [0,1]^{t-1} \times A^{t-1}$, for $t \geqslant 2$, while $H_1(1) = \varnothing$ denotes the null history at the beginning of the game. The second type is the history after a new offer has been made in a period, $H_2(t) = H_1(t) \times [0,1]$. The third type is the history after rejection, $H_3(t) = H_2(t) \times \{N\}$.

A strategy f_i for player i is a function which assigns an appropriate action to every possible history. For $i = 1,2$,

$$f_i\colon \bigcup_{t=1}^{\infty}\left[H_1(t)\cup H_2(t)\cup H_3(t)\right] \to A_i \cup [0,1] \cup \{Y,N\} \cup q.$$

[3] Since the game ends with an accepted offer, we do not define histories that have acceptances, and thus omit the sequence of rejections from the definition of a history.

In particular,

$$f_1\colon H_1(t) \to [0,1], \quad f_1\colon H_2(t) \to q, \quad f_1\colon H_3(t) \to A_1, \quad \text{for odd } t;$$
$$f_1\colon H_1(t) \to q, \quad f_1\colon H_2(t) \to \{Y, N\}, \quad f_1\colon H_3(t) \to A_1, \quad \text{for even } t;$$
$$f_2\colon H_1(t) \to q, \quad f_2\colon H_2(t) \to \{Y, N\}, \quad f_2\colon H_3(t) \to A_2, \quad \text{for odd } t;$$
$$f_2\colon H_1(t) \to [0,1], \quad f_2\colon H_2(t) \to q, \quad f_2\colon H_3(t) \to A_2, \quad \text{for even } t.$$

Here q denotes that a player cannot take an action, and it will be omitted in future specifications of particular strategies. A strategy profile $f = (f_1, f_2)$ is a strategy for each player, and gives players' instructions on how to play the negotiation game, conditional on history. For example, in some odd period t, $f_1(h_1(t))$ is player 1's offer, $f_2(h_2(t))$ is player 2's response to player 1's offer, and $f(h_3(t))$ is the disagreement game outcome. Any strategy profile induces a unique outcome path, and players' payoffs from a strategy profile are calculated directly from the induced outcome path in the negotiation game.

3. MOTIVATING EXAMPLES

Before the formal analysis of the negotiation game, we provide two simple examples which highlight the issues which have to be faced. For the first example, consider a negotiation game with the disagreement game shown in Figure 2. This disagreement game is a prisoners' dilemma game, which has a unique Nash equilibrium, (B, R), that yields the minimax payoff vector $(0, 0)$.

1\2	L	R
T	(0.4, 0.4)	(−0.2, 0.6)
B	(0.6, −0.2)	(0, 0)

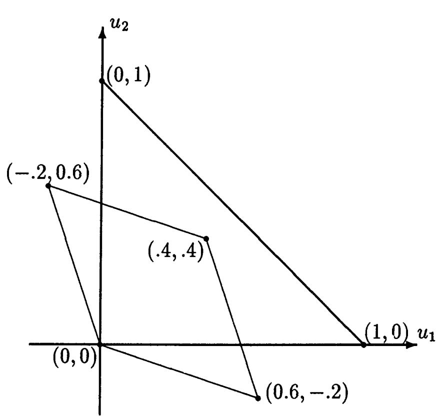

FIGURE 2

Consider the following strategy profile of the negotiation game. After any rejection the players play the Nash equilibrium (B, R) of the disagreement game. The disagreement payoff vector is therefore $(0,0)$ for all disagreement periods. The equilibrium proposals will thus be the same as those in the perfect equilibrium of Rubinstein's bargaining game with a disagreement payoff of $(0,0)$, in which the proposing player receives a share of $1/(1+\delta)$. Such an offer will be accepted in the first period, since the responding player either receives a share of $\delta/(1+\delta)$ from the first period on, or a share of $1/(1+\delta)$ from the next period on, which also leads to an average payoff of $\delta/(1+\delta)$. Should an offer be rejected, no player would deviate from (B, R) in the disagreement period, since (B, R) is the Nash equilibrium of the disagreement game and the continuation strategies are unaffected by a deviation. There are neither current nor future gains available from a deviation in the disagreement game, therefore. Immediate agreement on Rubinstein shares thus is a perfect equilibrium of the negotiation game.

The negotiation game also has other perfect equilibria, however. Necessarily, in these equilibria the players will not play the Nash equilibrium of the disagreement game after all rejections. Consider the following strategies. The strategy profile specifies the Nash outcome (B, R) only after rejections by player 1, and specifies the outcome (T, R) after rejections by player 2.[4] Subgame perfection requires that player 1 should not have an incentive to deviate from his dominated strategy T in the disagreement game. In order to support player 1 playing T, player 2 would therefore have to compensate player 1 in the next proposal. As long as player 2 offers $b_2 + 0.2 \cdot (1-\delta)/\delta$ if player 1 played T and b_2 otherwise, player 1 will be indifferent between playing T and B. These proposals in turn imply that player 2 will accept player 1's proposal b_1, if

$$\begin{aligned}(1-b_1) &= 0.6(1-\delta) + \delta\left[1 - b_2 - 0.2\frac{1-\delta}{\delta}\right] \\ &= 0.4(1-\delta) + \delta(1-b_2).\end{aligned}$$

Player 1, in turn, will accept player 2's offer b_2 if $b_2 = \delta b_1$, since after a rejection player 1 has a disagreement payoff of 0 and receives the share b_1 from the next period on. Together, these equalities imply that $b_1 = 0.6/(1+\delta)$ and $b_2 = \delta b_1$. Under this strategy profile player 2 obtains higher shares than from Rubinstein shares. As long as player 2 is patient enough, this fact can be exploited to deter deviations by player 2 from these strategies, if the perfect equilibrium with Rubinstein shares is implemented as soon as player 2 deviates.

The above strategies improve player 2's payoff by increasing his disagreement payoff after rejecting player 1's offers above his minimax payoff. If no disagreement game outcomes which do so are available, then it is also impossible to increase player 2's share. This will be the case if the difference between player 2's payoff and his minimax payoff is less than player 1's gain from deviation in

[4] The idea of alternating between disagreement payoffs in order to obtain extreme game payoffs is also exploited in Fernandez and Glazer (1991) and Haller and Holden (1990).

