Two-Party Competition with Persistent Policies*

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Abstract

This paper studies the Markov perfect equilibrium outcomes of a dynamic game of electoral competition between two policy-motivated parties. I model *incumbent policy persistence*: parties commit to implement a policy for their full tenure in office, and hence in any election only the opposition party renews its platform. In equilibrium, parties alternate in power and policies converge to symmetric alternations about the median voter's ideal policy. Parties' disutility from opponents' policies leads to alternations that display *bounded extremism*; alternations far from the median are never limits of equilibrium dynamics. Under a natural restriction on strategies, I find that *robust* long-run outcomes display *bounded moderation*; alternations close to the median are reached in equilibrium only if policy dynamics start there. I show that these results are robust to voters being forward-looking, the introduction of term limits, costly policy adjustments for incumbents, and office benefits.

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1 Introduction

If the parties are viewed in [a] temporal framework, one may better appraise the old saw that the parties offer the electorate only a choice between tweedledum and tweedledee. In fact, the differences between the parties vary from stage to stage in the conversion of controversy into new consensus. (Key (1958), p.247.)

Political parties are long-lived organisations that compete over sequences of elections, yet the overwhelming majority of theoretical work on electoral competition has focused on static models. In particular, little is known about the dynamics of party platforms and associated patterns in government policies and alternation. In this paper, I let the asymmetric roles of incumbent and opposition generate the key dynamic linkage of a model of two-party elections in which (a) governments alternate, (b) parties compromise, that is, starting from differentiated ideological positions, they gradually move towards proposing platforms which resemble one another, yet (c) they never become as indistinguishable as tweedledum and tweedledee; party labels matter and parties maintain distinct policy goals. These are novel and inherently dynamic insights into partisan competition which highlight conditions under which, as Key advocates above, two opposed sets of standard results from static models can be bridged: party competition leads to gradual but limited convergence away from divergent outcomes.[1]

I formulate a dynamic game of policy competition between two ideological parties that have ideal (single-dimensional) policies on each side of that of the median voter. Voters are myopic and support the party whose current policy yields them higher utility. Under *incumbent policy persistence*, parties commit to enact specific policies for their entire *tenure* in office, as opposed to their current *term*. In each election, incumbents champion (or rather defend) the policies they implemented in their previous term, while opposition parties, released from their past commitments by electoral defeat, are free to choose a new platform. Opposition parties are forward-looking and understand that the platforms that carry them to office will support their bids for reelection. The key insights of my model make precise how successive opposition parties trade off winning current elections with policies they prefer against committing to more moderate policies in order to constrain their future opponents.

I focus on equilibria in Markov strategies, which depend on the outcomes of previous elections only insofar as these affect the state: the identity of the incumbent party and its policy. While the model admits a complex set of Markov perfect equilibria, its *long-run policy outcomes*, which are the limit points of equilibrium paths given some initial state, can be simply described. I show that all equilibria have (a) alternation in power and (b) bounded extremism in the long-run, while robust equilibria have (c) bounded moderation. From initial states that are sufficiently distant from the median, two-party competition always leads to some convergence. The bound on long-run extremism, which is driven by parties' incen-

[1]See Osborne (1995) for a survey of results from static models on policy convergence and divergence.

tives to impose moderation on their future opponents, is tight. In particular, the indefinite repetition of the median policy can occur in the long-run. However, median convergence is not a *robust* outcome of the model. Under a natural refinement of the set of Markov perfect equilibria, I show that alternations close to the median occur in the long-run only if policy dynamics start there. That is, while convergence *towards* the median is dynamically robust, convergence *to* (or near) the median is not and ideological differentiation is persistent. The reason for this is that gradual policy convergence, which consists of an alternating sequence of compromises by both parties, must be self-reinforcing. The benefit of committing to more moderate policies is that future opponents commit to even more moderate policies, while its costs are foregone policy gains in the current election. The incentives to sustain convergence unravel as policies approach the median, since when parties champion similar policies, discounting wipes out the benefits of opponents' future compromise. Lastly, the bound on robust long-run moderation is tight.

I build upon static models of policy competition with policy-motivated candidates based on Wittman (1983) and Calvert (1985), which also produce equilibrium outcomes that are bounded away from both the median and the extremes. Since their key mechanism is a trade-off between preferred policies and probability of winning, of which my paper's central trade-off is a dynamic variant, our bounds on extremism and moderation do share some key ideas. An innovative aspect of my paper, and one which suggests further research, is focusing on both policy outcomes and on the qualitative features of the dynamics that lead to them. Static models can account for persistent trends in parties' policy choices only through corresponding trends in party and voter preferences. My approach fixes preferences and hence all insights gleaned from the model's policy dynamics are tied directly to parties' intertemporal equilibrium calculations. The determination of the bounds on long-run policies depends crucially on dynamic considerations, with the bound on moderation in particular reflecting an explicit constraint on parties' ability to sustain continued compromise over time.

Proposition 2 shows that the equilibrium policy paths of my model support two distinct patterns of power and alternation. In the first case, the initial policy is absorbing and the incumbent remains in power forever. These trivial policy dynamics arise only if a leftist (rightist) incumbent party is implementing a policy to the right (left) of the median in the initial state, sapping the competitive incentives of its opposition. Otherwise, the party system is competitive, both parties hold office and successive opposition parties win elections by committing to increasingly moderate policies. Such policy dynamics converge to an alternation at policies symmetric about the median, and in the long run, incumbents are defeated by opposition parties that are equally preferred by the median voter.[2] Policies that are supported as symmetric alternations in the long-run of some equilibrium are the *long-run policy outcomes* of the model.

[2]Predictable left-right alternation is not an essential feature of my results, but is due to my spare modelling of policy persistence and the absence of any source of exogenous noise.

The policy dynamics of my model display gradual policy convergence and plausible patterns of alternation that persist in the long-run. Such qualitative results on the evolution of partisan competition have not been a focus of existing dynamic models of elections, although they have garnered interest among political scientists.[3] Recently, gradual policy divergence has received the most attention. Poole and Rosenthal (2007) document a sharp and consistent increase in the polarisation across party lines in the voting behaviour of Democratic and Republican members of the U.S. Congress since the mid-1970's. While no consensus exists about its causes, the trend runs sharply counter to that for the period running from the 1920's to the 1970's, which saw a consistent decrease in polarisation until around 1950 followed by a persistent levelling off.[4] A separate but voluminous literature argues that periods separating what Key (1955) has termed 'critical elections' in the U.S. reflect a process of stabilisation in which 'polarization gives way to conciliation. As it does, the parties move from the poles toward the center and the distance between them narrows.'[5] Meanwhile, in the U.K., the term 'Consensus Politics' was coined to label the post-WWII period of perceived policy convergence between the Labour and Conservative parties thought to have ended with the election of Margaret Thatcher in 1979.[6] My model highlights conditions that render gradual policy convergence salient: parties' policies are sticky and their policy adjustments are constrained (here incumbent policy persistence) and there is broad and stable agreement regarding many underlying issues, with electoral competition focused mainly on incremental policy shifts within that accepted framework (here stable voter preferences and a single-dimensional policy space). In such environments, centripetal forces are strong and party competition involves cycles of successive moderation.

The most appropriate interpretation of the model is that of parties vying for control of entire legislatures or executive positions, where elections are decided by aggregate issues and conditions. Hence, while policy persistence puts incumbent governments at a disadvantage, it has little to say about the much-discussed empirical phenomenon of incumbency advantage. The latter reflects district-specific benefits to entrenched incumbent candidates that, if incumbency advantage is present, swamp any disadvantage stemming from reduced policy flexibility with respect to challengers.[7] Even if incumbent legislators of all parties are advan-

[3] Budge et al. (2001) report the findings of the Manifesto Reseach Project, which codes party platform data for dozens of democracies in all elections since 1945 and whose purpose is precisely to gather information about party platform dynamics.

[4] This trend is also documented in Poole and Rosenthal (2007). See Fiorina (1999) for one survey of the numerous competing explanations of increased polarisation. The earlier period of decreased and then stable polarisation has escaped such scrutiny.

[5] Sundquist (1983), p.319. Key (1955) defines critical elections as 'a type of election in which there occurs a sharp and durable electoral realignment between parties' (p.16). A oft-cited example is the presidential election of 1932 that brought F.D. Roosevelt and the New Deal to power. Documenting the aftereffects of that election, Sundquist (1983) notes that 'as the polarization of the electorate that had characterised the depression years dissolved into the moderation of more prosperous times, the conflict between the parties was somewhat muted. They remained anchored on either side of the activist-conservative line of cleavage, but the distance between them that had been so great in the early 1930's diminished.' (p.337)

[6] See Dutton et al. (1997) and Kavanagh and Morris (1994).

[7] For the U.S., see Erikson (1971) and Gelman and King (1990). Evidence for the existence of incumbency

taged in their individual races it does not follow that incumbency advantage applies to entire governments.[8] Similary, in electoral systems in which multi-party coalitions are frequent, my model can shed some light on the competitive dynamics between the main left and right blocs, abstracting from the distribution of power among the parties composing them.

The rich dynamics of my model vanish if instead incumbent policy persistence is dropped and elections are modelled as a sequence of independent contests. Proposition 1, extending a standard static result from Calvert (1985), shows that the resulting repeated game generates trivial dynamics and outcomes: it has a unique subgame perfect equilibrium, and in this equilibrium both parties commit to the median policy after all histories.[9] Incumbent policy persistence is meant to capture the fact that (a) parties are not free to take up *any* ideological position but are associated with policies they have championed in the past and that (b) this affects incumbent parties disproportionately, which are both natural features of elections. First, a party's policies are often attributed to the politicians that currently represent it and whose policy preferences are usually publicly known and stable over their political careers.[10] Meanwhile, politicians representing the incumbent party are rarely replaced between terms in office, while defeat at the polls typically leads to a renewal in a party's representation and leadership. Second, renouncing previous commitments or admitting policy mistakes, popularly known as 'flip-flops', can have large electoral costs.[11] Since incumbent governments have a longer list of recent policy achievements they face more constraints on adjustments to their policy positions. As a third example, when voters adopt retrospective strategies they choose to disregard incumbents' promises of policy change. My assumption of incumbent policy persistence then states that while challengers are evaluated on their *promises*, incumbents are evaluated on their *records*.[12] Full intertemporal commitment to policies by incumbents allows a simple characterisation of equilibrium outcomes, but it is the asymmetry between incumbent and opposition parties which is critical for my results. In Section 5.2, I show that my results hold if parties commit to policies for only two periods (alternatively if incumbent

advantage outside the U.S. is mixed. A small effect has been identified in Canadian federal elections by Krashinsky and Milne (1985), an insignificant one for the British House of Commons by Gaines (1998) and a disadvantage to incumbency has been found to prevail in Indian state legislature elections by Uppal (2009).

[8]In fact, Muller and Strom (2000) find that European parties that participate in government coalitions lose seats on average in the elections following their stay in government. In the U.S., it is well established that a sitting president's party typically fares poorly in mid-term elections (see Tufte (1975), Erikson (1988), or Alesina and Rosenthal (1989)).

[9]Other applications of repeated games to electoral competition are not designed to study policy dynamics. Duggan and Fey (2006) show that *any* policy path can be enforced by some subgame perfect equilibrium of the repeated two-party Downsian model with forward-looking voters. Alesina (1988) asks whether (constant) policy paths that maximise parties' joint payoffs can be supported in equilibrium when parties are policy-motivated and can renege on their campaign announcements when in office.

[10]This interpretation is in the spirit of the 'citizen-candidate' models of Besley and Coate (1997) and Osborne and Slivinski (1996). Poole and Rosenthal (2007) provide evidence that individual politicians' policy preferences do not change over time, at least in the case of U.S. members of Congress.

[11]In a recent paper, DeBacker (2010) finds that U.S. senators face both fixed and convex costs to changing their positions.

[12]See Fiorina (1981). Miller and Wattenberg (1985) and Nadeau and Lewis-Beck (2008) find evidence that voters participating in U.S. presidential elections tend to evaluate incumbents retrospectively and challengers prospectively.

party representatives face a term limit of two) or if only incumbent parties bear fixed policy adjustment costs.

Under different equilibria, policy dynamics can converge to alternations at different policies. Proposition 3 shows that the set of long-run policy outcomes consists of all sufficiently moderate policy alternations. That is, extreme policies are transient and are not observed on equilibrium paths after enough elections. Policy persistence along with discounting ensure that opposition parties prefer some alternation to the repetition of the median, as when alternations are sufficiently moderate their gains from enacting policies on their side of the median dominates the discounted disutility of opponents' policies. I show that a tight upper bound on the extremism of any alternating outcome reached in the long-run is given by the *most moderate* of the preferred alternations of each party. This bound on long-run extremism identifies those policies that are sufficiently extreme that they provide incentives for some party to enact more moderate policies in order to rein in its future opponents. In time, government policies reflect the preferences only of the most moderate party irrespective of the party in power. That the bound on long-run extremism is tight follows from equilibrium construction.

Of the long-run outcomes of the model, some are reached only if they occur in the initial state. That is, such outcomes are never reached from more extreme states through sequences of elections decided by increasingly moderate policies. A *robust long-run policy outcome* is a long-run policy outcome that can be reached from some initial state with a policy that differs from the policy outcome itself. To study robust outcomes, I require that parties' strategies be *consistent*, a natural equilibrium refinement that rules out parties' conditioning their policies on 'payoff-irrelevant' events that survive the Markov restriction and allows simple characterisations of payoffs and policies on equilibrium convergence paths. Proposition 4 shows that the set of robust long-run policy outcomes under equilibria in consistent strategies consists of all alternating outcomes that are sufficiently extreme. This tight bound on the moderation of robust long-run outcomes is derived explicitly and is strictly away from the median. Ideologically differentiated parties stay differentiated: alternating outcomes close to the median are never reached by consistent equilibrium policy dynamics that start from more extreme states. Contrary to the bound on extremism, the bound on robust moderation reflects the preferences of both parties. This follows since gradual moderation must be self-reinforcing and on an equilibrium convergence path, moderate policy commitments are supported by opponents' promises of further moderate commitments in future elections. Parties that prefer more extreme alternations cannot prevent opponents from unilaterally ensuring that only sufficiently moderate policies win elections. However, they can balk at the moderate policies they would need to implement to sustain continued compromise.

Irrespective of the two parties' preferences, the incentives compromise unravel as convergence paths approach the median. In particular, I construct bounds on how much policy moderation each party is willing to implement at each step of a convergence path in response to an opponent's proposed moderate move in the next election. When policy dynamics are sufficiently close to the median, parties' 'demands' for moderation are incompatible. Discounting is critical to this result. As policies approach the median, comparable moderate moves by an opposition party and its opponent have similar effects (in absolute value) on its payoffs, yet the opportunity cost of compromise is borne in full today while the gain is discounted. Hence, convergence breaks down near the median since both parties require their opponents to bear most of the cost of sustaining it. The bound on robust long-run moderation is shown to be tight through equilibrium construction.

Propositions 6 and 7 contain the results on term limits and costly policy adjustments described above. While myopic voting is a plausible assumption in large elections and it has the benefit of focusing attention solely on the competition between the parties, Proposition 5 shows that all the equilibrium outcomes studied in the model with myopic voters persist in the model with forward-looking voters. This requires, for all equilibrium outcomes with myopic voting, the construction of equilibrium strategies for the parties that give forward-looking voters the incentive to sustain this outcome. Intuitively, this is possible since equilibrium paths under myopic voting are convergent, and hence acceptable to a forward-looking median voter. Another extension considers the case in which parties are not solely policy-motivated by also derive direct benefits from holding office. A standard result in both static and dynamic models of elections[13] shows that politicians that value office are more willing to compromise, establishing a link between office benefits and policy moderation. Proposition 8 shows that (a) as office benefits vanish the set of long-run policy outcomes converges to that identified in Propsition 3 for the case of pure policy motivation, but that, more interestingly, (b) the same is true as office benefits become arbitrarily large. That is, the sets of long-run policy outcomes when parties are purely policy-motivated and when they value office and policies lexicographically coincide, which implies no consistent relationship between office benefits and policy moderation. The key to this result is the observation that policy moderation need not lead to longer tenure in office when opponents also value office keenly, as they will respond with moderate policies of their own. If no party allows its opponent to capture office for long stretches through policy moderation, policy dynamics behave as though office benefits were irrelevant.

Dynamic models of asynchronous policy competition can be traced back to Downs (1957) and were first formally presented in Kramer (1977) and Wittman (1977). They study models similar to mine in which, crucially, parties are myopic. Their models differ from each other only in their assumptions about parties' preferences. Kramer (1977) assumes that parties are office-motivated and maximise votes, while Wittman (1977) assumes that parties are policy-motivated.[14] Neither of the myopic strategies derived from these two models would

[13]See, for example, Calvert (1985) and Duggan (2000).

[14]Related to these papers is the literature on competition between myopic adaptive parties, such as Kollman et al. (1992), Kollman et al. (1998), de Marchi (1999) and Laver (2005). Kollman et al. (1992) generate policy dynamics that moderate over time yet stay bounded away from the median in the long-run.

equilibria of the dynamic game with forward-looking parties. Given a fixed incumbent the myopic policy-motivated parties of Wittman (1977) commit to their preferred (most extreme) winning policy. However, when faced with a myopic incumbent whose policy is sufficiently extreme, a forward-looking opposition party finds it optimal to sacrifice present payoffs and commit to a moderate policy in order to face more moderate opponents in future elections.[15] On the other hand, a naive extension of median convergence results to my model has opposition parties commit to the median policy in all states, which are the optimal actions of the myopic vote-maximising parties of Kramer (1977). However, a forward-looking policy-motivated opposition party expecting future opponents that always select median policies has no incentive to win the current election with the median policy: the sole cost of winning an election with non-median policies is the extremism it may generate in opponents' future policies.

The idea that forward-looking incumbents have incentives to strategically position current policies to affect future political outcomes has had numerous applications.[16] Closer to my paper are the infinite horizon models of dynamic legislative bargaining and spatial electoral competition. In dynamic legislative bargaining models,[17] a legislator is recognised each period to propose some policy which is put to a vote against the status quo. Current policies persist by becoming next period's status quo. As opposed to my characterisation of equilibrium outcomes, papers on dynamic legislative bargaining typically study specific equilibria. The model of Baron (1996) is most closely related to mine. He characterises an equilibrium in which all policy paths converge to the median policy, which contrasts with the non-robustness of policy outcomes near the median in my model. His result follows from the median legislator eventually being recognised, proposing the median policy and never supporting anything other than the status quo in future periods.[18]

Dynamic models of electoral competition between candidates with privately known policy preferences generate incentives to choose moderate policies to maintain a reputation for moderate preferences.[19] In these models, candidate selection by parties is nonstrategic and candidates' informational advantage is derived from having been drawn at random from the

See also Anesi (2010), who shows that sets of long-run Markov equilibrium outcomes of the game of Kramer (1977) coincide with von Neumann-Morgenstern stable sets of policies.

[15] In fact, Proposition 3 establishes the precise (yet restrictive) condition under which myopic behaviour may be dynamically optimal.

[16] Alesina and Tabellini (1990) show how incumbents accumulate excessive public debt in order to 'tie the hands' of future governments that may not share their preferences over public goods spending. For a review of this literature, consult Persson and Tabellini (2000). Bai and Lagunoff (2009) also present a useful discussion of this literature in the context of their more general infinite horizon model.

[17] See Baron (1996), Baron et al. (2008), Bowen and Zahran (2009), Duggan and Kalandrakis (2009), Fong (2008), Kalandrakis (2004) and Kalandrakis (2007). Also related are Battaglini and Coate (2007) and Battaglini and Coate (2008).

[18] It can be shown that it is the assumption of the existence of a median legislator that is critical for median convergence in Baron (1996): in Section 5.4 I show that in the legislative bargaining version of my model in which two non-median legislators are ever recognised, robust convergence outcomes are still bounded away from the median.