1\2	L	R
T	(0.4, 0.4)	(−1.1, 0.9)
B	(0.9, −1.1)	(0, 0)

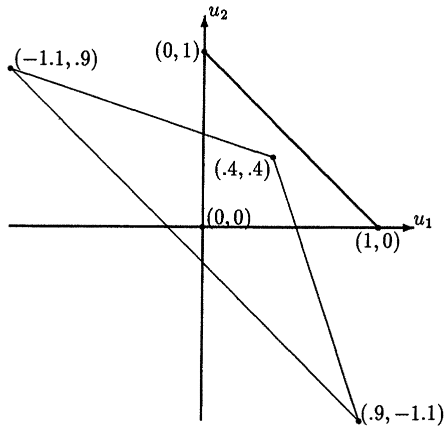

FIGURE 3

all outcomes of the disagreement game. Such is the case in the second example. Consider the disagreement game illustrated in Figure 3. This disagreement game is also a prisoners' dilemma, but in this game the difference between either player's payoff and his minimax payoff does not exceed his opponent's deviation gain. This is true for all actions in the disagreement game, including all probability mixtures over actions. Thus, it never pays player 2 to compensate player 1 for not deviating in the disagreement game, and vice versa. As a result, the only disagreement game outcome which can be supported in a disagreement period is the Nash equilibrium outcome of the disagreement game, and the negotiation game in turn has a unique perfect equilibrium. This uniqueness is robust to small perturbations in the disagreement game payoffs.[5]

4. SUBGAME PERFECT EQUILIBRIUM

In this section, we characterize the set of (subgame) perfect equilibria (Selten (1975)) of the negotiation game. In the spirit of the folk theorem for repeated games, we employ the idea of "equilibrium switching." A player who does not follow a proposed outcome path in the negotiation game will be "punished" by playing an equilibrium in which the player has a lower payoff than in the

[5] In particular, if all payoffs are perturbed by some small amount, the game will still be a prisoners' dilemma game with the Nash equilibrium giving rise to the mutual minimax payoffs. Furthermore, all deviation gains will continue to exceed the other player's payoff by at least the amount of the new minimax payoff—and thus the maximal payoff a player can obtain is his minimax payoff, just as before the perturbation.

proposed outcome path. We adopt the following three steps in order to characterize the set of equilibrium payoffs. First, we investigate the impact of a given sequence of disagreement payoffs on the perfect equilibrium outcome of an alternating-offers bargaining game. This analysis provides intuition on the structure of perfect equilibrium strategies in the negotiation game. As a corollary, we obtain the existence of perfect equilibrium in the negotiation game. Then, we derive a player's lowest equilibrium payoff in the negotiation game. Finally, we show that any feasible payoff vector of the negotiation game in which each player receives more than his lowest equilibrium payoff can be supported as the average payoff of a perfect equilibrium of the negotiation game, if players are sufficiently patient.

4.1. *Nonstationary Disagreement Payoffs in Bargaining*

It is well known that Rubinstein's (1982) bargaining game with discounting has a unique perfect equilibrium in which equilibrium offers are stationary. A similar result carries over to a bargaining game with any given nonstationary sequence of disagreement payoffs, as long as agreement dominates disagreement. In order to build some intuition on this point, we assume for the moment that players in the negotiation game are irrevocably committed to playing an outcome $a^t \in A$ of the disagreement game in period t. The negotiation game thus becomes a bargaining game with a nonstationary sequence of disagreement payoffs. The following proposition asserts that such a bargaining game has a unique perfect equilibrium payoff and that the equilibrium offers are history independent.

PROPOSITION 1: *If players are precommitted to play $a^t \in A$ in the disagreement game in period t in the absence of an agreement, when $\forall \delta$ and $i = 1, 2$, the negotiation game in which player i proposes first has a perfect equilibrium in which player i's offer b_i is accepted in the first period, where*

$$b_1 = \frac{1}{1+\delta} + (1-\delta) \sum_{j=0}^{\infty} \delta^{2j}\left[\delta u_1\left(a^{2j+2}\right) - u_2\left(a^{2j+1}\right)\right], \tag{1}$$

$$b_2 = \frac{\delta}{1+\delta} + (1-\delta) \sum_{j=0}^{\infty} \delta^{2j}\left[u_1\left(a^{2j+1}\right) - \delta u_2\left(a^{2j+2}\right)\right]. \tag{2}$$

Any other perfect equilibria are payoff equivalent.

The proof of this proposition closely follows Shaked and Sutton (1984) and is omitted. The possible multiplicity of equilibrium strategy profiles is introduced solely by the possibility that the disagreement payoffs are efficient (i.e., $u_1(a^t) + u_2(a^t) = 1$) in the first period under consideration, and thus both players are indifferent between immediate settlement and delay of one period. Proposition 1 highlights two important points about the bargaining process in the negotiation game. The first is that any fixed sequence of future disagreement payoffs determines a unique perfect equilibrium offer which is acceptable. The perfect equilibrium based on any fixed sequence of disagreement payoffs is therefore efficient. Assumption (A-5), that agreement weakly dominates disagreement, is crucial for Proposition 1 to hold. The equilibrium offer which just leaves the responding player indifferent between accepting and rejecting would not be made, if the proposing player would be worse off from having such an offer accepted rather than if his offer were to be rejected. Such a situation arises if disagreement dominates agreement, however.

The second point concerns the "bargaining power" which the disagreement payoffs bestow on the players. As equations (1) and (2) in Proposition 1 show, the equilibrium offers only depend on players' disagreement payoffs in periods when they are responding to an offer, not when they are making an offer. This is due to the fact that a responding player accepts all offers in which his payoff is not less than that from his disagreement payoff in the current period and his continuation payoff from the next period onward. Therefore, the higher his disagreement payoff, the higher must be his payoff in an acceptable offer. The proposing player, on the other hand, collects all of the residual surplus available from immediate agreement, which (weakly) exceeds his payoff if his offer is rejected by assumption (A-5), and his disagreement payoff is therefore irrelevant. Together, these two points show that any scope for varying the perfect equilibrium payoffs of the negotiation game must derive solely from the presence of endogenous disagreement payoffs. Only the ability to switch among different path's of future disagreement game payoffs can lead to multiple equilibrium payoffs in the negotiation game.