[19] See Banks and Duggan (2008), Bernhardt et al. (2004), Bernhardt et al. (2009), Duggan (2000) and Kalandrakis (2009).

voting population or the party's membership,[20] while in my model parties can commit to any policy. In the absence of signalling by privately informed candidates, Van Weelden (2009) shows that similar intuition and dynamics can obtain. The policy dynamics in these models have very different features. Typically, a succession of defeated incumbents' policies are drawn at random and bear no relation to one another. In their simplest variant, these models do not generate alternation in the long-run; successive extreme incumbents survive for one term in office until a sufficiently moderate candidate is elected and survives all challenges.[21] Meanwhile, all incumbents are replaced on most equilibrium paths of my model; moderation does not guarantee reelection, since opponents can respond by championing more moderate policies themselves.

2 Model

Two parties, L and R, contest an infinite sequence of elections at times $t = 0, 1, ...$ Each period starts with the incumbent party $I \in \{L, R\}$ in power, and the remaining party in opposition. An election consists of a vote over which party should form the next government, with the winning party determined by majority rule. The opposition party $-I = \{L, R\}\setminus\{I\}$ commits to implementing a policy in the policy space $X = [0, 1]$, if elected, and for as long as it remains in power: this is the assumption of incumbent policy persistence. Hence, in any election, the incumbent's policy commitment is inherited from the election that brought it to power. A party may also choose not to participate in the election.

An odd number of voters have symmetric single-peaked preferences over policies, and their ideal policies are distributed over policy space X. Some policy M corresponds to the median of voters' ideal policies. Distance preferences for all voters ensure that the median voter is decisive in single elections. Voters are myopic and in all voting subgames, I restrict attention to the equilibrium in weakly undominated strategies in which voters support the party that will enact a policy closest to their ideal policy if brought to power in this election. As the median voter is decisive, the party whose policy is closest to M wins the election. I assume for simplicity that ties are broken in favour of the opposition party.[22]

To formalise the dynamic game, define a *state* (I, x), with $I \in \{L, R\}$ and $x \in X$, which records the identity of the incumbent party along with its policy commitment. Given a state

[20] Bernhardt et al. (2009) show that drawing opponents from opposite sides of the political spectrum (i.e., from different parties) makes incumbents more willing to compromise by lowering their continuation value if they lose office. Related my assumption of incumbent policy persistence, Kalandrakis (2009) assumes that a party that a party that has recently lost an election is more likely to field a candidate of a different preference type.

[21] Notable exceptions are Kalandrakis (2009), in which incumbent parties previously believed to be moderate are replaced when their preferences become extreme and they implement extreme policies, and Bernhardt et al. (2004), in which incumbents are term-limited.

[22] When the incumbent champions any policy other than M, this is the only tie-breaking rule consistent (in the limit) with the equilibrium paths of the model. Any rule that selects the incumbent with positive probability would lead the opposition party to prefer committing to an arbitrarily more moderate policy that wins with probability 1.

(I, x), the corresponding stage game is a single-agent decision problem with the following timing:

- The opposition party $-I$ commits to a policy $z \in X$, or does not contest the election, written $z = Out$.
- Elections are held. Party I wins if and only if $|x - M| < |z - M|$.
- Parties L and R have single-peaked preferences over policies around 0 and 1 and represented by u_L and u_R respectively. Suppose, without loss of generality, that $M \leq \frac{1}{2}$, so that party L is (weakly) favoured by the median voter. Assume that $u_L(0) = u_R(1) = 0$, u_L (u_R) is strictly decreasing (increasing), twice continuously differentiable and strictly concave.

It is not critical that parties' ideal policies are located at the extremes of the policy space, only that these be on opposite sides of M. Concavity simplifies the results but can be relaxed. It captures two key features of parties' payoffs: the benefits of policy compromise by a party's opponent always more than offset its loss from its own compromise, and parties are more willing to compromise when facing extreme policies. Given state (I, x), let $W(I, x)$ be the *set of winning policies* for the opposition party. Note that for any $x \in X$ and $J \in \{L, R\}$, $W(J, x) = [\min\{2M - x, x\}, \max\{2M - x, x\}]$ and $W(J, x') \subset W(J, x)$ whenever $|x' - M| < |x - M|$.

Transitions between states are given as follows: the current period's winning party and policy become next period's incumbent party and incumbent policy, respectively. Formally, define the *state transition function* $\tau : (\{L, R\} \times X) \times (X \cup \{Out\}) \to \{L, R\} \times X$ by

$$\tau((I, x), z) = \begin{cases} (I, x) & \text{if } |x - M| < |z - M| \text{ or } z = Out, \\ (-I, z) & \text{if } |x - M| \geq |z - M|. \end{cases}$$

The dynamic game proceeds as follows: given some initial state (I, x), the two parties take part in an infinite sequence of elections, where the transition between stage games is given by τ. A *history starting from* (I, x) is a sequence $\{(I^i, x^i)\}_{i=1}^{N} \in (\{L, R\} \times X)^N$ with $N \leq \infty$ such that $(I^1, x^1) = \tau((I, x), z)$ and $(I^i, x^i) = \tau((I^{i-1}, x^{i-1}), z^i)$ for $i > 1$ for some $z, z^i \in X \cup \{Out\}$. The payoff to party J from terminal history $\{(I^i, x^i)\}_{i=1}^{\infty}$ starting from (I, x) is

$$\sum_{i=1}^{\infty} \delta_J^{i-1} u_J(x^i),$$

where $\delta_J < 1$ is party J's discount factor.

I restrict attention equilibria in Markov strategies. My aim is to shed light on the properties of parties' long-run interactions, for which it is natural to limit implicit equilibrium coordination and assume that challengers' behaviour depends on incumbents' policies only insofar as they affect available winning policies. Parties square off in elections that are years apart and often involve different politicians, so that strategies that with all else equal differentiate between events that occurred even a few elections ago would have problematic interpretations.

Definition 1. A *Markov strategy* for party J is a function $\sigma_J : \{L, R\} \times X \to X \cup \{Out\}$, with the restriction that $\sigma_J(J, x) = x$ for all $x \in X$.

The restriction captures the assumption of incumbent policy persistence. Let Σ_J be the set of Markov strategies for party J. Henceforth, the term strategy refers to a Markov strategy. While the restriction to pure strategies affects the set of equilibria of the game, it does not affect the set of long-run policy outcomes, as will be clear given the results of Proposition 2. With slight abuse of notation, the *state path* $\{(I^i, x^i)\}_{i=1}^{\infty}$ *induced by profile* (σ_L, σ_R) *starting from* (I, x) is defined recursively by

$$(I^1, x^1) = \tau((I, x), \sigma_{-I}(I, x)),$$
$$(I^i, x^i) = \tau((I^{i-1}, x^{i-1}), \sigma_{-I^{i-1}}(I^{i-1}, x^{i-1})).$$

The *policy path* $\{x^i\}_{i=1}^{\infty}$ *induced by* (σ_L, σ_R) *starting from* (I, x) is the policy sequence of the corresponding state path. Discounted payoffs to party $J \in \{I, -I\}$ from policy path $\{x^i\}_{i=1}^{\infty}$ induced by (σ_L, σ_R) starting from (I, x) are given by

$$V_J(\sigma_L, \sigma_R; (I, x)) = \sum_{i=1}^{\infty} \delta_J^{i-1} u_J(x^i).$$

Definition 2. A *Markov perfect equilibrium* is a strategy profile (σ_L, σ_R) such that, for each state (R, r),

$$\sigma_L(R, r) \in \arg\max_{\sigma'_L \in \Sigma_L} V_L(\sigma'_L, \sigma_R; (R, r)),$$

and for each state (L, ℓ)

$$\sigma_R(L, \ell) \in \arg\max_{\sigma'_R \in \Sigma_R} V_R(\sigma_L, \sigma'_R; (L, \ell)).$$

Henceforth, the term equilibrium refers to Markov perfect equilibrium.

3 Outcomes Without Incumbent Policy Persistence

When incumbents are primarily occupied with defending previous terms' policies, competition is transfered from *within* to *across* elections. This dampens the incentives that lead to median convergence in standard models. To illustrate this, consider the repeated game in which incumbent and opposition parties simultaneously commit to policies. This stage game is the standard model of electoral competition between policy-motivated parties and as is well known, it has a unique Nash equilibrium in which each party commit to the median policy.[23] Call the repeated simultaneous move game the *model without incumbent policy persistence*. Proposition 1 shows that only one of the long-run equilibrium outcomes of the model with incumbent policy persistence, that of full median convergence, arises in the absence of this assumption.

Proposition 1. *In the unique subgame perfect equilibrium of the model without incumbent policy persistence, parties commit to the median policy after all histories.*[24]

In the model without incumbent policy persistence, both parties can enforce continuation policy path $(M, M, ...)$ after all histories. In the model with incumbent policy persistence policy choices are asynchronous and party J can enforce policy path $(M, M, ...)$ *only* following histories in which it can commit to new policies. In an equilibrium in which parties alternate in office, this opportunity arises every other period. While parties can be worse off relative to policy path $(M, M, ...)$ as an incumbent, their gain from accessing office with their preferred policies is sufficient to balance this (discounted) loss. Proposition 1 shows that policy paths under policy persistence exhibit a form of dynamic inconsistency for incumbents since if they could, they would prefer to free themselves of their record and commit to the median policy.

4 Outcomes With Incumbent Policy Persistence

The restriction to Markov strategies does not eliminate equilibrium multiplicity, and the model's set of equilibria admits no simple description. I focus instead on characterising equilibrium outcomes, and in particular those that persist in the long-run. Long-run policy outcomes are defined, naturally, as limit points of sequences of policies induced by equilibrium dynamics from some initial state.

Definition 3. Policy y is a *long-run policy outcome under equilibrium* (σ_L, σ_R) *starting from* (I, x) if y is a limit point of the policy path induced by (σ_L, σ_R) starting from (I, x).

A policy that is a long-run policy outcome under some equilibrium starting from some state is called simply a *long-run policy outcome*.

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[23] See Calvert (1985).

[24] All proofs of my results are in the Appendix. The proof of Proposition 1 is not trivial since parties have different discount factors.

4.1 Equilibrium Policy Dynamics: Alternation

Proposition 2 characterises equilibrium dynamics along with the properties of their limit points.

Proposition 2. *Consider some equilibrium (σ_L, σ_R) and some state (I, x) along with the policy path $\{y^i\}$ induced by (σ_L, σ_R) starting from (I, x). Suppose that $(I, x) = (R, r)$.*

i. If $r \leq M$, then $y^i = r$ for all i.

ii. If $r > M$, then a) incumbents are always defeated on the equilibrium path, unless $y^i = M$ for some i, b) $\{y^i\}$ has a pair of limit points $(\hat{\ell}, 2M - \hat{\ell})$ for some $\hat{\ell} \leq M$, and c) $\sigma_L(R, 2M - \hat{\ell}) = \hat{\ell}$ and $\sigma_R(L, \hat{\ell}) = 2M - \hat{\ell}$.

The case of $(I, x) = (L, \ell)$ is symmetric.

In any equilibrium, party L will stay *Out*, or commit to some losing policy, whenever (R, r) is such that $r < M$, that is, when party R is on the left of the political spectrum. The policy path most favourable to L that can be sustained in any equilibrium from such a state is $(r, r, r, ...)$, which L can attain by failing to contest any election and trapping dynamics at the initial policy. Since item i of Proposition 2 shows that all policies can be reached by some equilibrium dynamics, I restrict attention to policies that can be reached by nontrivial dynamics. Call policy outcome $y \neq M$ *trivial* if it is a long-run policy outcome under (σ_L, σ_R) starting from (I, x) if and only if $y = x$ and the policy path $\{x^i\}$ induced by (σ_L, σ_R) from (I, x) is such that $x^i = y$ for all $i \geq 1$. From now on, the term long-run policy outcome refers to a long-run policy outcome that is not trivial.

Item ii of Proposition 2 ensures that nontrivial equilibrium dynamics entail alternation in power and convergence to symmetric pairs of policies of the form $(\ell, 2M - \ell)$ for some $\ell \leq M$. Figure 1 illustrates this result, depicting a possible policy path induced by some equilibrium profile from state (R, r) with $r > M$. On the equilibrium path, no party stays *Out* or commits to policies that either lose or are on their opponent's side of the median. The policy path alternates around the median and has at most a pair of limit points $(\hat{\ell}, \hat{r})$ since the sequences of each party's winning policies are monotone. The pair of long-run policies $(\hat{\ell}, \hat{r})$ need not be reached by the policy path. Furthermore, it must be that $\hat{r} = 2M - \hat{\ell}$. The final component of item ii of Proposition 2 states that limits of alternating equilibrium dynamics are absorbing; if the dynamics start at one of the limiting policies, they stay there.

The proofs of Proposition 2 and of the results to follow depend only on properties of parties' preferences over symmetric policy alternations, which vary according to the initial policy. To clarify this, define the functions $\{U_L^\theta : [0, M] \to \mathbf{R}\}_{\theta \in \{+,-\}}$ for party L as

$$U_L^+(\ell) = u_L(\ell) + \delta_L u_L(2M - \ell), \text{ and}$$
$$U_L^-(\ell) = u_L(2M - \ell) + \delta_L u_L(\ell).$$

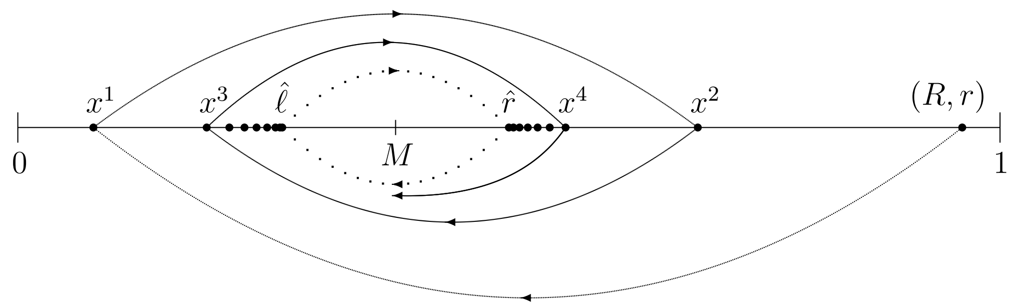

Figure 1: Illustration of Equilibrium Policy Dynamics.

The discounted sum $\frac{1}{1-\delta_L^2}U_L^+(\ell)$ is party L's payoff from alternation at policies $(\ell, 2M-\ell)$ *starting from* ℓ, while $\frac{1}{1-\delta_L^2}U_L^-(\ell)$ is its payoff to the same alternation when *starting from* $2M-\ell$. Functions $\{U_R^\theta : [0, M] \to \mathbf{R}\}_{\theta\in\{+,-\}}$ for party R are defined symmetrically. Strict concavity of parties' utility functions yields a natural preference order over symmetric alternations, whose properties are collected in the following lemma.

Lemma 1. *There exist uniquely defined policies* ℓ^* *and* r^* *such that*

$$\ell^* = \arg\max_{\ell\in[0,M]} U_L^+(\ell) \in [0, M), \text{ and}$$
$$r^* = 2M - \arg\max_{\ell\in[0,M]} U_R^+(\ell) \in (M, 2M].$$

U_L^- *(*U_R^-*) is strictly increasing (decreasing) and both* U_J^+ *and* U_J^- *are strictly concave for all* $J \in \{L, R\}$.

Given $\ell \in [0, M)$, the concavity of u_L ensures that the cost to L of a moderate move away from ℓ is dominated by the benefit of a moderate move away from $2M-\ell$. That U_L^+ is single-peaked around $\ell^* < M$, *L's favoured alternation*, follows from discounting. When the payoff to L from alternating pairs are evaluated starting from L's policy, a shift to a more moderate alternation ensures that party L suffers the full loss to moderation in its own policy, while the larger benefit of R's moderation is discounted. For any $\delta_L < 1$, ℓ^* is bounded away from the median as $\lim_{\ell\nearrow M} u_L(\ell) = \lim_{\ell\nearrow M} u_L(2M-\ell)$. Policies ℓ^* and r^* are key in the characterisation of long-run policy outcomes. Meanwhile, when the payoffs to L from alternations are evaluated starting from R's policy, L always prefers more moderate alternations. In particular, L's favoured alternation is that around M since L's loss from moderating its own policy, smaller than L's gain from R moderating its policy, is discounted.

4.2 Long-Run Policy Outcomes: Bounded Extremism

Proposition 3 shows that long-run policy outcomes admit a simple characterisation and display *bounded extremism*. That is, while sufficiently extreme policies can be observed on some equilibrium paths, they are transient.

Proposition 3. *Policy $\ell \leq M$ is a long-run policy outcome if and only if $\ell \in [\max\{\ell^*, 2M - r^*\}, M]$.*

Figure 2, illustrates Proposition 3 when $\ell^* \geq 2M - r^*$. The dotted section of the policy space indicates the set of long-run policy outcomes. All symmetric policy pairs more moderate than $(\ell^*, 2M - \ell^*)$, such as (ℓ, r) and (ℓ', r'), are long-run policy outcomes, with $(\ell^*, 2M - \ell^*)$ being the most extreme such pair.

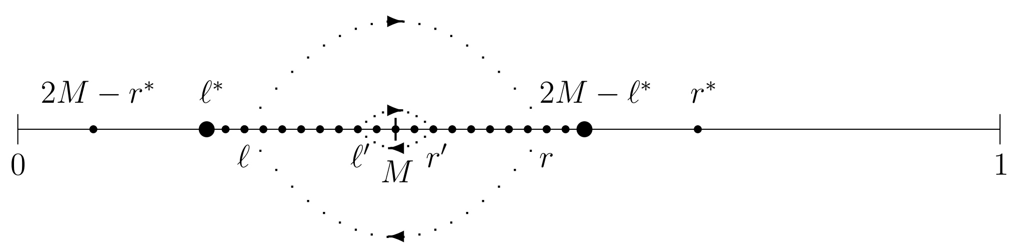

Figure 2: Set of Long-run Policy Outcomes.

The bound on long-run extremism follows since when facing a sufficiently extreme alternation (in the long-run), some party will prefer to rein in future opponents' policies by committing to more moderation. The policy $\max\{\ell^*, 2M - r^*\}$ indexes the most extreme alternation that is not subject to such incentives to unilateral moderation. The comparative statics of the set of long-run policy outcomes depend on the properties of policies ℓ^* and r^*.

Corollary 1. *The set of long-run policy outcomes has the following properties.*

i. *If v_J is obtained from u_J by a concave transformation, then* $[\max\{\ell^*, 2M - r^*\}, M]|_{v_J} \subseteq [\max\{\ell^*, 2M - r^*\}, M]|_{u_J}$.

ii. *If $\delta'_J > \delta_J$, then* $[\max\{\ell^*, 2M - r^*\}, M]|_{\delta'_J} \subseteq [\max\{\ell^*, 2M - r^*\}, M]|_{\delta_J}$.

iii. $\lim_{\delta_L \to 1}[\max\{\ell^*, 2M - r^*\}, M] = \lim_{\delta_R \to 1}[\max\{\ell^*, 2M - r^*\}, M] = \{M\}$.

iv. $\lim_{\delta_J \to 0}[\max\{\ell^*, 2M - r^*\}, M] = [\ell^* \mathbf{1}_{J=R} + 2M - r^* \mathbf{1}_{J=L}, M]$.

When $\ell^* > 0$, it is uniquely determined by $\frac{u_L'(\ell^*)}{u_L'(2M-\ell^*)} = \delta_L$, is increasing in δ_L and converges to M as δ_L converges to 1. As party L becomes less short-sighted, the cost of R's future policies increases and its preferred alternation comes closer to the median. Similarly, ℓ^* is increasing in L's disutility for policies away from its ideal point, captured by the concavity of u_L. The discount factor δ_L can be interpreted to reflect a host of institutional features that drive the 'farsightedness' of party L, such as (a) better control by party elites of either representatives' actions when in office or the party base at the candidate nomination stage, (b) longer terms in office, (c) the expected tenure of party leaders or (d) the tightness of the bonds between parties and their representatives following their terms in office (e.g., provision of employment within the party). Similarly, the 'concavity' of u_L can reflect the intensity of partisanship or institutional factors within the party that facilitate or inhibit compromise.

Proposition 3 and Corollary 1 make precise how long-run deviations from the median are driven by parties' trade-off between myopically leaning towards their preferred policies and farsightedly preempting their opponents' own leanings. The policies observed in the long-run are not determined symmetrically by both parties' preferences, but rather by the preferences of the party most willing to compromise. If some party is arbitrarily myopic, then the set of long-run outcomes is determined solely by the preferences of its opponent. If, on the other hand, only one party is arbitrarily far-sighted, policies are arbitrarily close to the median in the long-run.[25]

There are two steps to the proof of Proposition 3. The first establishes the existence of the bound on extremism, given by $\max\{\ell^*, 2M - r^*\}$. This step hinges on a useful lower bound on party L's equilibrium payoff: any equilibrium path following a commitment to some winning policy ℓ yields a payoff of at least $\frac{1}{1-\delta_L^2}U_L^+(\ell)$. To see this, consider a strategy for opposition party L which sets policy ℓ in the current election and responds myopically to all of R's subsequent policies. The payoff to L from this strategy is $u_L(\ell)$ in this election, along with a sequence of payoffs $\{U_L^-(r^i)\}$ in the subsequent pairs of elections, for some sequence of policies $\{r^i\}$ such that $\ell \leq 2M - r^i$ for all i. By Lemma 1, each payoff in this sequence is at least $U_L^-(\ell)$ and hence the payoff to selecting winning policy ℓ must be at least $\frac{1}{1-\delta_L^2}U_L^+(\ell)$. If $\ell < \ell^*$ were a long-run policy outcome, party L could win the election in state $(R, 2M - \ell)$ by committing to policy ℓ^* and guarantee itself a payoff of at least $\frac{1}{1-\delta_L^2}U_L^+(\ell^*)$, its preferred alternation. However, L's equilibrium payoff in state $(R, 2M - \ell)$ is $\frac{1}{1-\delta_L^2}U_L^+(\ell)$, yielding the desired contradiction.

The second step in the proof of Proposition 3 shows that the bound on long-run extremism is tight by constructing an equilibrium under which all policies $\ell \in [\max\{\ell^*, 2M - r^*\}, M]$

[25]Dynamic models of elections with private candidate preferences following Duggan (2000) also lead to more compromise as discount factors increase. In Alesina (1988), as in standard 'folk theorems', the set of equilibrium outcomes is larger for larger discount factors.

are long-run policy outcomes. Consider the strategy $\sigma_L^{\ell^*}$ such that

$$\sigma_L^{\ell^*}(R,r) = \begin{cases} \ell^* & \text{if } r \in [2M-\ell^*, 1], \\ 2M-r & \text{if } r \in [M, 2M-\ell^*), \\ Out & \text{if } r \in [0, M), \end{cases}$$

as well as myopically optimal strategy for party R, σ_R^{my}, such that

$$\sigma_R^{my}(L,\ell) = \begin{cases} 2M-\ell & \text{if } \ell \geq M, \\ Out & \text{if } \ell > M. \end{cases}$$

In the Appendix, I show that if $\ell^* \geq 2M - r^*$, then $(\sigma_L^{\ell^*}, \sigma_R^{my})$ is an equilibrium. If $\ell^* < 2M - r^*$, strategies $\sigma_R^{r^*}$ and σ_L^{my} can be defined by reversing the roles of the two parties and then $(\sigma_L^{my}, \sigma_R^{r^*})$ is an equilibrium.[26] Figure 3, depicting the interval $[0, M]$, illustrates equilibrium strategies $(\sigma_L^{\ell^*}, \sigma_R^{my})$. The directed curve above (below) the interval from point ℓ represents the equilibrium action of party L (R) in state $(R, 2M-\ell)$ ((L,ℓ)). In equilibrium, from any (L,ℓ) with $\ell < \ell^*$ or (R,r) with $r > 2M - \ell^*$, policies settle on alternation $(\ell^*, 2M - \ell^*)$ in at most two elections.

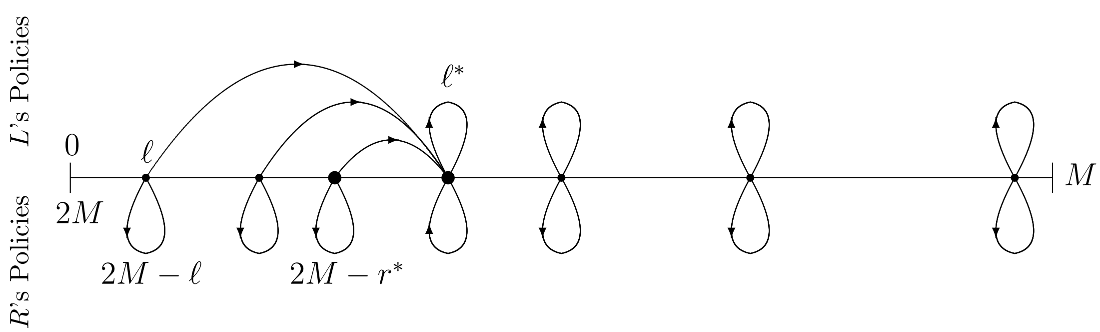

Figure 3: Policy Dynamics of Equilibrium $(\sigma_L^{\ell^*}, \sigma_R^{my})$.

In moderate states (L,ℓ) for some $\ell \geq \ell^*$ and (R,r) for some $r \leq 2M - \ell^*$, both parties respond myopically. In these states their preferences over alternations coincide; both prefer more extreme alternations when evaluated starting from their own policy. Parties' preferences over alternations also coincide in extreme states (L,ℓ) for some $\ell < 2M - r^*$ and (R,r) for some $r > r^*$. In these states, both parties prefer more moderate alternations starting from

[26]These equilibria provide the exact condition under which parties' behaviour in Wittman (1977) can be said to be dynamically rational: myopically optimal strategies form an equilibrium if and only if $\max\{\ell^*, 2M - r^*\} = 0$.

their own policy. However, having both parties committing to more moderate policies cannot be an equilibrium and some party, in this case L, must be responsible for bringing policy dynamics towards more moderate alternations. Since party R knows party L will commit to ℓ^* in the next election against any winning policy $r \in [2M - \ell^*, 2M - \ell]$ it champions in the current election, committing to myopic policy $2M - \ell$ is optimal. For intermediate states (L, ℓ) for some $\ell \in [2M - r^*, \ell^*)$ and (R, r) for some $r \in (2M - \ell^*, r^*]$, parties' preferences over alternations diverge and party L, which prefers more moderate pairs, ensures that policy paths converge.

4.3 Robust Long-run Policy Outcomes: Bounded Moderation

A long-run policy outcome y is the limit of equilibrium policy dynamics given *some* initial state. In this section, I investigate the qualitative properties of the equilibrium policy paths that support y as a long-run policy outcome. In particular, 'steady state' outcome y need not be dynamically stable in the following sense: given an initial state with policy more extreme than y, equilibrium policy dynamics need not have y as a limit point. For example, in the equilibrium $(\sigma_L^{\ell^*}, \sigma_R^{my})$, all policies $\ell \in (\ell^*, M]$ occur in the long-run only starting from (L, ℓ) or $(R, 2M - \ell)$.

Definition 4. Policy y is a *robust long-run policy outcome* if it is a long-run policy outcome under some equilibrium (σ_L, σ_R) starting from some state (I, x) such that x is not a long-run policy outcome under (σ_L, σ_R) starting from (I, x).

Long-run policy outcomes that are not robust are poor predictions of equilibrium play since they fail to arise given any different initial state. Robustness is a weak requirement of dynamic stability as it necessitates only the existence of a single policy x that lies on an equilibrium path that has y as a limit point.[27]

Verifying robustness for arbitrary Markov perfect equilibria is difficult as it requires a general characterisation of equilibrium convergence paths. To understand the difficulty, fix a particular equilibrium convergence path along with a state. The opposition party may have multiple best-responses following a deviation by the incumbent to a 'nearby' state which leaves its policy options 'essentially' unchanged. The opposition party can use such alternative best-responses to coordinate onto a new convergence path whose properties need not be closely related to those of the original convergence path. The restriction to Markov strategies should rule out such off-path equilibrium coordination, but fails to do so.[28] This suggests that a refinement of Markov equilibrium is called for.

[27]Note that trivial long-run policy outcomes are not robust.

[28]My model thus provides an example of a game in which taking the state to be the coarsest partition of strategically equivalent histories is not sufficient to rule out coordination on 'payoff-irrelevant' events (see Fudenberg and Tirole (1991)). The definition of the state cannot be refined through the coarsest common consistent partition of histories from Maskin and Tirole (2001). Since parties never move simultaneously after any history, they need not share a common consistent partition, and the results of Maskin and Tirole (2001) do not yield more than strategic equivalence in my model.

The asynchronous structure of my model suggests a natural refinement that eliminates coordination on 'payoff-irrelevant' events. In each state, the opposition party solves a single-agent decision problem, so that one can require that its strategy not lead to choice behaviour that would be labelled as inconsistent according to elementary concepts in decision theory. In particular, an opposition party should not condition on the exact policy of the incumbent when choosing policies in the interior of its set of winning policies. A party which commits to a moderate policy is unconstrained by the incumbent's policy, and hence facing a slightly more moderate incumbent should not lead it to change its policy choice. For example, suppose that party L chooses winning policy $\ell > 2M - r$ from set of winning policies $[2M - r, r]$ for some $r > M$. Consistency of choice as defined by the Weak Axiom of Revealed Preference requires that Party L choose the same policy from a set of winning policies $[2M - r', r']$ for some $r' \in [M, r)$ such that $2M - r' < \ell$. The choice of any other policy from the smaller set of winning policies could be justified by equilibrium considerations, but not by any fundamental political constraints. Requiring that parties' choices not display these types of inconsistencies is precisely what is needed to eliminate the patterns of equilibrium indifference that can lead to complex coordination off the equilibrium path. This, in turn, allows thorough treatment of robust long-run policy outcomes.

Definition 5. Markov strategy σ_{-I} is *consistent* if for any pairs of states (I, x) and (I, x'), whenever

i. $\tau((I, x), \sigma_{-I}(I, x)) = \tau((I, x'), \sigma_{-I}(I, x))$, and

ii. $\sigma_{-I}(I, x) \neq \sigma_{-I}(I, x')$,

then $\tau((I, x), \sigma_{-I}(I, x')) \neq \tau((I, x'), \sigma_{-I}(I, x'))$.

A *consistent Markov perfect equilibrium* is a Markov perfect equilibrium in consistent Markov strategies.[29]

Note that if $\tau((I, x), z) = \tau((I, x'), z)$ for some opposition party policy z that is winning in both states (I, x) and (I, x'), then the sequences of policies induced by z are the same in both states. Hence, Definition 5 states that if $\sigma_{-I}(I, x)$ induces identical outcomes in both states (I, x) and (I, x') and $\sigma_{-I}(I, x)$ is not chosen in state (I, x'), then $\sigma_{-I}(I, x')$ cannot induce identical outcomes in both states. Consistency defines a history to be 'payoff-irrelevant' if it is *revealed* to be irrelevant by a party's strategy at some other history. In the example above, party L *reveals*, through its choice of ℓ in state (R, r), that states (R, x) for $x \in [2M - \ell, r)$ are of no strategic importance.

Proposition 4 characterises robust long-run outcomes under consistent strategies and shows that they display *bounded moderation*. This does not contradict the results of Section 4; centripetal forces are present and policy paths tend to converge *toward* the median. However,

[29] A class to which, notably, the equilibrium $(\sigma_L^{\ell^*}, \sigma_R^{my})$ belongs.

policies do not converge *to* the median. The model admits median politics as a long-run policy outcome only if the initial incumbent party champions the median, otherwise parties remain differentiated and settle into clearly defined party identities.

Proposition 4. *There exists $\ell^{**} \in (\max\{\ell^*, 2M - r^*\}, M)$ such that policy $\ell \leq M$ is a robust long-run policy outcomes in consistent Markov strategies if and only if $\ell \in [\max\{\ell^*, 2M - r^*\}, \ell^{**}]$.*

Figure 4 illustrates Proposition 4. The dotted line indicates the set of long-run policy outcomes, while the dashed line indicates the subset of these pairs that are robust under equilibria in consistent strategies. For example, both policies in pair (ℓ, r) are robust, while policies in pair (ℓ', r'), more moderate than $(\ell^{**}, 2M - \ell^{**})$, are not.

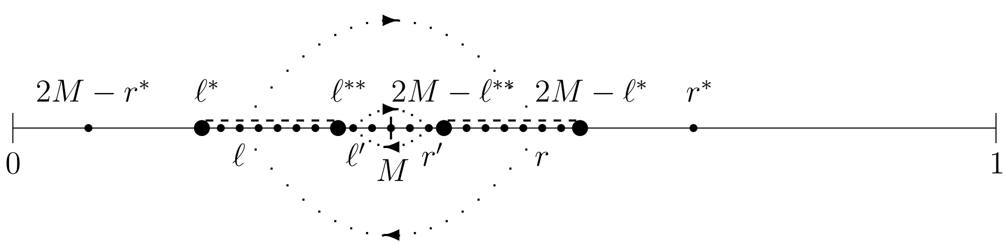

Figure 4: Set of Robust Long-run Policy Outcomes under Consistent Equilibria.

On equilibrium convergence paths a party's commitment to a more moderate policy must be reciprocated in future elections by its opponent. When converging to sufficiently moderate policy alternations, parties' value their opponents' (discounted) moderate moves so little that they are unwilling to commit to policies moderate enough to sustain convergence. Policy ℓ^{**}, whose properties are discussed below, is the most moderate policy that gives parties sufficient incentives to participate in these successive rounds of compromise.

Corollary 2. *The set of robust long-run policy outcomes in consistent Markov strategies has the following properties.*

i. *If v_J is obtained from u_J by a concave transformation, then $[\max\{\ell^*, 2M - r^*\}, \ell^{**}]|_{v_J} > [\max\{\ell^*, 2M - r^*\}, \ell^{**}]|_{u_J}$, where $\geq$ is the weak set order.*

ii. *If $\delta'_J > \delta_J$, then $[\max\{\ell^*, 2M - r^*\}, \ell^{**}]|_{\delta'_J} > [\max\{\ell^*, 2M - r^*\}, \ell^{**}]|_{\delta_J}$.*

iii. $\lim_{\delta_L \to 1}[\max\{\ell^*, 2M - r^*\}, \ell^{**}] = \lim_{\delta_R \to 1}[\max\{\ell^*, 2M - r^*\}, \ell^{**}] = \{M\}$.

iv. $\lim_{\delta_J \to 0}[\max\{\ell^*, 2M - r^*\}, \ell^{**}] = \{\ell^* \mathbf{1}_{J=R} + 2M - r^* \mathbf{1}_{J=L}\}$.

If a party is less myopic, its preferred alternation is more moderate but it is also more willing to compromise to achieve more moderate convergence outcomes, so that ℓ^{**} also moves towards the median. Hence, less myopic parties shifts the whole set of robust long-run policy outcomes toward the median. As for the set of long-run policy outcomes, the set of robust outcomes treats the preferences of the parties asymmetrically. If a single party is arbitrarily farsighted, the set of robust outcomes collapses onto the median policy. If, on the other hand, a single party is arbitrarily myopic, then the set of long-run policy outcomes is determined by the preferred alternation of the more farsighted party but the set of robust outcomes collapses to this alternation. While the myopic party cannot affect the set of long-run policy outcomes, it refuses to participate in any converging policy paths.

When studying the convergence outcomes of the model, it is convenient to focus on the symmetric images of party R's policies with respect to the median, mapping converging dynamics into a single increasing sequence of policies. The *convergence path* $\{y^i\}$ *to policy* $\hat{\ell} \in (0, M]$ *under equilibrium* (σ_L, σ_R) *starting from* (I, x) is a sequence such that

i. If $(I, x) = (R, r)$ for some $r > 2M - \hat{\ell}$, then $y^i = x^i$ for i odd and $y^i = 2M - x^i$ for i even, where $\{x^i\}$ is the sequence of policies induced by (σ_L, σ_R) starting from (I, x).

ii. If $(I, x) = (L, \ell)$ for some $\ell < \hat{\ell}$, then $y^i = x^i$ for i even and $y^i = 2M - x^i$ for i odd.

iii. $\{y^i\} \to \hat{\ell}$.

Consistent strategies allow simple characterisations of parties' policy choices and payoffs on equilibrium convergence paths. Lemma 3 in the Appendix characterises strategies along convergence paths in consistent strategies and is illustrated in Figure 5, showing a section of some convergence path $\{y^i\}$ initiated by party R committing to policy $2M - y^i$, to which L responds by moderating to y^{i+1}. By consistent strategies, $\sigma_L(R, r) = y^{i+1}$ for all $r \in (2M - y^i, 2M - y^{i+1}]$, that is, L moderates to y^{i+1} when facing an incumbent R championing a policy more moderate than $2M - y^i$. Furthermore, consistency implies that $\sigma_R(L, \ell) = 2M - \ell$ for all $\ell \in [y^i, y^{i+1})$, that is, R responds myopically whenever L stops short of moderating to y^{i+1}.[30]

As noted in the discussion of the myopic vote-maximising strategies of Kramer (1977), it is optimal to respond myopically to an opponent that always selects the median policy. Figure 6 shows that consistent equilibria display this behaviour locally. That is, consistent equilibrium convergence paths define alternating sets of policies in which a locally myopic party meets

[30]Note that if policy $\hat{\ell} \leq M$ is a robust long-run policy outcome under consistent equilibria and $y^i < \hat{\ell}$ is on a convergence path to $\hat{\ell}$, then all $\ell \in (y^i, \hat{\ell})$ are also on a convergence path to $\hat{\ell}$. In this sense, convergence outcomes under consistent equilibria can be said to be 'strongly' robust since convergence to $\hat{\ell}$ occurs from all more extreme states.

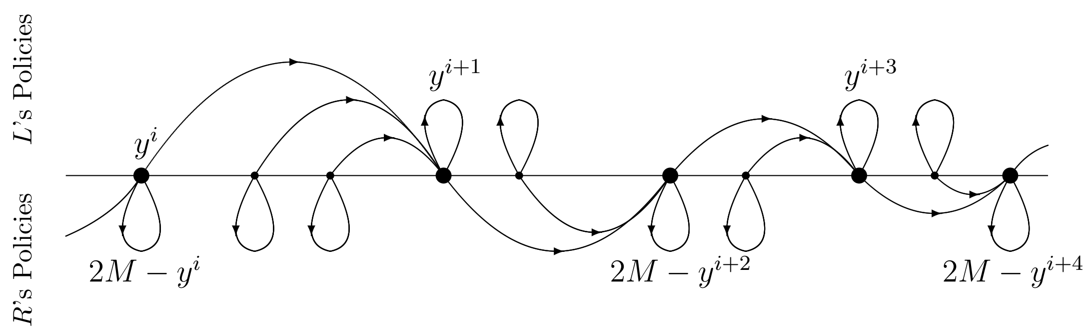

Figure 5: Convergence Paths under Consistent Equilibria.

a locally moderate party. Parties stake out non-negotiable 'core' issues and their opponents compromise on the corresponding policies on the other side of the median. The location of parties' core issues may seem idiosyncratic since they compromise over neighbouring policies. However, core issues are not due to parties' preferences for particular policies but arise endogenously as a tool to sustain policy convergence.