An immediate implication of Proposition 1 is that playing a Nash equilibrium of the disagreement game can be supported as part of a perfect equilibrium in the negotiation game. The reason is that there exist no profitable deviations from a Nash equilibrium, and that continuation payoffs can be history independent. Furthermore, since the same argument implies that any sequence of Nash equilibrium payoffs of the disagreement game determines a perfect equilibrium of the negotiation game, the negotiation game will have multiple perfect equilibria if the disagreement game has multiple Nash equilibrium payoffs. We summarize these results in the following corollary.

COROLLARY 1: *Suppose that a^t is a Nash equilibrium of the disagreement game G for all $t \geqslant 1$. Then for all $\delta \in (0,1)$, the negotiation game in which player i proposes first has a perfect equilibrium such that player i's offer b_i is accepted in the first period for $i = 1,2$, where b_1 is given by* (1) *and b_2 by* (2). *In particular, if a^* is a Nash equilibrium of G, then for all $\delta \in (0,1)$, the negotiation game in which player i proposes first has a perfect equilibrium such that player i's offer $b_i[u(a^*)]$ is accepted in the first period for $i = 1,2$, where*

$$b_1[u(a^*)] = \frac{1 + \delta u_1(a^*) - u_2(a^*)}{1+\delta}$$

and

$$b_2[u(a^*)] = \frac{\delta + u_1(a^*) - \delta u_2(a^*)}{1+\delta}.$$

4.2. *Punishment Equilibria*

Define an (optimal) punishment equilibrium for a player to be a perfect equilibrium in which this player receives his lowest perfect equilibrium payoff. Since the arguments are analogous for both players, only player 1's punishment equilibrium is derived explicitly in what follows. We first find a lower bound of player 1's perfect equilibrium payoffs as a function of the discount factor, and then construct a perfect equilibrium in which player 1's payoff equals this lower bound if the discount factor is large enough. The constructed perfect equilibrium is therefore player 1's punishment equilibrium.

As in the simple penal codes of Abreu (1988), a punishment equilibrium of the negotiation game must be self-enforcing. That is, a player who deviates in his own punishment can only be punished by restarting the same punishment equilibrium in the rest of the negotiation game. The (two-period) stationary structure of the negotiation game and Proposition 1 then imply a stationary structure for player 1's punishment equilibrium strategy. The strategy will specify one disagreement game outcome for all odd periods and another outcome for all even periods. Proposition 1 shows that player 1's equilibrium payoff depends positively on player 1's disagreement payoffs in even periods and negatively on player 2's disagreement payoffs in odd periods. Player 1 will therefore have his lowest equilibrium payoff in the negotiation game if the strategy profile specifies player 1's lowest supportable disagreement payoff for all even periods and player 2's highest supportable disagreement payoff for all odd periods.

Finding player 1's lowest disagreement payoff is straightforward. By definition, it is player 1's minimax payoff in the disagreement game. Player 1 has no incentive to deviate from his minimax outcome in the disagreement game. It can therefore be supported as part of a perfect equilibrium as long as player 2 can be forced to play the minimax outcome against player 1 (by switching to an equilibrium with a lower payoff for player 2 if he fails to do so).

Now consider the highest supportable disagreement payoff for player 2 in an odd period. Player 1 will not deviate from a proposed outcome in the disagreement game as long as player 2's next offer compensates player 1 for any foregone deviation gains. In such a scheme, player 2 receives the difference between his actual disagreement payoff and the compensation to player 1 as his effective disagreement payoff, while player 1 effectively receives the payoff from deviation without actually deviating. Player 2's highest supportable disagreement payoff in an odd period therefore is the maximum difference between player 2's disagreement payoffs and player 1's deviation gains, taken over all outcomes in the disagreement game. Let player 2's highest supportable disagreement payoff be denoted by w_2, defined as

$$w_2 = \max_{a \in A} \left\{ u_2(a) - \left[\max_{a_1' \in A_1} u_1(a_1', a_2) - u_1(a) \right] \right\}. \tag{3}$$

Assumptions (A-1) and (A-2) imply that such a w_2 is well defined. Assumptions (A-3) and (A-4) imply that $w_2 \geqslant 0$, while (A-5) implies that $w_2 \leqslant 1$.

Proposition 2: *For all $\delta \in (0,1)$, player 1's average perfect equilibrium payoffs in the negotiation game are not less than $(1-w_2)/(1+\delta)$, starting in an odd period, and not less than $[\delta(1-w_2)]/(1+\delta)$, starting in an even period.*

Proof: We prove the proposition by calculating the infimum of player 1's perfect equilibrium payoffs in the negotiation game. $\forall \delta \in (0,1)$, Corollary 1 and assumptions (A-1) and (A-3) imply that the set of perfect equilibrium payoffs of the negotiation game is nonempty and bounded. Let $L_1(\delta)$ be the infimum of player 1's perfect equilibrium payoffs in an odd period. Then player 1's perfect equilibrium payoffs in an even period are not less than $\delta L_1(\delta)$ by (A-4), since player 1 can guarantee himself his minimax payoff in the disagreement game after his rejection, and $L_1(\delta)$ from the next (odd) period on.