Section 4.2 noted that $\frac{1}{1-\delta_L^2}U_L^+(y^i)$ is a lower bound on L's payoff in state $(R, 2M - y^i)$. Lemma 4 in the Appendix shows that if $2M - y^i$ lies on a consistent equilibrium convergence path then this payoff is also an upper bound. That is, L's payoff at $(R, 2M - y^i)$ is computed 'as though' equilibrium dynamics were absorbed by an alternation at the symmetric pair of policies $(y^i, 2M - y^i)$. However, since in state $(R, 2M - y^{i+2})$ party L receives the payoff to an alternation at $(y^{i+2}, 2M - y^{i+2})$, its payoff upon gaining office on convergence paths to policies more moderate than ℓ^* is strictly decreasing and after each spell in opposition, parties regret their previous moderate policies. Lemma 4 also shows that L's payoff in state $(R, 2M - y^i)$ satisfies

$$U_L^+(y^i) - U_L^+(y^{i+1}) = \delta_L[U_L^-(y^{i+2}) - U_L^-(y^{i+1})]. \tag{1}$$

The left-hand side of (1) is the cost (computed in payoffs to alternations starting from L's policy) of choosing moderate policy y^{i+1} while the right-hand side is the (discounted) benefit (computed in payoffs to alternations starting from R's policy) of party R's subsequent moderate move to $2M - y^{i+2}$. These costs and benefits are balanced by the choice of y^{i+1}. Moderation is self-reinforcing: if parties anticipate an end to convergence in the future current incentives to choose moderate policies unravel. That is, if $y^i \geq \ell^*$, then (1) cannot be satisfied for $y^{i+2} = y^{i+1}$ unless $y^{i+1} = y^i$.[31] Equation (1) also explains why party L is willing to sustain

[31] In fact, this holds for all equilibria, not just those in consistent strategies. For the same reasons as above, but without relying on payoff condition (1), it can be shown that only the most extreme alternating

convergence paths to alternations more moderate than $(\ell^*, 2M - \ell^*)$, that is, why $\ell^{**} > \ell^*$. Around ℓ^*, the cost of moving to a more moderate alternation is of second-order importance, while the benefit of R's moderation is of first-order importance. Around ℓ^*, L is willing to bear almost all of the cost of sustaining convergence.

The recursive relationship in (1), along with the corresponding relationship for party R, allow the derivation of the bound on moderation $\ell^{**} \in (\ell^*, M)$. Fix one round of moderation from $(R, 2M - y^i)$ as the moves, first by L, then by R, that take the state to $(R, 2M - y^{i+2})$. Then (1) describes the share of the total moderation $y^{i+2} - y^i$ that L is willing to undertake. The bound ℓ^{**}, derived explicitly in the Appendix, is the most moderate policy for which the parties' 'supply' of moderation is consistent with convergence in the limit as $y^{i+2} \to y^i$. Convergence to moderate policies fails as the shares of any given round of moderation that parties are willing to undertake become too small. To see this, consider the polar case of convergence to the median. As a convergence path approaches M, moderate moves of similar sizes by parties L or R have similar effects on L payoffs, yet the gain from R's moderation is discounted. Since the same observation holds for R, both parties require their opponents to make larger moderate moves than they do, which contradicts convergence. As in section 4.2, the bound ℓ^{**} is shown to be tight through the construction of equilibria. In that section, a single equilibrium yields all long-run policy outcomes. Here, an equilibrium under which policy $\hat{\ell} \in (\max\{\ell^*, 2M - r^*\}, \ell^{**}]$ is a robust long-run policy outcome is constructed for each such $\hat{\ell}$. Given a policy path $\{y^i\}$ such that $y^0 = \ell^*$, $\{y^i\} \to \hat{\ell}$ and satisfying (1), the Appendix provides the equilibrium strategies and verifies their optimality. As in equilibrium $(\sigma_L^{\ell^*}, \sigma_R^{my})$, policies from any state more extreme than ℓ^* move rapidly to ℓ^*, and from there a convergence path ensures they approach $\hat{\ell}$. The key step is to show that the sequence $\{y^i\}$ exists, which follows by iterating the recursive relationship in (1) forward from $y^0 = \ell^*$ through the choice of y^1 and establishing the conditions under which this operation defines a converging policy path. Given any $\hat{\ell} \in (\ell^*, \ell^{**}]$, some policy $y^1 > \ell^*$ can be found such that $\{y^i\} \to \hat{\ell}$. From above, when $\hat{\ell} < \ell^{**}$ the share of moderation around $\hat{\ell}$ that parties are willing to undertake exceeds the amount of moderation that needs to be allocated to sustain convergence. The result hinges on the concavity of U_L^+ and U_L^-, as this ensures that parties become less willing to compromise as policies get closer to the median and hence the share of moderations that parties are willing to undertake at all ℓ' with $\ell' < \ell < \ell^{**}$ are larger than those they are willing to undertake at ℓ.

outcomes, $(\max\{\ell^*, 2M - r^*\}, 2M - \max\{\ell^*, 2M - r^*\})$, are ever reached from a more extreme state in a finite number of elections in any equilibrium.

5 Extensions

5.1 Forward-looking Voters

Myopic voting guarantees that all future office-holders are at least as moderate as the current incumbent. However, forward-looking voters may choose to elect opposition parties with more extreme platforms than incumbents if this generates preferred continuation play. First note that on any equilibrium convergence path of the model, the median voter has no incentive to support the incumbent since by voting against a (weakly) more moderate opposition, it is worse off in this election and faces the same choice in the next election. However, the consistent equilibria with myopic voters constructed in previous sections do not persist as equilibria of a game in which voters are forward-looking. The difficulties with myopic voting in these equilibria arise off equilibrium convergence paths.

Consider an extension of the model in which voters are forward-looking. I restrict attention to equilibria in which the median voter is decisive,[32] and consider a single representative median voter with utility function u_M and discount factor δ_M. A strategy for the voter is $\sigma_M : (\{L, R\} \times X) \times (X \times \{Out\}) \longrightarrow \{0, 1\}$, where $\sigma_M((I, x), z) = 0$ if and only if the median voter supports incumbent I with policy x in an election opposing it to $-I$ with policy z. Assume that the median voter never abstains so that in particular $\sigma_M((I, x), Out) = 0$ for all (I, x). Denote the set of strategies for M as Σ_M. As in Section 2, a profile of strategies $(\sigma_L, \sigma_R, \sigma_M)$ along with state (I, x) determines discounted payoff $V_J(\sigma_L, \sigma_R, \sigma_M; (I, x))$ for player $J \in \{L, R, M\}$.

Definition 6. A *Markov perfect equilibrium with forward-looking voters* is a strategy profile $(\sigma_L, \sigma_R, \sigma_M)$ such that for each state (I, x), (i) given σ_M, (σ_L, σ_R) form a Markov perfect equilibrium, and (ii) for any policy z,

$$\sigma_M((I, x), z) \in \arg\max_{\sigma'_M \in \Sigma_M} V_M(\sigma_L, \sigma_R, \sigma'_M; (I, x)).$$

To see that consistent equilibrium strategies under myopic voting are not equilibria with forward-looking voters, consider a consistent equilibrium convergence path $\{y^i\}$, a policy y^i such that $\sigma_L(R, 2M - y^i) = y^{i+1}$, a state (L, y') for some $y' \in (y^i, y^{i+1})$ and a deviation by R to $2M - y' + \epsilon$ for some $\epsilon < y' - y^i$. The median voter's myopic strategy, σ_M^{my}, calls for a vote against R. If it does so, its payoff V_M^L is given by

$$V_M^L = u_M(y') + \delta_M u_M(2M - y') + \delta_M^2 V_M(\sigma_L, \sigma_R, \sigma_M^{my}; (R, 2M - y^i)).$$

[32] In general, this entails more restrictive assumptions on voters' preferences than those used so far. Banks and Duggan (2006) show that sufficient conditions for median decisiveness is that all voters have quadratic utilities and a common discount factor.

If instead the median voter votes for R, its payoff V_M^R is given by

$$V_M^R = u_M(2M - y' + \epsilon) + \delta_M V_M(\sigma_L, \sigma_R, \sigma_M^{my}; (R, 2M - y^i)).$$

By symmetry of u_M,

$$\begin{aligned}\lim_{\epsilon \to 0} \left(V_M^L - V_M^R\right) &= \delta_M(1-\delta_M)\left[\frac{1}{1-\delta_M} u_M(2M - y^i) - V_M(\sigma_L, \sigma_R, \sigma_M^{my}; (R, 2M - y^i))\right] \\ &< 0,\end{aligned}$$

since the equilibrium path following $(R, 2M - y^i)$ consists of a converging path of policies all strictly closer to the median than $2M - y^i$. In the sections of the convergence path in which party R responds myopically, the median voter finds it costly to punish extreme deviations by party R. To do so, it must vote for the incumbent party L and keep it in power for another term, but this delays R's victory by one period and the resumption of convergence by two periods. Voting for deviating party R in this election lets a more moderate party L gain office in the next election.

Proposition 5 shows that given any alternating consistent equilibrium convergence path, it is possible to construct voter and party strategies that enforce this path in an equilibrium with forward-looking voters. Hence, the equilibrium outcomes of this paper are not due to myopic voting.

Proposition 5. *Consider consistent equilibrium (σ_L, σ_R) in the game with myopic voters. Consider state (I, x) such that $I = L$ and $x \leq M$ or $I = R$ and $x \geq M$, along with policy path $\{y^i\}$ induced from (I, x) by (σ_L, σ_R). Then there exist an equilibrium with forward-looking voters $(\sigma'_L, \sigma'_R, \sigma_M)$ such that the policy path $\{y'^i\}$ induced from (I, x) by $(\sigma'_L, \sigma'_R, \sigma_M)$ is such that $y^i = y'^i$ for all $i \geq 2$.*

In the equilibrium $(\sigma'_L, \sigma'_R, \sigma_M)$, the median voter sometimes votes against myopically preferred policies. In particular, in the sections of the convergence path in which party R responds myopically under consistent strategies, the median voter supports more extreme policies by R to ensure a quicker resumption of convergence.[33] The equilibrium strategies are illustrated in Figure 6. Consider policy y^i such that $\sigma_L(R, 2M - y^i) = y^{i+1}$. In the Appendix, for $\ell \in [y^i, y^{i+1})$ I define function $z^{i+1}(\ell) \in [y^i, \ell)$ such that $2M - z^{i+1}(\ell)$ is the most extreme policy by R supported by the median voter against ℓ in state (L, ℓ). Note that to 'resume' convergence, the median voter is never willing to support a policy by R that is more extreme than $2M - y^i$. Suppose, for example, that the median voter supported a proposal $r \in (2M - y^i, 2M - y^{i-1}]$. In the next election, L does not commit to a moderate policy (in fact it commits to $z^i(r) \in [y^{i-1}, r)$), and it takes two elections to return to state $(R, 2M - y^i)$. Against this, the median voter prefers to vote against R and wait two elections to arrive at the more moderate state (L, y^{i+1}).

[33] Parties strategies in equilibrium $(\sigma'_L, \sigma'_R, \sigma_M)$ are also consistent. They differ from consistent strategies under myopic voting since forward-looking voters induce different sets of winning policies.

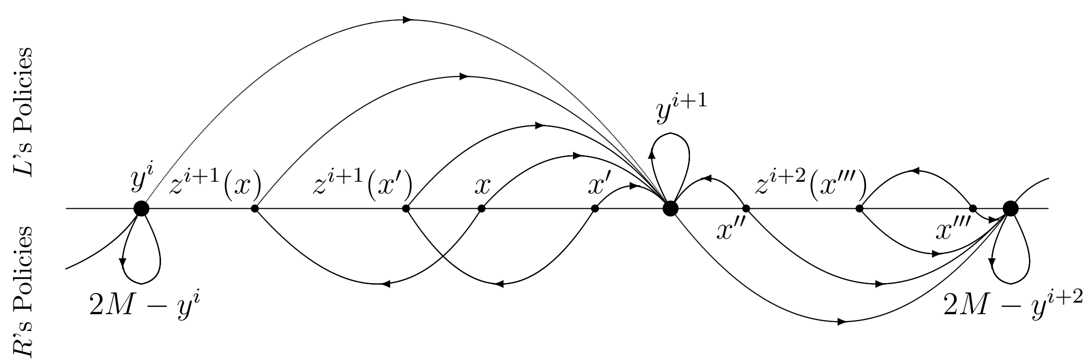

Figure 6: Policy Dynamics of Equilibria with Forward-looking Voters.

When voters are myopic, in states (L, ℓ) with $\ell > M$ party R does not participate in any elections and policy dynamics get trapped. When the median voter is forward-looking, it may vote in favour of policies by R that are more extreme than ℓ but lead to renewed convergence. Hence the result of Proposition 5 applies only to alternating convergence paths, and in the Appendix I show that parties on alternating convergence paths have no incentive to commit to a policy on their opponent's side of the median solely to have convergence eventually resume from a more extreme initial state.

5.2 Limited Policy Persistence

5.2.1 Term Limits

In this section, I show that my results are robust to two weaker versions of incumbent policy persistence. If incumbent policy persistence is interpreted as stemming from candidates having fixed policy preferences and being replaced by their parties only after having lost an election, then introducing term limits for incumbents allows parties to sometimes replace winning candidates. The following result shows that if incumbents can hold office for no more than $T \geq 2$ periods, the set of long-run policy outcomes of the model is the same as those identified by Proposition 3 for the model in which $T = \infty$.

Proposition 6. *Consider the model with term limits $T \geq 2$. Policy $\ell \leq M$ is a long-run policy outcome if and only if $\ell \in [\max\{\ell^*, 2M - r^*\}, M]$.*

Proposition 1 shows that in any equilibrium, the median is the only policy outcome when a term-limited incumbent steps down and parties compete simultaneously. However, with any term limit $T \geq 2$ it is still the case that in all equilibria first-term incumbents are always defeated, and hence never reach their term limits. Politicians can hope to reach their term limits only by implementing sufficiently moderate policies. However, in equilibrium, they gain by implementing policies they prefer even if they understand that this can lead to electoral defeat. In this model, electoral competition succeeds in dislodging incumbents that fail to cater to the median voter's preferences. Hence, it is not surprising that term limits should play no role.

5.2.2 Costly Policy Adjustment

The key feature of my model is the asymmetry of persistence between incumbent and opposition parties. Here, I relax the assumption of full incumbent policy persistence but maintain the feature that incumbent parties find it more costly than opposition parties to distance themselves from their records. Consider the following stage game indexed by (I, x). First, as before, party $-I$ commits to a policy y. Second, given y, party I can bear fixed adjustment c and change its policy to some alternative policy x', after which the election is held. Hence, the incumbent party can, at some cost, thwart opposition parties whose policies it particularly dislikes. Note that the model studied so far has $c = \infty$. Policy adjustment cost c is a reduced-form approach to capturing the various frictions that can keep incumbent parties from radically changing their policies between terms. These frictions could capture parties' reputational concerns, voter aversion towards 'flip-floppers' or the costs of intra-party strife involved with replacing incumbent representatives. The next result shows that as long as $c > 0$, some long-run alternation can still be sustained in equilibrium.

Proposition 7. *Consider the model with fixed cost $c > 0$ to policy adjustments for incumbents. There exists policies $\ell^c \in [0, M)$ and $r^c \in (M, 1]$ such that policy $\ell \leq M$ is a long-run policy outcome if and only if $\ell \in [\max\{\ell^c, 2M - r^c, \ell^*, 2M - r^*\}, M]$.*

The difference between the sets of long-run outcomes identified by propositions 3 and 7 varies monotonically in c, and their relationship is fleshed out in the following result.

Corollary 3. *The set of long-run policy outcomes with adjustment cost $c > 0$ has the following properties.*

i. *If $c' > c$, then $[\max\{\ell^{c'}, 2M - r^{c'}, \ell^*, 2M - r^*\}, M] \supseteq [\max\{\ell^c, 2M - r^c, \ell^*, 2M - r^*\}, M]$.*

ii. $\lim_{c \to 0}[\max\{\ell^c, 2M - r^c, \ell^*, 2M - r^*\}, M] = \{M\}$

iii. *There exists $\bar{c}$ such that $[\max\{\ell^c, 2M - r^c, \ell^*, 2M - r^*\}, M] = [\max\{\ell^*, 2M - r^*\}, M]$ for all $c \geq \bar{c}$.*

Intuitively, as the cost c decreases, incumbent parties are more willing to adjust their policies and, in the long-run, equilibrium alternations must get closer to the median. In the limit as $c \to 0$, parties propose policies simultaneously and, as per Proposition 1, only the median can be observed in the long-run. However, the case of full incumbent policy persistence ($c = \infty$) is not knife-edge. As the cost c increases, the set of long-run policy with adjustment costs eventually coincides with the set of long-run policy outcomes with full persistence.

5.3 Office-Motivated Parties

In this section, I allow the parties to have preferences over both implemented policies and office holding per se. More precisely, I consider the version of my model in which party J's stage game payoff to implemented policy y is the sum of $u_J(y)$ and an office benefit $b > 0$ that party J receives if it is the party actually implementing policy y.[34] The model studied so far has $b = 0$.

Existing static and dynamic models of elections find that parties that care more about holding office per se are more willing to compromise.[35] My model, on the other hand, provides a rationale for *not* expecting a clear-cut relationship between the strength of office-motivation and policy moderation. While a party that cares about holding office will be more willing to compromise *if* indeed compromise leads to longer tenure in office, the link between compromise and tenure is determined in equilibrium. If office benefits are high, future opponents are themselves more willing to compromise in order to gain access to office. In other words, to enjoy a longer tenure by implementing compromise policies a party needs the equilibrium consent of its opponent: these compromise policies must compensate its opponent for not holding office. Two features of my model generate this property. First, as opposed to static models, the benefits of office depend on the patterns of a party's tenure in office. Second, as opposed to existing models of dynamic elections, parties credibly commit to policies and hence the attributes of future opponents are not stationary but depend on current policy choices.

Proposition 8. *Consider the model with office benefits $b > 0$. There exist policies $\ell^{out} \leq \ell^{in} < M$ and $M > r^{in} \geq r^{out}$ such that either*

i. $\ell^{out} < 2M - r^{in}$, $r^{out} > 2M - \ell^{in}$ and policy $\ell \leq M$ is a long-run policy outcome if and only if $\ell \in [\max\{\ell^*, 2M - \ell^*\}, M]$, or

ii. $\ell^{out} \geq 2M - r^{in}$ and there exists policy $\ell^b \in [\max\{\ell^*, 2M - r^*\}, \max\{\max\{\ell^*, 2M - r^*\}, 2M - r^{in}\})$ such that policy $\ell \leq M$ is a long-run policy outcome supported by symmetric alternation only if $\ell \in [\max\{\ell^*, 2M - r^*\}, \ell^b] \cup [\max\{\max\{\ell^*, 2M - r^*\}, \ell^{out}\}, M]$.

[34] I maintain the assumption that ties at the median policy are broken in favour of the opposition party. This is no longer innocuous, as the parties now care about the pattern of office holding when the policy path is $\{M, M, ...\}$. However, having ties broken in favour of the opposition party would result in equilibrium if, for example, incumbents' policies were subject to perturbations.

[35] See Calvert (1985) and Duggan (2000).