Consider the players' perfect equilibrium payoffs in an odd period. Suppose that after player 2's rejection in the odd period, the players play a disagreement game outcome, say $a \in A$, and a perfect equilibrium with average payoff vector $v(a)$ from the next period on. Note that $v_1(a) + v_2(a) \leqslant 1$ by (A-5). Playing $a \in A$ in G in the odd period and the perfect equilibrium with payoff vector $v(a)$ in the next period has to be subgame perfect after player 2's rejection. This requires that player 1 have no incentive to deviate from the disagreement game outcome $a \in A$. Therefore, it must be true that

$$\begin{aligned}(1-\delta) \max_{a_1' \in A_1} u_1(a_1', a_2) + \delta^2 L_1(\delta) &\leqslant (1-\delta) u_1(a) + \delta v_1(a) \\ &\leqslant (1-\delta) u_1(a) + \delta[1 - v_2(a)],\end{aligned}$$

which yields that

$$\delta v_2(a) \leqslant \delta[1 - \delta L_1(\delta)] - (1-\delta)\left[\max_{a_1' \in A_1} u_1(a_1', a_2) - u_1(a) \right]. \tag{4}$$

Inequality (4) implies that player 2's continuation payoff from rejecting player 1's offer is not more than

$$\begin{aligned}&\max_{a \in A} \{(1-\delta) u_2(a) + \delta v_2(a)\} \\ &\quad \leqslant \max_{a \in A} \Big\{ (1-\delta) u_2(a) + \delta[1 - \delta L_1(\delta)] \\ &\qquad\qquad - (1-\delta)\left[\max_{a_1' \in A_1} u_1(a_1', a_2) - u_1(a) \right] \Big\} \\ &\quad = (1-\delta) \max_{a \in A} \left\{ u_1(a) + u_2(a) - \max_{a_1' \in A_1} u_1(a_1', a_2) \right\} \\ &\qquad + \delta[1 - \delta L_1(\delta)] \\ &\quad = (1-\delta) w_2 + \delta[1 - \delta L_1(\delta)].\end{aligned} \tag{5}$$

Hence, in the odd period player 2 will certainly accept player 1's offer as long as his share is not less than (5). The infimum of player 1's perfect equilibrium payoff $L_1(\delta)$ thus satisfies the following inequality,

$$L_1(\delta) \geqslant 1 - \left[(1-\delta)w_2 + \delta(1 - \delta L_1(\delta))\right].$$

After simplification, the last inequality reduces to $L_1(\delta) \geqslant (1-w_2)/(1+\delta)$. *Q.E.D.*

The outstanding question at this point is if the lower bound in Proposition 2 can actually be attained in a perfect equilibrium of the negotiation game. The following proposition demonstrates that there does exist a perfect equilibrium in which player 1's payoff attains this lower bound, provided that the players are patient enough. The proposed equilibrium strategy profile specifies a disagreement outcome which solves (3) for all odd periods, and player 1's minimax outcome $m^1 \in A$ for all even periods. Based on this sequence of disagreement game outcomes, the equilibrium offers are computed in accordance with Proposition 1, except that player 2's offer compensates player 1, if player 1 did not deviate in the last disagreement game.

PROPOSITION 3: *There exists a* $\underline{\delta} \in (0,1)$ *such that,* $\forall \delta \in (\underline{\delta}, 1)$, *the offer* $(1 - w_2)/(1+\delta)$ *is accepted in a perfect equilibrium of a negotiation game in an odd period, and the offer* $\delta(1-w_2)/(1+\delta)$ *is accepted in a perfect equilibrium in an even period. The resulting average payoff vectors are, respectively,*

$$\left(\frac{1-w_2}{1+\delta}, \frac{\delta + w_2}{1+\delta}\right) \quad \textit{and} \quad \left(\frac{\delta(1-w_2)}{1+\delta}, \frac{1+\delta w_2}{1+\delta}\right).$$

Note that the equilibrium offers in Proposition 3 are the same as those in the perfect equilibrium of a Rubinstein bargaining game with a fixed disagreement payoff of $(0, w_2)$ in every period. The necessity for the restriction on the discount factor in Proposition 3 arises from the need to enforce player 2's adherence to the proposed equilibrium strategy. If the discount factor is large enough, a perfect equilibrium in accordance with Corollary 1 is sufficient to get player 2 to follow his strategy. Note that the perfect equilibrium according to Corollary 1 may not be player 2's punishment equilibrium. The value of $\underline{\delta}$ therefore depends on the choice of the particular equilibrium used to punish player 2. The formal equilibrium strategy profile and the proof are given in the Appendix.

Proposition 2 implies that the equilibria constructed in Proposition 3 are player 1's punishment equilibria in the negotiation game. By analogous arguments, we can find player 2's punishment equilibria. Let

$$w_1 = \max_{a \in A} \left\{ u_1(a) - \left[\max_{a_2' \in A_2} u_2(a_1, a_2') - u_2(a) \right] \right\}$$

be player 1's highest supportable disagreement payoff in an even period of player 2's punishment equilibrium. As before, $0 \leqslant w_1 \leqslant 1$. Player 2's minimax payoff in the disagreement game is his lowest supportable disagreement payoff in an odd period of his punishment equilibrium. As the following proposition asserts, the offers in player 2's punishment equilibria are the same as the equilibrium offers of a bargaining game with a fixed disagreement payoff of $(w_1, 0)$ in all periods.

PROPOSITION 4: *There exists a* $\underline{\delta} \in (0,1)$ *such that*, $\forall \delta \in (\underline{\delta}, 1)$, *the offer* $(1 + \delta w_1)/(1+\delta)$ *is accepted in a perfect equilibrium of a negotiation game in an odd period, and the offer* $\delta(\delta + w_1)/(1+\delta)$ *is accepted in a perfect equilibrium in an even period. The resulting average payoff vectors are, respectively,*

$$\left(\frac{1+\delta w_1}{1+\delta}, \frac{\delta(1-w_1)}{1+\delta}\right) \quad \textit{and} \quad \left(\frac{\delta + w_1}{1+\delta}, \frac{1-w_1}{1+\delta}\right).$$

Together, Propositions 3 and 4 imply that the negotiation game has a unique perfect equilibrium if and only if $(w_1, 0) = (0, w_2)$, i.e., $w_1 = 0$ and $w_2 = 0$. This implies that any Nash equilibrium payoff vector has to be $(0,0)$, which is the minimax point. If w_1 and w_2 equal the players' minimax payoffs, only the minimax payoff vector can be supported in disagreement periods in a perfect equilibrium of the negotiation game.[6] The equilibrium offers will have to be stationary in this case, and coincide with those of a Rubinstein bargaining game. On the other hand, the negotiation game has multiple equilibria if at least one of w_1 or w_2 exceeds the respective player's minimax payoff. In that case, a player's disagreement payoff after a rejection can depend on the history of play, and this in turn leads to equilibrium offers which depend on the history of play.