Furthermore, policy x is a non-trivial long-run policy outcome not supported by symmetric alternation only if $x \in [2M - r^{in}, \ell^{out}]$, or

iii. $r^{out} \leq 2M - \ell^{in}$*, and the statement is symmetric to ii.*

Policy ℓ^{out} is defined such that if $\ell^{out} > 0$, then party L is indifferent between never holding office and having policy ℓ^{out} implemented forever and gaining office every second election and having policies alternate at $(\ell^{out}, 2M - \ell^{out})$. Policy ℓ^{in} is defined such that if $\ell^{in} > 0$, then party L is indifferent between holding office forever and implementing policy $2M - \ell^{in}$ and holding gaining office every second election and having policies alternate at $(\ell^{in}, 2M - \ell^{in})$. Policies r^{out} and r^{in} can be defined similarly for party R. Hence, if $\ell^{out} \geq 2M - r^{in}$, there is scope for a policy $\ell \in [2M - r^{in}, \ell^{out}]$ to simultaneously give incentives (a) to party R to commit to it knowing that party L will fail to contest all future elections and (b) to opposition party L in state (R, ℓ) not to commit to some winning policy just to gain office. Case i above covers the case in which no such 'bargains' can be sustained. In this case, since parties understand that any attempt to hold office forever will be thwarted, none is made and the set of long-run policy outcomes is as though $b = 0$. Note that cases ii and iii offer only necessary conditions on the sets of long-run policy outcomes with office benefits. Partial converses are derived through equilibrium construction in the Appendix. They permit the following comparative statics results.

Corollary 4. *The set of long-run policy outcomes $\mathcal{L}^b$ with office benefit $b > 0$ has the following properties.*

i. $\lim_{b \to 0} \mathcal{L}^b = [\max\{\ell^*, 2M - r^*\}, M]$.

ii. There exits $\bar{b}$ such that $\mathcal{L}^b = [\max\{\ell^*, 2M - r^*\}, M]$ for all $b \geq \bar{b}$.

Unsurprisingly, as $b \to 0$ the set of long-run policy outcomes converges to that identified in proposition 3 for the model with $b = 0$. The result in ii is more surprising. Office benefits give parties incentives to commit to moderate policies in order to have longer tenure only if they can coordinate onto policies that (a) the incumbent is willing to champion in exchange for office and (b) the opposition party is happy receiving in exchange for non-participation. In the limit as $b \to \infty$, parties rank office and policies lexicographically. However, the set of long-run policy outcomes is exactly the same as in the limit as $b \to 0$ when they are purely office-motivated. Parties that ranks office and policies lexicographically can never offer a compromise policy that induces their opponents to allow them to enjoy long tenures in office. Hence, in equilibrium, both parties are resigned to one-term tenures, and their actions are guided solely by their policy preferences.[36]

[36] It can also be shown from the results in the Appendix that if the game is symmetric, that is, if $u_L(x) = u_R(1-x)$ for all $x \in [0,1]$, $\delta_L = \delta_R$ and $M = \frac{1}{2}$, then $\mathcal{L}^b = [\max\{\ell^*, 2M - r^*\}, M]$ for all b. If parties are identical, then there can be no 'wedge' between the policies office-holding parties are willing to offer and those that non-participating parties are willing to accept.

5.4 Legislative Bargaining

This section discusses in detail the relationship between my paper and the dynamic legislative bargaining model of Baron (1996), which features a single-dimensional policy space. There are three crucial differences between my paper and Baron (1996). First, I do not assume that the median voter is represented by a party that shares its preferences over policies. Second, incumbent policy persistence generates a history-dependent proposer recognition rule. Finally, the equilibria I construct in Section 4.3 have the median voter strictly prefer to support opposition parties on the equilibrium path.

To view my model as a legislative bargaining model, reinterpret voters as legislators, with M denoting the ideal policy of the median legislator. However, in contrast to Baron (1996), only two legislators can be recognised to propose policies; these are legislators L and R that have ideal policies 0 and 1. For simplicity, assume that they are recognised each period with equal probability. In the *legislative bargaining model*, a state (I, x) consists of the current proposer along with the status quo. A proposal strategy for party I is $\sigma_I : \{I\} \times X \longrightarrow X$.[37] As above, I assume that the median legislator is decisive in equilibrium.[38] Consider voting strategies σ_M for the median legislator, where now $\sigma_M((I, x), z) = 0$ if and only if M supports the status quo. An equilibrium of the legislative bargaining game is as in Definition 6, with the relevant reinterpretations.

A convergence path $\{y^i\}$ in the legislative bargaining game is as defined above but its description no longer corresponds to the realised equilibrium policy path. Given a strategy for the median legislator, consistent proposal strategies for voters are as in Definition 5. In the Appendix, I show that myopic voting is optimal for the median legislator when facing consistent strategies. Since it is without loss of generality to assume that in any equilibrium the median legislator supports proposal y^{i+1} in state (L, y^{i+1}), consistent Markov proposal strategies along with $\sigma_M = \sigma_M^{my}$ imply that if $\sigma_L(L, 2M - y^i) = y^{i+1}$, then $\sigma_L(L, y^{i+1}) = y^{i+1}$. Hence under consistent proposal strategies a convergence path describes a lottery over equilibrium policy paths; policy dynamics are staggered and the status quo may remain unchanged for some time while the same legislator is recognised several periods in a row. When a new legislator is recognized, the status quo resumes its convergence.

Proposition 9 shows that the nonconvergence result of Proposition 4 is due to the median legislator never being recognised.

Proposition 9. *In any equilibrium of the legislative bargaining model in consistent proposal strategies, any limit point of some convergence path from state (I, x) with $x \neq M$ is bounded away from M.*

[37]It is the norm in legislative bargaining models to describe the state as solely the status quo, before a new proposer is drawn. I model the state as being described after a proposer has been drawn simply to maintain consistency in notation with the earlier sections. This also explains the use of the redundant notation $\sigma_I(I, x)$ for party I's strategy.

[38]As above, the sufficient conditions of Banks and Duggan (2006) can be called upon.

The proof shows that the main features of the results of Section 4.3, in particular those concerning convergence path payoffs under consistent strategies, can be reproduced in the legislative bargaining setting. I do not derive the conditions for the existence of convergence paths, but these hinge on assumptions about parties' preferences over the staggered versions of alternating outcomes. Discounting ensures that parties have a preferred such staggered alternation that is bounded away from the median. As in my main model, convergence beyond these preferred staggered alternations requires convergence paths satisfying conditions like those of (1). It is also clear that convergence paths cannot approach the median for the same reasons as in my model.

Baron (1996) characterises an equilibrium in which the median voter is indifferent between supporting the status quo and the new proposal in all periods. On the convergence paths of consistent equilibria, the median voter strictly prefers to vote against the status quo. The equilibrium of Baron (1996) is in fact closely related to the equilibrium $(\sigma_L^{\ell^*}, \sigma_R^{my})$ of Section 4.2. In that equilibrium, when play has reached a symmetric alternation, the median voter is indifferent between both parties' policies. Given continuation play, it would vote for any more moderate policy, since this leads to more moderate alternations, and vote against all more extreme policies.

An *iid* recognition rule makes it easier to verify that myopic voting is optimal for the median legislator. Consider, for example, the problematic states for myopic voting under incumbent policy persistence. Take y^i such that $\sigma_L(L, 2M - y^i) = y^{i+1}$, and consider state (R, ℓ) for $\ell \in (y^i, y^{i+1})$. R is expected to propose $r = 2M - \ell$. Suppose it deviates to $r' \in (r, 2M - y^i]$. If the median legislator supports R, policy r' is passed and in the next period the median legislator faces a lottery between a freezing of convergence at r' and a resumption of convergence by L proposing y^{i+1}. If instead it supports the status quo, in the next period the median legislator faces a lottery between a freezing of convergence at ℓ and a resumption of convergence by L proposing y^{i+1}. The median legislator supports the status quo since $|M - \ell| < |M - r'|$. Since the median legislator does not affect the lottery over future proposers by its vote, it faces no cost to punish deviations.

6 Conclusion

This paper has studied the policy dynamics of a game of electoral competition between two policy-motivated parties. Although incumbent policy persistence allows opposition parties to win elections with extreme policies, an incentive to commit to more moderate policies is generated by the benefits of *imposing* moderation on future opponents. At some opportunity cost which consists of foregone policy gains in the current election, parties can, and in equilibrium do, commit to more moderate future electoral outcomes by championing moderate policies. Furthermore, since the incentives to moderate vanish as policies approach the median, convergence *toward* the median is a dynamically robust phenomenon, while convergence *to* the median is not.

The rich policy dynamics of the model are generated by incumbent policy persistence. It is not unrealistic to suggest that incumbents and challengers are subjected to different standards by voters. In an election, incumbent politicians typically have little choice but to 'run on their record'. Their performance in office is fresh in the minds of voters, who have had years to derive information about incumbents' aptitudes and preferences from their decisions. Compounding this effect, opposition candidates or parties often elaborate and expound their platforms relative to the policies enacted by incumbents. Whatever the accepted evaluation of a politician's or party's term in office, incumbents can only have marginal success in drawing voters' attention away from their record. As a consequence, their ability to propose policies to voters that differ considerably from those they championed while in office is constrained. Office-holding politicians are acutely aware of this and act accordingly. In a recent example, while less than a year into his first term, Barack Obama already frames his efforts to pass a health care reform bill through its effects on a bid for reelection which is more than three years away: 'I intend to be president for a while and once a bill passes, I own it. And if people look and say, 'You know what? This hasn't reduced my costs[, ...] insurance companies are still jerking me around,' I'm the one who's going to be held responsible.'[39]

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A Appendix

A.1 No Policy Persistence

Proof of Proposition 1. As noted in the text, $\frac{1}{1-\delta_J}u_J(M)$ is a subgame perfect equilibrium payoff for party J following any history. Since party J can always enforce this payoff by committing to policy M following any history, this payoff is the lowest SPE payoff for J. Hence a policy path $\{y^i\}$ is a subgame perfect equilibrium policy path only if $\sum_{i=0}^{\infty}\delta^i u_J(y^i) \geq \frac{1}{1-\delta_J}u_J(M)$ for all J and all i.

The first step in the proof shows that the game's only subgame perfect equilibrium policy path following any history is the indefinite repetition of the median policy. Strict concavity is needed to ensure that if $y \neq M$ is strictly on party J's side of the median, then $u_J(y) - u_J(M) < u_{-J}(M) - u_{-J}(y)$.[40] This holds since any strictly concave functions u_L and u_R defined on $[0,1]$ with u_L strictly decreasing and u_R strictly increasing can be normalised such that $|u'_L(M)| = |u'_R(M)|$. Suppose $y < M$. By strict concavity, for all $\ell \in [y, M)$ we have $|u'_L(\ell)| < |u'_L(M)| = |u'_R(M)| < |u'_R(\ell)|$, and hence $u_L(y) - u_L(M) < u_R(M) - u_R(y)$.

Consider subgame perfect equilibrium policy path $\{y^i\}$ following some history with $y^0 \neq M$, and suppose that y^0 is on J's side of the median. Define

$$D_J^0 = 0,$$
$$D_{-J}^0 = \frac{u_{-J}(M) - u_{-J}(y^1)}{\delta_{-J}}.$$

For any $i \geq 1$ and y^i (weakly) on J's side of the median, define D_J^i and D_{-J}^i recursively as

$$D_J^i = \max\left\{0, \frac{D_J^{i-1} + [u_J(M) - u_J(y^i)]}{\delta_J}\right\},$$
$$D_{-J}^i = \frac{D_{-J}^{i-1} + [u_{-J}(M) - u_{-J}(y^i)]}{\delta_{-J}}.$$

That is, interpret $D_J^i \geq 0$ as the payoff 'debt' for party J at stage i of subgame perfect equilibrium policy path $\{y^i\}$ relative to path $(M, M, ...)$. This debt collects all deviations from payoff $u_J(M)$; if party J makes a loss with respect to $u_J(M)$ at y^i, then the equilibrium

[40] Any assumptions that yields this property are sufficient for the result of Proposition 1. For example, if u_L and u_R are weakly concave but strictly concave in a neighbourhood of M.

payoff from y^{i+1} needs to yield an excess of at least D_J^i over $\frac{1}{1-\delta_J}u_J(M)$. Debts grow by factor $\frac{1}{\delta_J}$ each period since they are incurred in the current period and reimbursed in later periods. Negative debts are never incurred since party J must be guaranteed the payoff $\frac{1}{1-\delta_J}u_J(M)$ after all histories.

Since $y^0 \neq M$, debts (D_L^0, D_R^0) are such that $D_J^0 > 0$ for some J. Suppose without loss of generality that $\delta_L \leq \delta_R$. First note that for all $i > 0$, it cannot be that $D_L^i = D_R^i = 0$, since $D_J^0 > 0$ and whenever $D_J^i < D_J^{i-1}$, it must be that y^i is strictly on J's side of the median and hence that $D_{-J}^i > D_{-J}^{i-1}$. Next, note that for all J, we have that $\liminf_{i\to\infty} D_J^i = 0$, and also that $D_J^i = 0$ infinitely often. To see this, suppose that there exists some k such that $D_J^i > 0$ for all $i \geq k$. Then the equilibrium value to party J from subgame perfect equilibrium policy path $\{y^i\}_{i=k}^{\infty}$ is strictly less than $\frac{1}{1-\delta_J}u_J(M)$, a contradiction.

Suppose now that $y^0 < M$, and hence that $D_L^0 = 0 < D_R^0$. Then either

i. $D_L^i = 0$ for all $i > 0$.

ii. $D_L^i > 0$ for some $i > 0$.

In case i, it must be that $y^i \leq M$ for all $i > 0$, and hence that $\lim_{i\to\infty} D_R^i \geq \lim_{i\to\infty} \frac{D_R^0}{\delta_R^i} = \infty$, a contradiction. We now see that assuming $y^0 < M$ is without loss of generality. First, any subgame perfect equilibrium policy path that deviates from the median policy after some history must have some subsequence that begins at stage k with debt levels $D_J^k = 0 < D_{-J}^k$. Second, assume instead that $D_L^0 > 0 = D_R^0$. Then either $D_R^i = 0$ for all i, which leads to contradiction, or there exists k such that $D_L^k = 0$, in which case we must have $D_R^k > 0$. Now consider case ii above. There must exist $n > m \geq 0$ with $n - m > 1$ such that $D_R^m > 0$, $D_L^m = D_L^n = 0$ and $D_L^i > 0$ for $i \in \{m+1, ..., n-1\}$. We want to show that $D_R^m < D_R^n$. Consider the sequence $\{\hat{y}^i\}_{i=m+1}^n$ that solves the following minimisation problem.

$$\min_{\{y^i\}_{i=m+1}^n \in X^{n-m}} D_R^n \quad \text{subject to } D_L^m = D_L^n = 0, \text{ given } D_R^m > 0. \tag{2}$$

$\{\hat{y}^i\}_{i=m+1}^n$ exists since D_R^n is continuous and X^{n-m} is compact. Suppose that $\{\hat{y}^i\}_{i=m+1}^n$ is such that $\hat{D}_L^{n-1} > 0$, where $\hat{D}_J^i$ is the debt of party J under $\{\hat{y}^i\}_{i=m+1}^n$. Hence since $D_L^n = 0$ it must be that $\hat{y}^n < M$. Suppose that $\hat{D}_R^{n-2} + [u_R(M) - u_R(\hat{y}^{n-1})] < 0$, which implies that $\hat{D}_R^{n-1} = 0$ and that $\hat{y}^{n-1} > M$. For $\epsilon > 0$, consider $\bar{y}^{n-1} = \hat{y}^{n-1} - \epsilon$ and $\bar{y}^n = \hat{y}^n + \eta_\epsilon$, where η_ϵ is chosen such that $\bar{D}_L^n = 0$. For sufficiently small ϵ, we have that $\bar{D}_R^{n-1} = \hat{D}_R^{n-1} = 0$ and $\bar{D}_R^n < \hat{D}_R^n$, a contradiction. Now suppose that $\hat{D}_R^{n-2} + [u_R(M) - u_R(\hat{y}^{n-1})] \geq 0$. $\hat{D}_R^n$ is strictly increasing in $\hat{y}^{n-1}$ if

$$-\frac{u_R'(\hat{y}^{n-1})}{\delta_R^2} - \frac{u_R'(\hat{y}^n)}{\delta_R}\frac{\mathrm{d}\hat{y}^n}{\mathrm{d}\hat{y}^{n-1}} > 0, \tag{3}$$

where $\frac{\mathrm{d}\hat{y}^n}{\mathrm{d}\hat{y}^{n-1}}$ is given by

$$\frac{u_L'(\hat{y}^{n-1})}{\delta_L^2} - \frac{u_L'(\hat{y}^n)}{\delta_L}\frac{\mathrm{d}\hat{y}^n}{\mathrm{d}\hat{y}^{n-1}} = 0,$$

or $\frac{\mathrm{d}\hat{y}^n}{\mathrm{d}\hat{y}^{n-1}} = -\frac{1}{\delta_L}\frac{u_L'(\hat{y}^{n-1})}{u_L'(\hat{y}^n)}$, which comes from partially differentiating the constraint $D_L^n = 0$ with respect to y^{n-1} and y^n. We can rewrite (3) as

$$\frac{u_L'(\hat{y}^{n-1})}{u_L'(\hat{y}^n)} > \frac{\delta_L}{\delta_R}\frac{u_R'(\hat{y}^{n-1})}{u_R'(\hat{y}^n)}.$$

Say $\hat{y}^{n-1} \geq M$. Then $|u_L'(\hat{y}^{n-1})| \geq |u_R'(\hat{y}^{n-1})|$, $\frac{\delta_L}{\delta_R} \leq 1$ and $|u_L'(\hat{y}^n)| < |u_R'(\hat{y}^n)|$ (since $y^n < M$) imply that (3) holds, and hence that $\{\hat{y}\}_{i=m+1}^n$ does not solve (2), a contradiction. Hence it must be that $\hat{y}^{n-1} < M$.

This pairwise necessary condition for optimality can be used all along the sequence $\{\hat{y}\}_{i=m+1}^n$ to show that a solution to (2) with $\hat{y}^n < M$ must have $\hat{y}^i < M$ for all $i \in \{m+1, ..., n-1\}$. But consider instead sequence $\{\tilde{y}\}_{i=m+1}^n$ with $\tilde{y}^i = M$ for all i. This sequence satisfies the constraints of (2), and is such that $\tilde{D}_R^n = \frac{D_R^m}{\delta_R^{n-m}} < D_R^n$ for any $\{y^i\}_{i=m+1}^n$ with $D^{n-1} < M$. Hence, for the purported equilibrium sequence from above, we have as desired that $D_R^n > D_R^m$. Considering the full policy sequence, we have that whenever $D_L^i > 0$ for $i \in \{m+1, n-1\}$, then $D_R^n > D_R^m$. Furthermore, whenever $D_L^i = 0$ for $i \in \{m+1, n-1\}$, then again $D_R^n > D_R^m$ since $D_L^i = 0$ only if $y^i \leq M$, and as shown above if $D_L^m = 0$, then $D_R^m > 0$. Hence, given the SPE path $\{y^i\}$ following some history for which $D_R^k > 0$, we have that $\lim_{i\to\infty} D_R^i = \infty$, a contradiction.