If the negotiation game has multiple efficient perfect equilibria, then it will also have some inefficient perfect equilibria which feature delay in reaching agreement. In fact, any feasible payoff vector of the negotiation game is a perfect equilibrium payoff vector in the negotiation game if and only if every player receives a higher payoff than in his punishment equilibrium. This is formally shown in the following subsection.

4.3. *Perfect Equilibria of the Negotiation Game*

In the spirit of the folk theorem, we characterize the set of perfect equilibrium payoffs of the negotiation game as the discount factor approaches unity. As δ tends to one, player 1's and 2's payoffs in their respective punishment equilibria converge to

$$\underline{v}_1 = \frac{1-w_2}{2} \quad \text{and} \quad \underline{v}_2 = \frac{1-w_1}{2}.$$

The following theorem asserts that any feasible payoff vector of the negotiation game which strictly dominates $(\underline{v}_1, \underline{v}_2)$ is a perfect equilibrium payoff in the negotiation game with a sufficiently large discount factor.

[6] Notice that, in general, it is not required that the minimax payoffs of players equal each other, only that each player's highest supportable payoff also be his lowest supportable payoff (i.e. $w_i = u_i(m^i)$). The fact that everything is equal here is an artifact of the normalization in (A-4).

THEOREM: *For any feasible payoff vector* (v_1, v_2) *of the negotiation game such that* $v_i > \underline{v}_i$, *for* $i = 1, 2$, *there exists* $\underline{\delta} \in (0,1)$ *such that* $\forall \delta \in (\underline{\delta}, 1)$, (v_1, v_2) *is a perfect equilibrium payoff vector in the negotiation game with discount factor* δ.

The proof of the Theorem is given in the Appendix. The equilibrium strategy profile used in the proof is very intuitive. First, we find an outcome path which leads to the average payoff (v_1, v_2). Such an outcome path is in general not unique. For example, if (v_1, v_2) is a feasible payoff vector of the disagreement game, then the outcome path of the negotiation game could involve infinitely repeated play of the disagreement game without agreement, or a sequence of disagreement outcomes followed by an agreement. The only restriction on the outcome path is that both players' continuation payoffs are higher than their respective punishment equilibrium payoffs in every subgame. The following strategy profile then implements the chosen outcome path. In every period before the agreement, the proposing player demands the whole surplus, and any other offer will be considered a deviation. The responding player, of course, rejects these offers. In the disagreement game, the players play the appropriate outcome as specified in the outcome path. Deviations by any player in either the offers or the disagreement games will be punished by implementation of the deviating player's punishment equilibrium for the rest of the negotiation game. In the last period (if there is one), the offer specified in the outcome path will be proposed and accepted; otherwise the player who deviates first will be punished.

This strategy profile is not unique. In particular, one could construct a strategy profile which has offers that converge to the final agreement. Observationally, this would be more plausible, since such strategies would lead to a converging sequence of offers and counter-offers culminating in agreement, compared to the "stubborn" behavior followed by "sudden" agreement which is implied by the strategies used in the proof.

5. DISCUSSION

We have presented a model of dynamic idiosyncratic allocation which combines aspects of Rubinstein bargaining and repeated games. In this negotiation model two players alternate in proposing a binding allocation of a recurring surplus, while they repeatedly play a disagreement game which determines their payoffs before agreement. The results we have obtained for this model lead to two main insights. The first is the fact that negotiation processes which involve payoffs from a strategic relationship outside the bargaining process itself will in general lead to multiple perfect equilibria. Some of these equilibria feature delay and are inefficient. This result obtains even though there is perfect information in the negotiation model, and is due to disagreement payoffs which can vary in response to players' past actions. The power of history dependent strategies, which in bargaining is lost to the dynamics of the alternating-offers process, is therefore reinstated by the endogenous disagreement payoffs of the negotiation model.

The second insight that the negotiation model generates is that the set of average payoffs which can be supported in perfect equilibria of a negotiation game is not the set of all feasible and individually rational payoffs of the disagreement game. The concurrency of the bargaining process with the repeated play of the disagreement game restricts the disagreement payoffs, in particular with respect to the maximal payoff achievable by a player. This result is due to the fact that extreme payoffs which are not one-shot Nash equilibria of the disagreement game cannot be supported by drawn out future reward phases. Instead, they have to be supported by the immediately following bargaining offer, since the bargaining process will end the game in the next period. Since the bargaining frontier is negatively sloped, an immediate tradeoff between the two players' payoffs is implied. This restriction becomes especially important in the strategies supporting a player's punishment, and the severity of the punishment will only be determined by the structure of the disagreement game, and is independent of the bargaining surplus available.

These results depend principally on two assumptions. One is that agreement dominates disagreement at least weakly. If, to the contrary, there exist disagreement payoffs which dominate agreement strictly, then all such payoffs can be supported by standard repeated game arguments, and multiplicity of equilibrium is guaranteed. The second critical assumption is that all actions are observable, including probability mixtures in the disagreement game. This makes all deviations detectable. While similar assumptions do not affect the equilibrium set in the repeated game literature, in the negotiation game this assumption enlarges the set of equilibrium payoffs. Nevertheless, qualitatively the results remain unchanged if observability is relaxed.[7] The reason for this is as follows. The actions which are used in the disagreement game to minimize a player's payoff or maximize his opponent's payoff will have to satisfy an additional constraint if they are unobservable. These constraints will, in general, reduce the maximal disagreement payoff which can be supported as part of a perfect equilibrium, and increase the minimal payoff. Actions that satisfy the constraints will exist, however.

In closing, we would like to point out two aspects of the model. One is that the negotiation model applies to the case where the Pareto frontier of the disagreement game coincides with the bargaining frontier. The model thus encompasses the case of repeated games for which binding agreements can be written, as long as the players alternate in proposing the final allocation. The second point concerns the economic benefit which agreement yields over repeated play. While the position of the bargaining frontier itself does not affect the range of allocations which can be supported in equilibrium, it will affect the maximal time of delay which can be supported. As the economic importance of agreement increases, agreement will have to occur more quickly, and in the

[7] See Busch and Wen (1993), available upon request from the authors.

limit, as the disagreement game becomes irrelevant to players' payoffs, the equilibrium set converges to that of a pure bargaining game. On the other hand, if all agreements are dominated by some disagreement payoff, agreement is economically unimportant, and the folk theorem for infinitely repeated games with discounting applies. The bargaining model and the infinitely repeated game model can thus be viewed as limiting cases of the negotiation model.