The previous argument shows that the unique SPE policy path following any history is $(M, M, ...)$. It remains to be shown that both parties' strategies must call for them to commit to the median following any history. If party J's strategy calls for some policy $y \neq M$ after some history, then party $-J$ must win the election with policy M. Since $y \neq M$, party $-J$ can win the election with a policy it prefers to M, say y'. Since following any deviation, party $-J$ payoffs revert to $\frac{1}{1-\delta_{-J}}{}_{-J}(M)$, deviating to y' is profitable for $-J$. □

A.2 Policy Dynamics

Proof of Proposition 2. Consider state (R, r) and policy path $\{x^i\}$ induced by (σ_L, σ_R) from (R, r). First note that the policy path following state (R, M) can only be $(M, M, ...)$. To prove the rest of point i and part of point ii, consider the following claim: *In any MPE (σ_L, σ_R), $\sigma_L(R, r) \in X \setminus W(R, r) \cup \{Out\}$ for all $r < M$ and $\sigma_L(R, r) \leq M$ for all $r > M$. The corresponding claims for party R are symmetric.* To show this, consider some MPE (σ_L, σ_R) with $\sigma_L(R, r) \in [r, 2M - r]$ for some $r < M$. Consider a one-shot deviation by L at state (R, r) to *Out*. The payoff to this deviation is

$$u_L(r) + \delta_L V_L(\sigma_L, \sigma_R; (R, r)),$$

while the payoff to $\sigma_L(R, r)$ is $V_L(\sigma_L, \sigma_R; (R, r))$. Hence the deviation is unprofitable if and only if

$$V_L(\sigma_L, \sigma_R; (R, r)) \geq \frac{1}{1-\delta_L} u_L(r). \tag{4}$$

Since $r < M$, the policy path following (R, r) most favourable to L is $(r, r, ...)$. Hence we have that

$$V_L(\sigma_L, \sigma_R; (R, r)) \leq \frac{1}{1-\delta_L} u_L(r). \tag{5}$$

(4) and (5) imply that $V_L(\sigma_L, \sigma_R; (R, r)) = \frac{1}{1-\delta_L} u_L(r)$, which holds if and only if $\sigma_L(R, r) = r$ and $\sigma_R(L, r) = r$. Now consider a deviation for R in state (L, r) to $r^d \in (r, 2M - r]$. Any policy path $\{x^i\}$ induced by (σ_L, σ_R) from (R, r^d) must be such that $x^i > r$ for all i. Hence the payoff to r^d is

$$\begin{aligned} u_R(r^d) + \sum_{i=1}^{\infty} \delta_R^{2i-1} [u_R(x^i) + \delta_R u_R(x^{i+1})] \\ > \frac{1}{1-\delta_R^2} u_R(r), \end{aligned}$$

a contradiction. For the second part of the claim, take (R, r) for some $r > M$ such that $\sigma_L(R, r) > M$. Consider a deviation to some $\ell^d \in (M, \sigma_L(R, r))$. By the first part of the claim, the payoff to ℓ^d is given by

$$\begin{aligned} \frac{1}{1-\delta_L} u_L(\ell^d) &> \frac{1}{1-\delta_L} u_L(\sigma_L(R, r)) \\ &= V_L(\sigma_L, \sigma_R; (R, r)), \end{aligned}$$

a contradiction. In a similar manner, if (R, r) for some $r > M$ is such that $\sigma_L(R, r) = Out$, considering a deviation to some $\ell^d \in (M, r)$ yields the desired contradiction.

For point ii of Proposition 2, note that by the previous claim, the sequence $\{x^i\}_{i \text{ odd}}$ is weakly increasing and bounded by x^1 and M, and hence converges to some limit $\hat{\ell}$. The sequence $\{x^i\}_{i \text{ even}}$ is weakly decreasing and bounded by M and x^2, and hence converges to some limit $\hat{r}$. Furthermore, it must be that $\hat{\ell} = 2M - \hat{r}$. Suppose instead that $\hat{\ell} - (2M - \hat{r}) = \epsilon > 0$. Consider $n \in \mathbf{N}$ such that $\hat{\ell} - x^i < \epsilon$ for all $i \geq n$ odd. Then for $j \geq n$ odd

$$\begin{aligned} 2M - \ell^j &< 2M - \hat{\ell} + \epsilon \\ &= \hat{r} \\ &\leq x^{j+1} \end{aligned}$$

and hence $x^{j+1} \notin W(L, x^j)$ and there can be no $\sigma_R(L, x^j)$ such that $\tau((L, x^j), \sigma_R(L, x^j)) = x^{j+1}$, a contradiction. A similar argument shows that it cannot be that $\hat{\ell} < 2M - \hat{r}$. Hence $\hat{r} = 2M - \hat{\ell}$.

To complete the proof of Proposition 2, it remains to be shown that $\sigma_L(R, \hat{r}) = \hat{\ell}$ and $\sigma_R(L, \hat{\ell}) = \hat{r}$. Suppose first that $x^i = \hat{\ell}$ for some i odd. Then $x^j = \hat{\ell}$ for all $j > i$ odd and it must be that $\sigma_L(R, \hat{r}) = \hat{\ell}$ and $\sigma_R(L, \hat{\ell}) = \hat{r}$. Suppose now that $x^i \neq \hat{\ell}$ for all i, and that $\sigma_R(L, \hat{\ell}) = r < \hat{r}$. Consider $\Delta > 0$ such that

$$u_L(\hat{\ell}) + \frac{\delta_L}{1-\delta_L^2} U_L^-(2M - r) > \frac{1}{1-\delta_L^2} U_L^+(\hat{\ell}) + \Delta. \tag{6}$$

Such a Δ exists by Lemma 1 since $r < \hat{r}$. Since u_L is continuous and $\{x^i\}_{i \text{ odd}} \to \hat{\ell}$, there exists $n \in \mathbf{N}$ and $\epsilon > 0$ such that for all $i \geq n$ odd, $\hat{\ell} - x^i < \epsilon$ and $u_L(x^i) - u_L(\hat{\ell}) < \Delta$. Now, for any $j \geq n$ odd

$$\begin{aligned} V_L(\sigma_L, \sigma_R; (R, x^{j-1})) &= u_L(x^j) + \sum_{i=1}^{\infty} \delta_L^{2i-1}[u_L(x^{j+2i-1}) + \delta_L u_L(x^{j+2i})] \\ &\leq u_L(x^j) + \sum_{i=1}^{\infty} \delta_L^{2i-1} U_L^-(x^{j+2i-1}) \\ &\leq u_L(x^j) + \frac{\delta_L}{1-\delta_L^2} U_L^-(\hat{\ell}) \\ &< \frac{1}{1-\delta_L^2} U_L^+(\hat{\ell}) + \Delta. \end{aligned} \tag{7}$$

The first inequality follows from the fact that $x^{j+2i+1} \geq 2M - x^{j+2i}$ for all i. The second inequality follows by Lemma 1 from the fact that $x^{j+2i} \geq \hat{r}$ for all i. In state (R, x^{j-1}), consider a deviating strategy by L, σ_L^d, with the properties

$$\begin{aligned} &\sigma_L^d(R, x^{j-1}) = \hat{\ell} \text{ and} \\ &\quad \sigma_L^d(R, r') = 2M - r' \text{ for all } r' \leq \hat{r}. \end{aligned}$$

Consider the policy path $\{x'^i\}$ induced by (σ_L^d, σ_R) from (R, x^{j-1}). The payoff to σ_L^d is

$$\begin{aligned} u_L(\hat{\ell}) + \sum_{i=1}^{\infty} \delta_L^{2i-1} U_L^-(2M - x'^{2i}) &\geq u_L(\hat{\ell}) + \frac{\delta_L}{1-\delta_L^2} U_L^-(2M - \hat{r}) \\ &> \frac{1}{1-\delta_L^2} U_L^+(\hat{\ell}) + \Delta \\ &> V_L(\sigma_L, \sigma_R; (R, x^{j-1})), \end{aligned}$$

a contradiction. The first inequality follows from Lemma 1 and the fact that $x^{2i} \leq \hat{r}$ for all i, the second from (6) and the third from (7). The same proof applies to show that $\sigma_L(R, \hat{r}) = \hat{\ell}$. □

A.3 Bounded Extremism

Proof of Proposition 3. The following lemma provides a lower bound on equilibrium payoffs.

Lemma 2. *Consider MPE (σ_L, σ_R). In state (R, r) with $r > M$, the payoff to party L from policy $\ell \in W(R, r)$ for some $\ell \leq M$ is at least $\frac{1}{1-\delta_L^2} U_L^+(\ell)$. The statement for party R is symmetric.*

Proof of Lemma 2. Given state (R, r) with $r > M$, consider the strategy σ'_L for L with the properties

$$\sigma'_L(R, r) = \ell \in W(R, r) \text{ and}$$
$$\sigma'_L(R, r') = 2M - r' \text{ for all } r' \leq 2M - \ell.$$

Consider the policy path $\{x^i\}$ induced by (σ'_L, σ_R) from (R, r). The payoff to σ'_L is

$$u_L(\ell) + \sum_{i=1}^{\infty} \delta_L^{2i-1} U_L^-(x^{2i}) \geq \frac{1}{1-\delta_L^2} U_L^+(\ell),$$

where the inequality follows by Lemma 1 since $x^{2i} \leq 2M - \ell$ for all i. □

The following claim establishes the bound on the extremism of long-run policy outcomes: *If policy ℓ is a long-run policy outcome, then $\ell \geq \max\{\ell^*, 2M - r^*\}$.* To show this, suppose that $\ell^* \geq 2M - r^*$ and that $\ell < \ell^*$ is a long-run policy outcome under (σ_L, σ_R) starting from some state. By Lemma 2, party L can obtain a payoff of at least $\frac{1}{1-\delta_L^2} U_L^+(\ell^*)$ by committing to ℓ^* in state (R, r). However, $V_L(\sigma_L, \sigma_R; (R, r)) = \frac{1}{1-\delta^2} U_L^+(\ell) < \frac{1}{1-\delta_L^2} U_L^+(\ell^*)$ by Lemma 1 since $\ell < \ell^*$, a contradiction.

To complete the proof of Proposition 3, the following claim verifies the equilibrium construction of Section 4.2: *If $\ell^* \geq 2M - r^*$, the strategy profile $(\sigma_L^{\ell^*}, \sigma_R^{my})$ forms an equilibrium. If $\ell^* < 2M - r^*$, the strategy profile $(\sigma_L^{my}, \sigma_R^{r^*})$ forms an equilibrium.* To show this, suppose that $\ell^* \geq 2M - r^*$. First verify the optimality of L's proposed strategy. Given $(\sigma_L^{\ell^*}, \sigma_R^{my})$ compute

$$V_L(\sigma_L^{\ell^*}, \sigma_R^{my}; (R, r)) = \begin{cases} \frac{1}{1-\delta_L^2} U_L^+(\ell^*) & \text{for } r \in [2M - \ell^*, 1], \\ \frac{1}{1-\delta_L^2} U_L^+(2M - r) & \text{for } r \in [M, 2M - \ell^*), \\ \frac{1}{1-\delta_L^2} u_L(r) & \text{for } r \in [0, M). \end{cases}$$

Note that for all r, r' such that $r > r'$, $\sigma_L(R, r) \in W(R, r)$ and $\sigma_L(R, r) \neq \sigma_L(R, r') \in W(R, r')$,

$$V_L(\sigma_L^{\ell^*}, \sigma_R^{my}; (R, r)) > V_L(\sigma_L^{\ell^*}, \sigma_R^{my}; (R, r')).$$

Hence, at any state (R, r) such that $\sigma_L(R, r) \in W(R, r)$, party L cannot profit from one-shot deviation ℓ^d such that $\sigma_L(R, r') = \ell$ for some $r' \neq r$. Hence only one-shot deviations $\ell^d \in [0, \ell^*) \cup (M, 1]$ can be profitable for L at some state.

The value of setting $\ell^d \in [0, \ell^*)$ if winning at (R, r) is

$$u_L(\ell^d) + \delta_L u_L(2M - \ell^d) + \delta_L^2 V_L(\sigma_L^{\ell^*}, \sigma_R^{my}; (R, 2M - \ell^d))$$
$$= U_L^+(\ell^d) + \frac{\delta_L^2}{1-\delta_L^2} U_L^+(\ell^*).$$

$\ell^d \in [0, \ell^*)$ is winning only in states (R, r) with $r \in [2M - \ell^d, 1] \cup [0, \ell^d]$. For $r \in [2M - \ell^d, 1]$

$$\begin{aligned} V_L(\sigma_L^{\ell^*}, \sigma_R^{my}; (R, r)) &= \frac{1}{1 - \delta_L^2} U_L^+(l^*) \\ &> U_L^+(\ell^d) + \frac{\delta_L^2}{1 - \delta_L^2} U_L^+(\ell^*), \end{aligned}$$

where the inequality follows from Lemma 1 since $\ell^d < \ell^*$. For $r \in [0, \ell^d]$

$$\begin{aligned} V_L(\sigma_L^{\ell^*}, \sigma_R^{my}; (R, r)) &= \frac{1}{1 - \delta_L^2} u_L(r) \\ &> U_L^+(\ell^d) + \frac{\delta_L^2}{1 - \delta_L^2} U_L^+(\ell^*), \end{aligned}$$

where the inequality follows since $r \leq \ell^d$.

The value of setting $\ell^d \in (M, 1]$ if winning at (R, r) is

$$\frac{1}{1 - \delta_L^2} u_L(\ell^d).$$

$\ell^d \in (M, 1]$ is winning only in states (R, r) with $r \in [2M - \ell^d, M] \cup [\ell^d, 1]$. For $r \in [2M - \ell^d, M]$

$$\begin{aligned} V_L(\sigma_L^{\ell^*}, \sigma_R^{my}; (R, r)) &= \frac{1}{1 - \delta_L^2} u_L(r) \\ &> \frac{1}{1 - \delta_L^2} u_L(\ell^d), \end{aligned}$$

where the inequality follows since $r < \ell^d$. For $r \in [\ell^d, 1]$

$$\begin{aligned} V_L(\sigma_L^{\ell^*}, \sigma_R^{my}; (R, r)) &> \frac{1}{1 - \delta_L^2} u_L(M) \\ &> \frac{1}{1 - \delta_L^2} u_L(\ell^d), \end{aligned}$$

where the first inequality follows since $r > M$, and the second since $\ell^d > M$. Hence, no profitable deviation for L exists and $\sigma_L^{\ell^*}$ is optimal when facing σ_R^{my}.

Now verify the optimality of R's proposed strategy. Given $(\sigma_L^{\ell^*}, \sigma_R^{my})$ compute

$$V_R(\sigma_L^{\ell^*}, \sigma_R^{my}; (L, \ell)) = \begin{cases} u_R(2M - \ell) + \frac{\delta_R}{1 - \delta_R^2} U_R^-(\ell^*) & \text{for } \ell \in [0, \ell^*), \\ \frac{1}{1 - \delta_R^2} U_R^+(\ell) & \text{for } \ell \in [\ell^*, M), \\ \frac{1}{1 - \delta_R^2} u_R(\ell) & \text{for } \ell \in [M, 1). \end{cases}$$

Again, note that for all $\ell < \ell'$, $\sigma_R(L, \ell) \in W(L, \ell)$ and $\sigma_R(L, \ell) \neq \sigma_R(L, \ell') \in W(L, \ell')$

$$V_R(\sigma_L^{\ell^*}, \sigma_R^{my}; (L, \ell)) > V_R(\sigma_L^{\ell^*}, \sigma_R^{my}; (L, \ell')).$$

Hence, at any state (L, ℓ) such that $\sigma_R(L, \ell) \in W(L, \ell)$, party R cannot profit by deviating to any r^d such that $\sigma_R(L, \ell') = r^d$ for some $\ell' \neq \ell$. Hence only one-shot deviations $r^d \in [0, M)$ can be profitable for R at some state. That these cannot be profitable for R follows from a verification similar to that for deviations $\ell^d \in (M, 1]$ for L above. Hence, no profitable deviation for R exists and σ_R^{my} is optimal when facing $\sigma_L^{\ell^*}$. □

Proof of Corollary 1. Consider v_L obtained from u_L by applying some increasing concave transformation. Then for any $\ell \in (0, M)$, $\frac{v_L'(\ell)}{v_L'(2M-\ell)} < \frac{u_L'(\ell)}{u_L'(2M-\ell)}$, and hence ℓ^* approaches M as parties' utilities become more concave. The rest of the claim follows from the discussion in the text. □

A.4 Consistent Markov Perfect Equilibria

The following Lemma characterises convergence paths under consistent strategies.

Lemma 3. *Consider consistent Markov strategies σ_L and σ_R.*

i. If $\sigma_L(R, r) = \ell \in (\max\{2M - r, 0\}, M]$ for some $r > M$, then $\sigma_L(R, r') = \ell$ for all $r' \in [2M - \ell, r)$.

ii. Suppose (σ_L, σ_R) form a consistent equilibrium. If $\sigma_L(R, r) = \ell \in (\max\{2M - r, 0\}, M]$ for some $r > M$, then $\sigma_R(L, \ell') = 2M - \ell'$ for all $\ell' \in [\max\{2M - r, 0\}, \ell)$.

Both statements for R are symmetric.

Proof of Lemma 3. Part i is immediate from the definition of consistent Markov strategies. For part ii, consider consistent equilibrium (σ_L, σ_R), $r > M$ and $\sigma_L(R, r) = \ell > \max\{2M - r, 0\}$. Suppose for some $\ell' \in [\max\{2M - r, 0\}, \ell)$, $\sigma_R(L, \ell') = r' < 2M - \ell'$. There are two cases. First, suppose that $r' \geq 2M - \ell$. Consider the one-shot deviation by R to $2M - \ell'$ in state (L, ℓ'). The payoff to this deviation is

$$\begin{aligned} &u_R(2M - \ell') + \delta_R V_R(\sigma_L, \sigma_R; (R, 2M - \ell')) \\ &\quad > u_R(r') + \delta_R V_R(\sigma_L, \sigma_R; (R, r')) \\ &\quad = V_R(\sigma_L, \sigma_R; (L, \ell')). \end{aligned}$$

a contradiction. The inequality follows since $\sigma_L(R, r') = \ell$ for all $r' \in [2M - \ell, r]$ and $r' < 2M - \ell'$.

Second, suppose $r' < 2M - \ell$. Then by the part i of the lemma it must be that $\sigma_R(L, \ell'') = r'$ for all $\ell'' \in [\ell', 2M - r']$. By reversing the roles in the proof of the first case above, it can be seen that L can profitably deviate to $2M - r'$ at (R, r'). □

The following lemma characterises payoffs on consistent equilibrium convergence paths.

Lemma 4. *Consider long-run policy outcome $\hat{\ell} > \max\{\ell^*, 2M - r^*\}$, associated consistent equilibrium (σ_L, σ_R) and convergence path $\{y^i\} \to \hat{\ell}$ starting from some state. Take state $(R, 2M - y^i)$ such that $\sigma_L(R, 2M - y^i) = y^{i+1}$ with $i > 1$. Then*

$$V_L(\sigma_L, \sigma_R; (R, 2M - y^i)) = \frac{1}{1 - \delta_L^2} U_L^+(y^i). \tag{8}$$

Furthermore,

$$\frac{1}{1 - \delta_L^2} U_L^+(y^i) = u_L(y^{i+1}) + \frac{\delta_L}{1 - \delta_L^2} U_L^-(y^{i+2}). \tag{9}$$

The case of state (L, y^i) such that $\sigma_R(L, y^i) = 2M - y^{i+1}$ with $i > 1$ is symmetric.

Proof of Lemma 4. Consider state $(R, 2M - y^i)$ such that $\sigma_L(R, 2M - y^i) = y^{i+1}$ with $i > 1$. Since $\hat{\ell} > \max\{\ell^*, 2M - r^*\}$, we have that $y^i < y^{i+1}$ for all i. Since $i > 1$, by Lemma 3 there exists $\epsilon > 0$ such that for all $\ell \in (y^i - \epsilon, y^i]$, $\sigma_R(L, \ell) = 2M - y^i$. For any $\bar{\epsilon} \in (0, \epsilon)$, consider one-shot deviation by L at $(R, 2M - y^i + \bar{\epsilon})$ to $y^{i+1} = \sigma_L(R, 2M - y^i)$. The value to this deviation is given by

$$\begin{aligned} V_L(\sigma_L, \sigma_R; (R, 2M - y^i)) &\leq V_L(\sigma_L, \sigma_R; (R, 2M - y^i + \bar{\epsilon})) \\ &= u_L(y^i - \epsilon) + \delta_L u_L(2M - y^i) \\ &\quad + \delta_L^2 V_L(\sigma_L, \sigma_R; (R, 2M - y^i)), \end{aligned}$$

where the inequality follows from equilibrium. This yields

$$V_L(\sigma_L, \sigma_R; (R, 2M - y^i)) \leq \frac{1}{1 - \delta_L^2} [u_L(y^i - \bar{\epsilon}) + \delta_L u_L(2M - y^i)]$$

for any $\bar{\epsilon} \in (0, \epsilon)$, and hence by the continuity of u_L

$$V_L(\sigma_L, \sigma_R; (R, 2M - y^i)) \leq \frac{1}{1 - \delta_L^2} U_L^+(y^i).$$

Lemma 2 yields the opposite inequality and hence

$$V_L(\sigma_L, \sigma_R; (R, 2M - y^i)) = \frac{1}{1 - \delta_L^2} U_L^+(y^i).$$

The final claim of the lemma follow since

$$\begin{aligned} V_L(\sigma_L, \sigma_R; (R, 2M - y^i)) &= u_L(y^{i+1}) + \delta_L u_L(2M - y^{i+2}) \\ &\quad + \delta_L^2 V_L(\sigma_L, \sigma_R; (R, 2M - y^{i+2})). \end{aligned}$$

□

A.5 Bounded Moderation

To construct the bound on long-run moderation, define mappings $\alpha_L : [\max\{\ell^*, 2M - r^*\}, M] \to (0,1]$ and $\alpha_R : [\max\{\ell^*, 2M - r^*\}, M] \to (0,1]$ such that

$$\begin{aligned} \frac{u_L'(\ell)}{u_L'(2M-\ell)} &= \frac{\delta_L}{\delta_L^2 + \alpha_L(\ell)(1-\delta_L^2)} \text{ and} \\ \frac{u_R'(2M-\ell)}{u_R'(\ell)} &= \frac{\delta_R}{\delta_R^2 + \alpha_R(\ell)(1-\delta_R^2)}. \end{aligned} \tag{10}$$

Define ℓ^{**} such that $\alpha_L(\ell^{**}) + \alpha_R(\ell^{**}) = 1$. First show that α_L, α_R and $\ell^{**} \in (\max\{\ell^*, 2M - r^*\}, M)$ are well-defined. To see this, note that since u_L is concave $\frac{u_L'(\ell)}{u_L'(2M-\ell)}$ is strictly increasing in $\ell \in [\ell^*, M]$, with a minimum of δ_L and a maximum of 1. Now $\frac{\delta_L}{\delta_L^2+\alpha_L(1-\delta_L^2)}$ is strictly decreasing in $\alpha_L \in [0,1]$, with a minimum of δ_L and a maximum of $\frac{1}{\delta_L}$. $\alpha_L(\ell)$ is well-defined for $\ell \in [\max\{\ell^*, 2M - r^*\}, M]$ since $\frac{u_L'(\max\{\ell^*,2M-r^*\})}{u_L'(2M-\max\{\ell^*,2M-r^*\})} \geq \delta_L$. Also, $\alpha_L(\ell) \in (0,1]$ for all ℓ since $\alpha_L(M) = \frac{\delta_L}{1+\delta_L}$ and $\alpha_L(\ell^*) = 1$. Similarly, $\alpha_R(l)$ is well-defined. Furthermore, $\alpha_L(\ell) + \alpha_R(\ell)$ is strictly decreasing in $\ell \in [\max\{\ell^*, 2M - r^*\}, M]$, with $\alpha_L(M) + \alpha_R(M) < 1$ and $\alpha_L(\max\{\ell^*, 2M - r^*\}) + \alpha_R(\max\{\ell^*, 2M - r^*\}) > 1$. Thus $\ell^{**} \in (\max\{\ell^*, 2M - r^*\}, M)$.

To understand the derivation of α_L and α_R, consider y^i, $y^{i+2} = y^i + \Delta$ for some $\Delta > 0$ and $\alpha_L \in [0,1]$ such that

$$\frac{1}{1-\delta_L^2} U_L^+(y^i) = u_L(y^i + \alpha_L \Delta) + \frac{\delta_L}{1-\delta_L^2} U_L^-(y^i + \Delta). \tag{11}$$

α_L is well-defined since evaluating (11) at $\alpha_L = 0$ yields

$$\begin{aligned} \frac{1}{1-\delta_L^2} U_L^+(y^i) &= u_L(y^i) + \frac{\delta_L}{1-\delta_L^2} U_L^-(y^i) \\ &< u_L(y^i) + \frac{\delta_L}{1-\delta_L^2} U_L^-(y^i + \Delta), \end{aligned}$$

while evaluating (11) at $\alpha_L = 1$ yields

$$\begin{aligned} \frac{1}{1-\delta_L^2} U_L^+(y^i) &> \frac{1}{1-\delta_L^2} U_L^+(y^i + \Delta) \\ &= u_L(y^i + \Delta) + \frac{\delta_L}{1-\delta_L^2} U_L^-(y^i + \Delta), \end{aligned}$$

where both inequalities follow from Lemma 1. The limit of (11) as $\Delta \to 0$ yields that α_L is determined by (10) evaluated at y^i.

Proof of Proposition 4. The following claim establishes the bound on the moderation of robust long-run policy outcomes: *If policy $\hat{\ell} \leq M$ is a robust long-run policy outcome under some consistent equilibrium, then $\hat{\ell} \leq \ell^{**}$.* To show this, the following lemma establishes the properties of the recursive equation (1) that determine consistent equilibrium convergence path policies that allow us to determine possible convergence points.

Lemma 5. *Consider robust long-run policy outcome $\hat{\ell}$ under consistent equilibrium (σ_L, σ_R) and associated convergence path $\{y^i\}$ starting from some state.*

i. Suppose that

$$\frac{u_L'(\hat{\ell})}{u_L'(2M-\hat{\ell})} < \frac{\delta_L}{\delta_L^2 + \alpha_L(1-\delta_L^2)} \tag{12}$$

for some $\alpha_L \in [0,1]$ and that $\sigma_L(R, 2M - y^{i-1}) = y^i$ for some i. Then $y^i - y^{i-1} > \frac{\alpha_L}{1-\alpha_L}(y^{i+1} - y^i)$.

ii. Conversely, suppose that

$$\frac{u_L'(y^j)}{u_L'(2M-y^j)} > \frac{\delta_L}{\delta_L^2 + \alpha_L(1-\delta_L^2)} \tag{13}$$

for some $\alpha_L \in [0,1]$ and that $\sigma_L(R, 2M - y^{j-1}) = y^j$. Then $y^i - y^{i-1} < \frac{\alpha_L}{1-\alpha_L}(y^{i+1} - y^i)$ for all $i \geq j$.

The case for party R is symmetric.

Proof of Lemma 5. To prove part i of the lemma, first prove the following claim: *Suppose that for some $\alpha_L \in [0,1]$ and y, Δ such that $y - \Delta \in [\ell^*, M]$*

$$U_L^+(y-\Delta) - U_L^+(y-(1-\alpha_L)\Delta) \leq \delta_L[U_L^-(y) - U_L^-(y-(1-\alpha_L)\Delta)], \tag{14}$$

then for any $y' \leq y$ and $n \in \mathbf{N}$ such that $y' - 2^n\Delta \in [\ell^*, M]$

$$U_L^+(y'-2^n\Delta) - U_L^+(y'-2^n(1-\alpha_L)\Delta) \leq \delta_L[U_L^-(y') - U_L^-(y'-2^n(1-\alpha_L)\Delta)] \tag{15}$$

with the inequality strict if $y' \neq y$ or $n > 0$. Note that (14) implies that on an infinite convergence path for some consistent equilibrium for which $\sigma_R(L, \ell) = 2M - (y - \Delta)$, $\sigma_L(R, 2M - (y-\Delta)) - y \geq \alpha_L\Delta$. The claim states that if party R's successive policy choices on some consistent equilibrium convergence path are $2M - (y - \Delta)$ and $2M - y$ and party L is (weakly) willing to moderate to $y - (1-\alpha_L)\Delta$ when in state $(R, 2M - (y - \Delta))$,[41] then in another consistent equilibrium convergence path in which party R's successive policies are $2M - (y' - \Delta')$ and $2M - y'$ with $y' \leq y$, then party L is strictly willing to moderate to $y' - (1-\alpha_L)\Delta'$ in state $(R, 2M - (y' - \Delta'))$, where $\Delta' = 2^n\Delta$ for some $n \in \mathbf{N}$.

To prove the claim, note first that, for $y' \leq y$

$$\begin{aligned} U_L^+(y'-\Delta) - U_L^+(y'-(1-\alpha_L)\Delta) &\leq U_L^+(y-\Delta) - U_L^+(y-(1-\alpha_L)\Delta) \\ &\leq \delta_L[U_L^-(y) - U_L^-(y-(1-\alpha_L)\Delta)] \\ &\leq \delta_L[U_L^-(y') - U_L^-(y'-(1-\alpha_L)\Delta)], \end{aligned}$$

[41] That is, moderate by $\alpha_L\Delta$.

with the first and third inequalities strict if $y' \neq y$. The first inequality follows from the strict concavity of U_L^+, the second from (14), and the third from the strict concavity of U_L^-. Given (14), the above shows that

$$U_L^+(y-2\Delta) - U_L^+(y-(2-\alpha_L)\Delta) < \delta_L[U_L^-(y-\Delta) - U_L^-(y-(2-\alpha_L)\Delta)],$$

and

$$U_L^+(y-(2-\alpha_L)\Delta) - U_L^+(y-2(1-\alpha_L)\Delta) < \delta_L[U_L^-(y-(1-\alpha_L)\Delta) - U_L^-(y-2(1-\alpha_L)\Delta)]. \quad (16)$$

Hence we have that

$$\begin{aligned}
\delta_L[U_L^-(y) - U_L^-(y-2(1-\alpha_L)\Delta)] &= \delta_L[U_L^-(y) - U_L^-(y-(1-\alpha_L)\Delta)] \\
&\quad + \delta_L[U_L^-(y-(1-\alpha_L)\Delta) - U_L^-(y-2(1-\alpha_L)\Delta)] \\
&> U_L^+(y-\Delta) - U_L^+(y-(1-\alpha_L)\Delta) \\
&\quad + U_L^+(y-(2-\alpha_L)\Delta) - U_L^+(y-2(1-\alpha_L)\Delta) \\
&> U_L^+(y-2\Delta) - U_L^+(y-\Delta(2-\alpha_L)) \\
&\quad + U_L^+(y-(2-\alpha_L)\Delta) - U_L^+(y-2(1-\alpha_L)\Delta) \\
&= U_L^+(y-2\Delta) - U_L^+(y-2(1-\alpha_L)\Delta).
\end{aligned}$$

The first inequality follows from (14) and (16), and the second inequality follows from Lemma 1 since $y-(1-\alpha_L)\Delta = y-\Delta(2-\alpha_L)-(y-2\Delta) = \alpha_L\Delta$. The claim follows by applying the above argument recursively.

To complete the proof of part i of Lemma 5, consider (12). This condition guarantees that for arbitrarily small Δ, party L is willing to take up share $\alpha_L\Delta$ of moderation Δ from $y-\Delta$ to y. Hence, there exists some $\tilde{\Delta}$ such that for all $\Delta < \tilde{\Delta}$,

$$U_L^+(\hat{\ell}-\Delta) - U_L^+(\hat{\ell}-(1-\alpha_L)\Delta) < \delta_L[U_L^-(\hat{\ell}) - U_L^-(\hat{\ell}-(1-\alpha_L)\Delta)].$$

Thus, by the earlier claim, for all $y < \hat{\ell}$ and Δ such that $y-\Delta > \ell^*$,

$$U_L^+(y-\Delta) - U_L^+(y-(1-\alpha_L)\Delta) < \delta_L[U_L^-(y) - U_L^-(y-(1-\alpha_L)\Delta)].$$

This implies that for y^i such that $\sigma_L(R, 2M-y^{i-1}) = y^i$, $y^i - y^{i-1} > \frac{\alpha_L}{1-\alpha_L}(y^{i+1}-y^i)$.

The proof of part ii of Lemma 5 follows along the lines of part i. While part i is backward-looking, part ii is forward-looking. That is, part i establishes that if at the limit point of a consistent equilibrium convergence path party L is willing to undertake share α_L of all marginal moderations, then it was also willing to undertake share α_L of all past moderate moves. In contrast. part ii shows that if at some point on a convergence path, party L would be unwilling to undertake share α_L of marginal moderations, then it will undertake less than share α_L of all future moderations on the convergence path. Evidently, part ii is useful to establish conditions for nonconvergence, while part i helps establish conditions for convergence. □

Now to show that moderation is bounded by ℓ^{**}, consider a robust long-run policy outcome $(\hat{\ell}, 2M - \hat{l})$ with $\hat{\ell} > \ell^{**}$ and associated consistent equilibrium (σ_L, σ_R). Consider state (R, r) with $2M - r < \hat{\ell}$ and convergence path $\{y^i\} \to \hat{\ell}$ given (R, r) with $\sigma_L(R, 2M - y^0) = y^1$. Fix n such that $y^n > \ell^{**}$ and $\sigma_L(R, 2M - y^n) = y^{n+1}$. Hence

$$\begin{aligned}\frac{u_L'(y^n)}{u_L'(2M - y^n)} &> \frac{u_L'(\ell^{**})}{u_L'(2M - \ell^{**})} \\ &= \frac{\delta_L}{\delta_L^2 + \alpha_L(\ell^{**})(1 - \delta_L^2)},\end{aligned}$$

and hence by part i of Lemma 5, for all $j \geq n$,

$$y^{j+1} - y^j < \frac{\alpha_L(\ell^{**})}{1 - \alpha_L(\ell^{**})}(y^{j+2} - y^{j+1}).$$

Similarly, if $j \geq n + 1$ and $\sigma_R(L, y^j) = 2M - y^{j+1}$ then

$$y^{j+1} - y^j < \frac{\alpha_R(\ell^{**})}{1 - \alpha_R(\ell^{**})}(y^{j+2} - y^{j+1}).$$

This yields that for all $j \geq n + 1$,

$$\begin{aligned}y^{j+1} - y^j &< \frac{\alpha_L(\ell^{**})}{1 - \alpha_L(\ell^{**})} \frac{\alpha_R(\ell^{**})}{1 - \alpha_R(\ell^{**})}(y^{j+3} - y^{j+2}) \\ &< (y^{j+3} - y^{j+2}).\end{aligned}$$

Hence the convergence path $\{y^i\} \to \hat{\ell}$ contains a nonconverging subsequence, a contradiction.

To show that the bound on long-run moderation is tight, given a strictly increasing sequence $\{y^i\} \to \hat{\ell}$ with $y^0 = \ell^*$ and y^i, y^{i+1} and y^{i+2} satisfying the conditions of Lemma 4 for all $i \geq 1$, consider the following strategies

$$\sigma_{L^*}^{\hat{\ell}}(R, r) = \begin{cases} \ell^* & \text{for all } r \geq 2M - \ell^*, \\ 2M - r & \text{for all } r \in (2M - y^i, 2M - y^{i-1}) \text{ with } i > 0 \text{ odd}, \\ y^i & \text{for all } r \in [2M - y^i, 2M - y^{i-1}] \text{ with } i > 0 \text{ even}, \\ 2M - r & \text{for all } r \in [M, 2M - \hat{\ell}], \\ Out & \text{for all } r < M. \end{cases}$$

$$\sigma_R^{\hat{\ell}}(L, \ell) = \begin{cases} 2M - \ell & \text{for all } \ell < \ell^*, \\ y^i & \text{for all } \ell \in [y^{i-1}, y^i] \text{ with } i > 0 \text{ odd}, \\ 2M - \ell & \text{for all } \ell \in (y^{i-1}, y^i) \text{ with } i > 0 \text{ even}, \\ 2M - \ell & \text{for all } \ell \in [\hat{\ell}, M], \\ Out & \text{for all } \ell > M. \end{cases}$$

If instead $\ell^* < 2M - r^*$, then for robust long-run policy outcome $(\hat{\ell}, 2M - \hat{\ell})$ with $\hat{\ell} > 2M - r^*$, strategies $(\sigma_L^{\hat{\ell}}, \sigma_{R^*}^{\hat{\ell}})$ can be constructed in a similar manner with the roles of the parties reversed.

The following claim verifies that these strategies form an equilibrium: *Suppose that* $\ell^* \geq 2M - r^*$. *Given* $\hat{\ell} > \ell^*$ *and a strictly increasing sequence* $\{y^i\} \to \hat{\ell}$ *with* $y^0 = \ell^*$ *and* y^i, y^{i+1} *and* y^{i+2} *satisfying the conditions of Lemma 4 for all* $i \geq 1$, *strategies* $(\sigma_{L^*}^{\hat{\ell}}, \sigma_R^{\hat{\ell}})$ *form a form a consistent equilibrium under which* $\hat{\ell}$ *is a robust long-run policy outcome. The equilibrium* $(\sigma_L^{\hat{\ell}}, \sigma_{R^*}^{\hat{\ell}})$ *in the case of* $\ell^* < 2M - r^*$ *can be determined similarly.* To show this, suppose $\ell^* \geq 2M - r^*$. First verify the optimality of L's proposed strategy. Given $(\sigma_{L^*}^{\hat{\ell}}, \sigma_R^{\hat{\ell}})$ and the conditions of the lemma for $\{y^i\}$, compute

$$V_L(\sigma_{L^*}^{\hat{\ell}}, \sigma_R^{\hat{\ell}}; (R, r)) = \begin{cases} u_L(\ell^*) + \frac{\delta_L}{1-\delta_L^2} U_L^-(y^1) \\ \qquad \text{for } r \in [2M - \ell^*, 1], \\ u_L(2M - r) + \delta_L u_L(2M - y^i) + \frac{\delta_L^2}{1-\delta_L^2} U_L^+(y^{i+1}) \\ \qquad \text{for } r \in (2M - y^i, 2M - y^{i-1}) \text{ with } i > 0 \text{ odd}, \\ u_L(y^i) + \frac{\delta_L}{1-\delta_L^2} U_L^-(y^{i+1}) \\ \qquad \text{for } r \in [2M - y^i, 2M - y^{i-1}] \text{ with } i > 0 \text{ even}, \\ \frac{1}{1-\delta_L^2} U_L^+(2M - r) \text{ for } r \in [M, 2M - \bar{\ell}], \\ \frac{1}{1-\delta_L^2} u_L(r) \text{ for } r \in [0, M). \end{cases}$$

Note that for all r, r' such that $r > r'$, $\sigma_L(R, r) \in W(R, r)$ and $\sigma_L(R, r) \neq \sigma_L(R, r') \in W(R, r')$,

$$V_L(\sigma_{L^*}^{\hat{\ell}}, \sigma_R^{\hat{\ell}}; (R, r)) > V_L(\sigma_{L^*}^{\hat{\ell}}, \sigma_R^{\hat{\ell}}; (R, r')).$$

Hence, at any state (R, r) such that $\sigma_L(R, r) \in W(R, r)$, party L cannot profit by deviating to any ℓ^d such that $\sigma_L(R, r') = \ell$ for some $r' \neq r$. Hence only one-shot deviations $\ell^d \in [0, \ell^*) \cup \left(\bigcup_{i>0, i \text{ even}} [y^{i-1}, y^i)\right) \cup (M, 1]$ can be profitable for L at some state. The value to setting $\ell^d \in [0, \ell^*)$ if winning at (R, r) is

$$U_L^+(\ell^d) + \delta_L^2 V_L(\sigma_{L^*}^{\hat{\ell}}, \sigma_R^{\hat{\ell}}; (R, 2M - \ell^d)) = U_L^+(\ell^d) + \delta_L^2 V_L(\sigma_{L^*}^{\hat{\ell}}, \sigma_R^{\hat{\ell}}; (R, 2M - \ell^*)).$$