Dept. of Economics, University of Waterloo, Waterloo, Ontario N2L 3G1, Canada,

and

Dept. of Economics, University of Windsor, Windsor, Ontario N9B 3P4, Canada.

Manuscript received July, 1991; final revision received July, 1994.

APPENDIX

PROOF OF PROPOSITION 3: Note the fact that the payoff vector in the proposition corresponds to the perfect equilibrium in the bargaining game with disagreement payoff $(0, w_2)$. If $(0, w_2)$ is a Nash equilibrium payoff of the disagreement game, then Proposition 3 follows directly from Corollary 1. If $(0, w_2)$ is not a Nash equilibrium payoff of the disagreement game, the proof is constructive. We will first derive a necessary $\underline{\delta}$, then provide a strategy profile which implements the payoff vector, and verify the subgame perfection of the proposed strategy profile in the negotiation game with $\delta \in (\underline{\delta}, 1)$.

Let a^* be a Nash equilibrium of the disagreement game G where $u(a^*) \neq (0, w_2)$, and let $b_1[u(a^*)]$ and $b_2[u(a^*)]$ be the equilibrium offers in the bargaining game with disagreement payoff $u(a^*)$ from Corollary 1. Also, let $a^1 \in A$ be a solution to (3), let $m^1 \in A$ be a minimax strategy combination against player 1, and let $r_1(a^1)$ be defined as follows:

$$w_2 = u_1(a^1) + u_2(a^1) - \max_{a'_1 \in A_1} u_1(a'_1, a^1_2), \tag{A.1}$$

$$r_1(a^1) = \frac{1-\delta}{\delta}\left[\max_{a'_1} u_1(a'_1, a^1_2) - u_1(a^1)\right]. \tag{A.2}$$

$r_1(a^1) \geqslant 0$ is the present value of player 1's gain by deviating from a^1. Define

$$b_1(0, w_2) = \frac{1 - w_2}{1+\delta} \quad \text{and} \quad b_2(0, w_2) = \frac{\delta(1 - w_2)}{1+\delta}.$$

See Figure 4 for an illustration.

Let B and W be the maximum and the minimum payoffs to a player in G, respectively. Both B and W are finite by assumptions (A-1) and (A-2). Since $w_2 > u_2(a^*) - u_1(a^*)$— due to $w_2 \geqslant u_2(a^*)$, $u_1(a^*) \geqslant 0$, and $(0, w_2) \neq u(a^*)$—there exists $\underline{\delta} \in (0,1)$ such that the following inequalities hold for all $\delta \in (\underline{\delta}, 1)$:

$$0 < b_2(0, w_2) + r_1(a^1) < b_2[u(a^*)], \tag{A.3}$$

$$(1-\delta)B + \delta[1 - b_2[u(a^*)]] \leqslant (1-\delta)W + \delta \min\{1 - b_1(0, w_2), 1 - b_2(0, w_2) - r_1(a^1)\}. \tag{A.4}$$

Consider the following strategy profile: In an odd period player 1 offers $b_1(0, w_2)$, and player 2 will reject player 1's offer only if player 1 demands more than $b_1(0, w_2)$, after which the players will

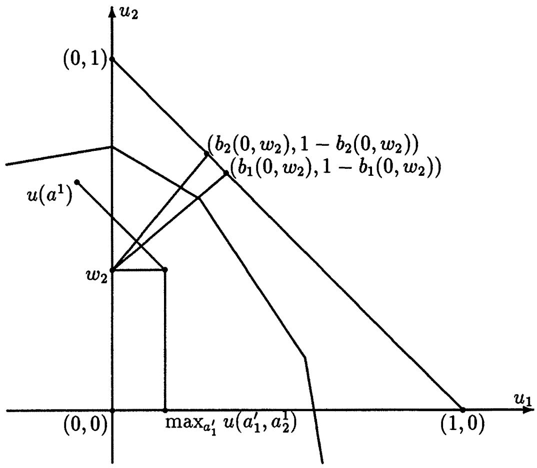

FIGURE 4.—$b_1(0,w_2)=(1-w_2)/(1+\delta)$ and $b_2(0,w_2)=\delta b_1(0,w_2)$.

play a^1 in G. In an even period, player 2 will offer $b_2(0,w_2)$ if player 1 deviated from a^1 in the last period, and $b_2(0,w_2)+r_1(a^1)$ otherwise. In either case, player 1 will reject only if player 2 offers less. If player 1 rejects an offer by player 2 which should have been accepted, the players play m^1 in the following disagreement game. If player 2 deviates, however, the players will follow the equilibrium strategies in Corollary 1, which are the perfect equilibrium of the negotiation game with a disagreement payoff vector $u(a^*)$. The formal strategy functions are as follows. In an odd period t,

$$f_1(h_1(t))=\begin{cases} b_1[u(a^*)] & \text{if player 2 has deviated,}\\ b_1(0,w_2) & \text{otherwise,}\end{cases}$$

$$f_2(h_2(t))=\begin{cases} Y & \text{if } b^t\leqslant f_1(h_1(t)),\\ N & \text{otherwise,}\end{cases}$$

$$f(h_3(t))=\begin{cases} a^* & \text{if player 2 has deviated,}\\ a^1 & \text{otherwise,}\end{cases}$$

and in an even period t,

$$f_2(h_1(t))=\begin{cases} b_2[u(a^*)] & \text{if player 2 has deviated,}\\ b_2(0,w_2) & \text{if player 1 deviated in } a^1 \text{ in the last period,}\\ b_2(0,w_2)+r_1(a^1) & \text{otherwise,}\end{cases}$$

$$f_1(h_2(t))=\begin{cases} Y & \text{if } b^t\geqslant f_2(h_1(t)),\\ N & \text{otherwise,}\end{cases}$$

$$f(h_3(t))=\begin{cases} a^* & \text{if player 2 has deviated,}\\ m^1 & \text{otherwise.}\end{cases}$$

The rest of the proof shows the subgame perfection of this strategy profile for $\delta\in(\underline{\delta},1)$. In the case that player 2 has deviated, the players play the perfect equilibrium from Corollary 1 with the Nash equilibrium payoff $u(a^*)$ as the disagreement payoff. Therefore, we only have to consider the other cases.

First, consider players' strategies in the disagreement game following a rejection. Player 1 will not deviate from m^1 since m^1 is a minimax strategy against him. Player 1 will not deviate from a^1 either.

According to the strategy profile, if player 1 deviates from a^1, player 2 will propose $b_2(0,w_2)$ instead of $b_2(0,w_2)+r_1(a^1)$ in the next period. (A.2) implies that player 1 cannot increase his payoff by deviating from a^1, i.e., player 1 receives the same payoff from deviating or not,

$$(1-\delta)\max_{a_1'\in A_1} u_1(a_1', a_2^1) + \delta b_2(0,w_2) = (1-\delta)u_1(a^1) + \delta\left[b_2(0,w_2)+r_1(a^1)\right].$$

On the other hand, if player 2 deviates from either m^1 or a^1 the players will play the perfect equilibrium in Corollary 1 from the next period onward. Therefore player 2's payoff is, at most,

$$(1-\delta)B+\delta\left[1-b_2\left[u(a^*)\right]\right],$$

which is less than his equilibrium payoff in either case,

$$(1-\delta)W+\delta\min\left\{1-b_1(0,w_2), 1-b_2(0,w_2)-r_1(a^1)\right\},$$

due to (A.4). Therefore both players will not deviate from the proposed strategies in the disagreement game.

Next, consider players' offers and responses. In an odd period, player 2's payoff from rejecting $b_1(0,w_2)$ is $1-b_1[u(a^*)]$, which is less than $1-b_1(0,w_2)$ by Proposition 2. If player 1 deviates to demand more than $b_1(0,w_2)$, player 2's payoff from rejecting is

$$(1-\delta)u_2(a^1)+\delta\left[1-b_2(0,w_2)-r_1(a^1)\right]=1-b_1(0,w_2).$$

Therefore player 2 will follow his response strategies, and reject player 1's offer only if player 1 demands more than $b_1(0,w_2)$. As a result, if player 1 demands more in his offer, player 2 will reject, the players will play a^1 in the current period, and player 2's offer $b_2(0,w_2)+r_1(a^1)$ will be accepted in the next period. The following equalities state that player 1 cannot receive more than $b_1(0,w_2)$ by demanding more in his offer:

$$\begin{aligned}(1-\delta)u_1(a^1)&+\delta\left[b_2(0,w_2)+r_1(a^1)\right]\\ &=(1-\delta)\left[u_1(a^1)+u_2(a^1)\right]+\frac{\delta^2-w_2}{1+\delta}\\ &\leqslant(1-\delta)+\frac{\delta^2-w_2}{1+\delta}=b_1(0,w_2),\end{aligned}$$

where we have made use of (A.1) and (A.2). Therefore the players will follow their offer and response strategies in an odd period.

In an even period, player 1's payoff from rejecting an offer which should be accepted is $b_2(0,w_2)$, resulting from the disagreement game outcome m^1 in the current period and agreement $b_1(0,w_2)$ the next period. Note that player 1's payoff from accepting an offer which should be accepted is not less than $b_1(0,w_2)$. On the other hand, player 1's payoff from rejecting an offer which should be rejected is $b_2[u(a^*)]$, resulting from player 2's punishment. $b_2[u(a^*)]$ is player 1's highest payoff in an even period. Therefore, player 1 will follow his response strategies to reject only if player 2 demands more than he should. As a result, if player 2 demands more, he will be punished and his payoff will be $1-b_2[u(a^*)]$, which is not more than his payoffs in any offer in the strategy profile. Hence, player 2 will not deviate in his proposal strategies.

The proposed strategy profile therefore constitutes a subgame perfect equilibrium in the negotiation game with discount factor $\delta\in(\underline{\delta},1)$. The equilibrium outcome is that player 1's offer $b_1(0,w_2)$ is accepted in the first period when player 1 makes the first offer. Player 1's punishment equilibrium in the negotiation game when player 2 offers first is induced by our proposed perfect equilibrium after player 1 demanded more or deviated in the disagreement game in the first period.

Q.E.D.

PROOF OF THE THEOREM: We prove the theorem when player 1 makes the first offer. A similar proof can be constructed to prove the theorem when player 2 makes the first offer. We first find an outcome path $\pi(T)$ with average payoff vector v and a necessary $\underline{\delta}$, then provide a strategy profile with outcome path $\pi(T)$, and show the subgame perfection of the strategy profile for $\delta\in(\underline{\delta},1)$.

Let B be the maximum payoff to a player in G. B is finite by (A-1) and (A-2). The fact that $\underline{v}_i<v_i$ and the definition of $\underline{v}_i$ for $i=1,2$ imply that $\exists\underline{\delta}$ such that for $\delta\in(\underline{\delta},1)$ the optimal

punishment equilibria in Propositions 3 and 4 exist, and the following inequalities hold:

$$(A.5) \qquad (1-\delta)B + \frac{1-w_2}{1+\delta} \leqslant v_1 \quad \text{and} \quad (1-\delta)B + \frac{1-w_1}{1+\delta} \leqslant v_2.$$

Since v is feasible, $\forall \delta \in (\underline{\delta}, 1)$ there is an outcome $\hat{a} \in A$, and offer $\hat{b}_1$, and an integer T, such that

$$(A.6) \qquad v_1 = (1-\delta^T)u_1(\hat{a}) + \delta^T \hat{b}_1, \quad \text{and} \quad v_2 = (1-\delta^T)u_2(\hat{a}) + \delta^T \hat{b}_2,$$

$$(A.7) \qquad u_1(a) < v_1 < \hat{b}_1, \quad \text{and} \quad u_2(a) < v_2 < \hat{b}_2,$$

where $\hat{b}_2 = 1 - \hat{b}_1$. Let $\pi(T)$ be the outcome path where players play $\hat{a}$ for $(T-1)$ periods and agree on the offer $\hat{b}_1$ in period T. Equation (A.6) states that the average payoff to player i from the outcome path $\pi(T)$ is v_i. The following proof is also applicable if the players never reach an agreement, if $u_i(\hat{a}) = v_i$ and T is infinite.

Consider the following strategy profile: Before period T, the proposing player i demands the whole value of 1, the responding player j will accept any proposal if his share is not less than $(\delta + w_j)/(1+\delta)$ for $j \neq i$, and the players play $\hat{a}$ after rejection. In period T, the offer $\hat{b}_1$ is accepted. If at any point a player i, $i = 1, 2$, deviates, the players will immediately start to play player i's optimal punishment from Proposition 3 or 4, beginning in the appropriate subgame. For example, if player 1 does not demand the whole value of 1, the players will play according to the strategies after player 1 deviated from his offer in his optimal punishment. The outcome path of this strategy profile is $\pi(T)$ with average payoff vector v. The rest of this proof verifies the subgame perfection of the strategy profile for $\delta \in (\underline{\delta}, 1)$.

For $\delta \in (\underline{\delta}, 1)$, after a player deviates from the proposed outcome path $\pi(T)$, the continuation equilibrium is one of the optimal punishment equilibria from Propositions 3 and 4, depending on the deviator. Therefore we only have to verify the subgame perfection of the strategy profile along the outcome path $\pi(T)$. In the rest of the proof, let player i be the proposing player and j be the responding player.

In period $t < T$, after rejection, if player i deviates from $\hat{a}$ in G, for $i = 1, 2$, he will be punished from the next period on. (A.5) and (A.7) imply that player i's payoff from deviating is less than that from the strategy profile, since for $j \neq i$,

$$(1-\delta)B + \delta\frac{1-w_j}{1+\delta} < (1-\delta)B + \frac{1-w_j}{1+\delta} \leqslant v_i \leqslant (1-\delta^{T-t})u_i(\hat{a}) + \delta^{T-t}\hat{b}_i.$$

If player i demands less than 1, he will be punished and player j's payoff from rejecting will be equal to $1 - (1-w_j)/(1+\delta) = (\delta + w_j)/(1+\delta)$. Hence, player j will accept an offer only if his share is not less than $(\delta + w_j)/(1+\delta)$. As a result, if player i demands less, his payoff will be, at most, $(1-w_j)/(1+\delta)$. (A.5) and (A.7) yield that

$$\frac{1-w_j}{1+\delta} < v_i \leqslant (1-\delta^{T-t})u_i(\hat{a}) + \delta^{T-t}\hat{b}_i.$$

The last inequality indicates that player i will not demand less than 1. Therefore, both players will follow their strategies before period T.

In period T, the players should agree on $\hat{b}_i$. If player j rejects $\hat{b}_i$, he will be punished and his payoff will be $\delta(1-w_i)/(1+\delta)$, which is less than $\hat{b}_j$ from (A.5) and (A.7). However, player j's payoff from rejecting a nonequilibrium offer is $(\delta - w_j)/(1+\delta)$, which is not less than $\hat{b}_j$. Therefore, player j will reject only if player i demands more. As a result, if player i demands more, player j will reject and player i will be punished with payoff $(1-w_j)/(1+\delta) < \hat{b}_i$. Player i therefore will propose $\hat{b}_1$. Therefore, both players will not deviate in period T. *Q.E.D.*

REFERENCES

Abreu, D. (1988): "On The Theory of Infinitely Repeated Games with Discounting," *Econometrica*, 56, 383–396.

Admati, A. R., and M. Perry (1987): "Strategic Delay in Bargaining," *Review of Economic Studies*, 54, 345–364.

Ausubel, L. M., and R. J. Deneckere (1989): "Reputation in Bargaining and Durable Goods Monopoly," *Econometrica*, 57, 511–532.

BENOIT, J. P., AND V. KRISHNA (1985): " Finitely Repeated Games," *Econometrica*, 53, 905–922.
BUSCH, L-A., AND Q. WEN (1993): "A Note on Unobservable Mixed Strategies in Negotiation Games," Working Paper 9311, Department of Economics, University of Waterloo.
CHATTERJEE, K., AND L. SAMUELSON (1987): "Bargaining with Two-Sided Incomplete Information: An Infinite Horizon Model with Alternating Offers," *Review of Economic Studies*, 54, 175–192.
FERNANDEZ, R., AND J. GLAZER (1991): "Striking for a Bargain Between Two Completely Informed Agents," *American Economic Review*, 81, 240–252.
FUDENBERG, D., AND E. MASKIN (1986): "The Folk Theorem in Repeated Games with Discounting or with Incomplete Information," *Econometrica*, 54, 533–554.
GUL, F., AND H. SONNENSCHEIN (1988): "On Delay in Bargaining with One-Sided Uncertainty," *Econometrica*, 56, 601–612.
HALLER, H., AND S. HOLDEN (1990): "A Letter to the Editor on Wage Bargaining," *Journal of Economic Theory*, 52, 232–236.
NASH, J. (1950): "The Bargaining Problem," *Econometrica*, 18, 155–162.
——— (1953): "Two-Person Cooperative Games," *Econometrica*, 21, 128–140.
RUBINSTEIN, A. (1982): "Perfect Equilibrium in a Bargaining Model," *Econometrica*, 50, 97–109.
SELTEN, R. (1975): "Reexamination of the Perfectness Concept for Equilibrium Points in Extensive Games," *International Journal of Game Theory*, 4, 25–55.
SHAKED, A., AND J. SUTTON (1984): "Involuntary Unemployment as a Perfect Equilibrium in a Bargaining Model," *Econometrica*, 52, 1351–1364.
STÅHL, S. (1972): "Bargaining Theory," Economic Research Institute, Stockholm School of Economics.
WILSON, R. (1987): "Game-Theoretic Analyses of Trading Processes," in *Advances in Economic Theory*, ed. by T. Bewley. Cambridge: Cambridge University Press.