$\ell^d \in [0, \ell^*)$ is winning only in states (R, r) with $r \in [2M - \ell^d, 1] \cup [0, \ell^d]$. For $r \in [2M - \ell^d, 1]$

$$\begin{aligned} V_L(\sigma_{L^*}^{\hat{\ell}}, \sigma_R^{\hat{\ell}}; (R, r)) &> U_L^+(\ell^d) + \delta_L^2 V_L(\sigma_{L^*}^{\hat{\ell}}, \sigma_R^{\hat{\ell}}; (R, 2M - \ell^d)) \\ &= U_L^+(\ell^d) + \delta_L^2 V_L(\sigma_{L^*}^{\hat{\ell}}, \sigma_R^{\hat{\ell}}; (R, 2M - r)). \end{aligned}$$

since

$$\begin{aligned} V_L(\sigma^{\hat\ell}_{L^*},\sigma^{\hat\ell}_R;(R,r)) &= u_L(\ell^*)+\frac{\delta_L}{1-\delta_L^2}U_L^-(y^1)\\ &> \frac{1}{1-\delta_L^2}U_L^+(\ell^*)\\ &> \frac{1}{1-\delta_L^2}U_L^+(\ell^d). \end{aligned}$$

The first inequality follows from Lemma 1 and the fact that $y^1>\ell^*$, and the second inequality from Lemma 1 and the fact that $\ell^d<\ell^*$. That a deviation to $\ell^d\in[0,\ell^*)$ in states (R,r) with $r\in[0,\ell^d]$ is not profitable follows from an argument similar to that in Lemma 4. The value of setting $\ell^d\in[y^{i-1},y^i)$ for $i>0$ odd if winning at (R,r) is

$$U_L^+(\ell^d)+\delta_L^2 V_L(\sigma^{\hat\ell}_{L^*},\sigma^{\hat\ell}_R;(R,2M-\ell^d)).$$

$\ell^d\in[y^{i-1},y^i)$ is winning only in states (R,r) with $r\in[2M-\ell^d,1]\cup[0,\ell^d]$. Consider

$$\begin{aligned} V_L(\sigma^{\hat\ell}_{L^*},\sigma^{\hat\ell}_R;(R,2M-y^i)) &= \frac{1}{1-\delta_L^2}U_L^+(y^{i-1})\\ &= U_L^+(y^{i-1})+\delta_L^2 V_L(\sigma^{\hat\ell}_{L^*},\sigma^{\hat\ell}_R;(R,2M-y^{i-1}))\\ &\geq U_L^+(\ell^d)+\delta_L^2 V_L(\sigma^{\hat\ell}_{L^*},\sigma^{\hat\ell}_R;(R,2M-\ell^d)), \end{aligned}$$

where the inequality follows from Lemma 1 and the fact that $\ell^*<y^{i-1}\leq\ell^d$ and the fact that $V_L(\sigma^{\hat\ell}_{L^*},\sigma^{\hat\ell}_R;(R,2M-y^{i-1}))=V_L(\sigma^{\hat\ell}_{L^*},\sigma^{\hat\ell}_R;(R,2M-\ell^d))$. Hence, the value to ℓ^d is weakly smaller than the value following action $y^i=\sigma_L(R,2M-y^i)$, and hence for all states (R,r) with $r\in[2M-\ell^d,1]$ deviation to ℓ^d by L cannot be profitable. That a deviation to $\ell^d\in[y^{i-1},y^i)$ in states (R,r) with $r\in[0,\ell^d]$ is not profitable follows from an argument similar to that in the case of equilibrium $(\sigma_L^{\ell^*},\sigma_R^{my})$, as does the argument that there is no profitable deviation to $\ell^d\in(M,1]$.

Arguments very similar to those for L above can determine R's payoffs under $(\sigma^{\hat\ell}_{L^*},\sigma^{\hat\ell}_R)$ and verify that it constitutes an equilibrium. Clearly $\hat\ell$ is a robust long-run policy outcome under $(\sigma^{\hat\ell}_{L^*},\sigma^{\hat\ell}_R)$ since policy dynamics have $\hat\ell$ as a limit point starting from all more extreme states.

To complete the proof of Proposition 4, let Y be the set of increasing extended real-valued sequences.

Definition 7. Define mapping $B:(\ell^*,M]\to Y$ such that $B(y)^0=\ell^*$, $B(y)^1=y$, for each $i\geq 2$ with i even $B(y)^i$ solves

$$U_L^+(B(y)^{i-2})-U_L^+(B(y)^{i-1})=\delta_L[U_L^-(B(y)^i)-U_L^-(B(y)^{i-1})], \tag{17}$$

and for each $i\geq 3$ with i odd, $B(y)^i$ solves

$$U_R^+(B(y)^{i-2})-U_R^+(B(y)^{i-1})=\delta_R[U_R^-(B(y)^i)-U_R^-(B(y)^{i-1})], \tag{18}$$

if solutions $B(y)^i \leq M$ exist to (17) and/or (18). If not, set $B(y)^i = \infty$ for all $j \geq i$. Define mapping $\Gamma : (\ell^*, M] \to \mathbf{R} \cup \{\infty\}$ such that $\Gamma(y) = \lim_{i\to\infty} B^i(y)$.

Equations (17) and (18) restate the payoff conditions of Lemma 4. Suppose that $\ell^* \geq 2M - r^*$ and that there exists a consistent equilibrium under which $\hat{\ell} \in (\ell^*, \ell^{**}]$ is a robust long-run policy outcome. In that case, there exists a convergence path $\{y^i\} \to \hat{\ell}$ from state (L, ℓ^*). Suppose that in state (L, ℓ^*) party R selects policy $2M - y$ for $y \in (\ell^*, M]$. The mapping B recovers the full sequence of equilibrium convergence path policies. When no such path exists, we have $B(y)^i = \infty$ for some i. Iteration on B yields a candidate for the sequence posited in the claim for equilibrium $(\sigma^{\hat{\ell}}_{L^*}, \sigma^{\hat{\ell}}_R)$, which is acceptable if the limit of of $B(y)$, that is $\Gamma(y)$, is contained in $(\ell^*, \ell^{**}]$. The following claim makes this precise: *Mapping B is such that*

i. The mapping Γ is well-defined, increasing, strictly increasing on $\{y : \Gamma(y) < \infty\}$, right-continuous on $\{y : \Gamma(y) < \ell^{**}\}$ and left-continuous on $\{y : \Gamma(y) < \infty\}$.

ii. For any $\hat{\ell} \in (\ell^*, \ell^{**}]$, there exists y such that $\Gamma(y) = \hat{\ell}$.

iii. A strictly increasing sequence $\{y^i\} \to \hat{\ell}$ with $y^0 = \ell^*$ and y^i,y^{i+1} and y^{i+2} satisfying the conditions of Lemma 4 for all $i \geq 1$.

To show this, not that for $y^1 \in (\ell^*, M]$, $\Gamma(y^1)$ is the limit an increasing extended real-valued sequence and hence is well-defined. For the monotonicity of Γ, consider $y^1, \tilde{y}^1 \in (\ell^*, M]$ such that $y^1 < \tilde{y}^1$, along with induced sequences $\{B(y^1)^i\} = \{y^i\}$ and $\{B(\tilde{y}^1)^i\} = \{\tilde{y}^i\}$. First show that for $i \geq 1$, whenever $\infty > \tilde{y}^{i-1} \geq y^{i-1}$, $\infty > \tilde{y}^i > y^i$, $\tilde{y}^i - \tilde{y}^{i-1} > y^i - y^{i-1}$, and $y^{i+1}, \tilde{y}^{i+1} < \infty$, it is the case that $\tilde{y}^{i+1} - \tilde{y}^i > y^{i+1} - y^i$ and $\tilde{y}^{i+1} > y^{i+1}$. Suppose $\tilde{y}^{i-1} - \epsilon = y^{i-1}$, where $\epsilon \geq 0$. Hence

$$\begin{aligned} U^+_L(\tilde{y}^{i-1} - \epsilon) - U^+_L(\tilde{y}^i - \epsilon) - &\delta_L[U^-_L(y^{i+1}) - U^-_L(\tilde{y}^i - \epsilon)] \\ &> U^+_L(y^{i-1}) - U^+_L(y^i) - \delta_L[U^-_L(y^{i+1}) - U^-_L(y^i)] \\ &= 0, \end{aligned}$$

where the inequality follows by Lemma 1 since $\tilde{y}^i - y^i > \epsilon$. Define $\bar{y}^{i+1}$ such that

$$U^+_L(\tilde{y}^{i-1} - \epsilon) - U^+_L(\tilde{y}^i - \epsilon) - \delta_L[U^-_L(\bar{y}^{i+1}) - U^-_L(\tilde{y}^i - \epsilon)] = 0.$$

It must be that $\bar{y}^{i+1} > y^{i+1}$. By Lemma 1, it is also the case that

$$\begin{aligned} U^+_L(\tilde{y}^{i-1}) - U^+_L(\tilde{y}^i) - &\delta_L[U^-_L(\bar{y}^{i+1} + \epsilon) - U^-_L(\tilde{y}^i)] \\ &> U^+_L(\tilde{y}^{i-1} - \epsilon) - U^+_L(\tilde{y}^i - \epsilon) - \delta_L[U^-_L(\bar{y}^{i+1}) - U^-_L(\tilde{y}^i - \epsilon)] \\ &= 0, \end{aligned}$$

and hence $\tilde{y}^{i+1} > \bar{y}^{i+1} + \epsilon > y^{i+1}$ and $\tilde{y}^{i+1} - \tilde{y}^i > \bar{y}^{i+1} - \tilde{y}^i - \epsilon > y^{i+1} - y^i$. By induction, if $y^1, \tilde{y}^1 \in \{y : \Gamma(y) < \infty\}$, this implies that for each $i \geq 1$, $\tilde{y}^i > y^i$, and

$$\begin{aligned}
\Gamma(\tilde{y}^1) &= \lim_{i\to\infty} \tilde{y}^i \\
&> \lim_{i\to\infty} y^i \\
&= \Gamma(y^1).
\end{aligned}$$

The above argument also shows that if $y^1 < \tilde{y}^1$, then $y^i < \tilde{y}^i$ for all i such that $\tilde{y}^i < \infty$, and hence that $\Gamma(y^1) \leq \Gamma(\tilde{y}^1)$.

Suppose Γ is not right-continuous at y^1, and that $\Gamma(y^1) < \ell^{**}$. Then there exists $\epsilon > 0$ such that for any $\delta > 0$, $\Gamma(y^1 + \delta) - \Gamma(y^1) > \epsilon$. Take $\bar{\epsilon} \in (0, \min\{\epsilon, \ell^{**} - \Gamma(y^1)\})$. Hence $\Gamma(y^1) + \tilde{\epsilon} < \ell^{**}$ Consider $\tilde{y}^1 \in (y^1, y^1 + \delta)$ and associated sequence $\{\tilde{y}^i\}$. Since $\Gamma(y^1) + \bar{\epsilon} < \ell^{**}$, by part ii of Lemma 5 there exist α_L and α_R with $\alpha_L + \alpha_R > 1$ such that for any $\{\bar{y}^i\} \to \Gamma(\bar{y}^1)$ with $\Gamma(\bar{y}^1) \leq \Gamma(y^1) + \bar{\epsilon}$, $\bar{y}^{i+1} - \bar{y}^i < \frac{\alpha_L}{1-\alpha_L}(\bar{y}^i - \bar{y}^{i-1})$, $\bar{y}^i - \bar{y}^{i-1} < \frac{\alpha_R}{1-\alpha_R}(\bar{y}^{i-1} - \bar{y}^{i-2})$ and

$$\lim_{i\to\infty} \bar{y}^i < \bar{y}^0 + (\bar{y}^1 - \bar{y}^0)\frac{\frac{\alpha_L}{1-\alpha_L}(1 + \frac{\alpha_R}{1-\alpha_R})}{1 - \frac{\alpha_L}{1-\alpha_L}\frac{\alpha_R}{1-\alpha_R}}.$$

Conversely, if $\bar{y}^0 + (\bar{y}^1 - \bar{y}^0)\frac{\frac{\alpha_L}{1-\alpha_L}(1+\frac{\alpha_R}{1-\alpha_R})}{1-\frac{\alpha_L}{1-\alpha_L}\frac{\alpha_R}{1-\alpha_R}} \leq \Gamma(y^1) + \bar{\epsilon}$, then it must be that $\Gamma(\tilde{y}^1) < \Gamma(y^1) + \bar{\epsilon}$. Since $\{y^i\} \to \Gamma(y^1)$, there exists $n \in \mathbf{N}$ such that

$$y^i + (y^{i+1} - y^i)\frac{\frac{\alpha_L}{1-\alpha_L}(1 + \frac{\alpha_R}{1-\alpha_R})}{1 - \frac{\alpha_L}{1-\alpha_L}\frac{\alpha_R}{1-\alpha_R}} < \Gamma(y^1) + \frac{\bar{\epsilon}}{2}$$

for all $i \geq n$. Fix $j \geq n$. Since for all $i \geq 1$, $\tilde{y}^{i+1}$ is a continuous function of $\tilde{y}^i$ and $\tilde{y}^{i-1}$, $\tilde{y}^1$ can be found such that $\tilde{y}^j - y^j < \frac{\bar{\epsilon}}{4}$ and $(\tilde{y}^{j+1} - \tilde{y}^j) - (y^{j+1} - y^j) < \frac{\bar{\epsilon}}{4}\frac{1-\frac{\alpha_L}{1-\alpha_L}\frac{\alpha_R}{1-\alpha_R}}{\frac{\alpha_L}{1-\alpha_L}(1+\frac{\alpha_R}{1-\alpha_R})}$. Then it follows that

$$\begin{aligned}
\tilde{y}^j + (\tilde{y}^{j+1} - \tilde{y}^j)\frac{\frac{\alpha_L}{1-\alpha_L}(1 + \frac{\alpha_R}{1-\alpha_R})}{1 - \frac{\alpha_L}{1-\alpha_L}\frac{\alpha_R}{1-\alpha_R}} &< y^j + \frac{\bar{\epsilon}}{4} + (y^{j+1} - y^j)\frac{\frac{\alpha_L}{1-\alpha_L}(1 + \frac{\alpha_R}{1-\alpha_R})}{1 - \frac{\alpha_L}{1-\alpha_L}\frac{\alpha_R}{1-\alpha_R}} + \frac{\bar{\epsilon}}{4} \\
&< \Gamma(y^1) + \bar{\epsilon}.
\end{aligned}$$

Hence $\Gamma(\tilde{y}^1)$ is such that $\Gamma(\tilde{y}^1) < \Gamma(y^1) + \bar{\epsilon}$, a contradiction.

Suppose Γ is not left-continuous at y^1, and that $\Gamma(y^1) < \infty$. Then there exists $\epsilon > 0$ such that for any $\delta > 0$, $\Gamma(y^1) - \Gamma(y^1 - \delta) > \epsilon$. Take $j \in \mathbf{N}$ such that $y^j > \Gamma(y^1) - \epsilon + \eta$ for $\eta \in (0, \epsilon)$. Fix $\tilde{y}^1$ such that $y^j - \tilde{y}^j < \eta$. Hence $\tilde{y}^j > y^j - \eta > \Gamma(y^1) - \epsilon$, and hence $\Gamma(\tilde{y}^1) > \Gamma(y^1) - \epsilon$, since $\{\tilde{y}^i\}$ is increasing, a contradiction.

The set $\{y : \Gamma(y) < \ell^{**}\}$ is nonempty since $\lim_{y^1\to\ell^*} \Gamma(y^1) = \ell^*$, and hence by continuity of Γ on $\{y : \Gamma(y) < \ell^{**}\}$, for each ℓ with $\ell < \ell^{**}$, there exists y such that $\Gamma(y) = \ell$. Finally, since Γ is left-continuous on $\{y : \Gamma(y) < \infty\}$, there exist a y such that $\Gamma(y) = \ell^{**}$. □

Proof of Corollary 2. Corollary 2 follows from 1 and the properties of ℓ^{**} established above. □

A.6 Forward-looking Voters

Proof of Proposition 5. Consider consistent equilibrium convergence path $\{y^i\}$ with associated consistent equilibrium strategies (σ_L, σ_R). Assume for now that on convergence paths, the median voter votes according to σ_M^{my}. To construct strategies (σ'_L, σ'_R) in the game with forward-looking voters, the profile (σ_L, σ_R) needs to be modified in two ways. First, consider policy y^i such that $\sigma_L(R, 2M - y^i) = y^{i+1}$. For $x \in [y^i, y^{i+1})$, define $z^{i+1}(x) \in [y^i, x)$ such that

i. If

$$u_M(x) - u_M(y^i) > \delta_M \left[V_M(\sigma_L, \sigma_R, \sigma_M^{my}; (R, 2M - y^i)) - \frac{1}{1-\delta_M} u_M(x) \right],$$

then $z^{i+1}(x)$ solves

$$u_M(x) - u_M(z^{i+1}(x)) = \delta_M \left[V_M(\sigma_L, \sigma_R, \sigma_M^{my}; (R, 2M - y^i)) - \frac{1}{1-\delta_M} u_M(x) \right].$$

ii. If

$$u_M(x) - u_M(y^i) \leq \delta_M \left[V_M(\sigma_L, \sigma_R, \sigma_M^{my}; (R, 2M - y^i)) - \frac{1}{1-\delta_M} u_M(x) \right],$$

then $z^{i+1}(x) = y^i$.

That is, R commits to $2M - z^{i+1}(x)$ as 'punishment' for L being in power with policy x as opposed to y^{i+1} and $z^{i+1}(x)$ is the most extreme such punishment that the median voter supports. For y^i such that $\sigma_R(L, y^i) = 2M - y^{i+1}$ and for $x \in (2M - y^{i+1}, 2M - y^i]$, $z^{i+1}(x) \in [y^i, 2M - x)$ can be defined symmetrically.

Second, given some σ_M and $\ell > M$, let $\bar{r}(\ell) > \ell$ be the most extreme commitment by R in state (L, ℓ) that the median voter supports and that R has the incentive to make. If the median voter accepts $\bar{r}(\ell)$, then policy dynamics are 'freed' from the policy traps of equilibria with myopic voters and, after at most one period, the equilibrium path rejoins convergence path $\{y^i\}$. For $r < M$, define $\bar{\ell}(r) < r$ symmetrically. Note that, as with the functions $\{z^{i+1}(\cdot)\}$, $\bar{r}(\cdot)$ and $\bar{\ell}(\cdot)$ are determined only by how parties and the median voter evaluate convergence paths under $(\sigma_L, \sigma_R, \sigma_M^{my})$. Now define strategy σ'_R as

$$\sigma'_L(R, r) = \begin{cases} z^{i+1}(r) & \text{if } r \in (2M - y^{i+1}, 2M - y^i] \text{ for } y^i \text{ such that } \sigma_R(L, y^i) = 2M - y^{i+1}, \\ \bar{\ell}(r) & \text{if } r < M \text{ and } u_L(\bar{\ell}(r)) + \delta_L V_L(\sigma_L, \sigma_R; (L, \bar{\ell}(r))) \geq \frac{1}{1-\delta_L} u_L(r) \\ \sigma_L(R, r) & \text{otherwise.} \end{cases}$$

σ'_R can be defined symmetrically. Let σ_M be a best-response to (σ'_L, σ'_R) in which the median voter supports the opposition party when indifferent. Given the parties' strategies, the median voter has no incentive to vote for the incumbent on a convergence path.

Hence, given convergence path policy y^i such that $\sigma_L(R, 2M - y^i) = y^{i+1}$, we have that $V_K(\sigma'_L, \sigma'_R, \sigma_M; (R, 2M - y^i)) = V_K(\sigma_L, \sigma_R, \sigma_M^{my}; (R, 2M - y^i))$ for $K \in \{L, R, M\}$. I do not describe the median voter's equilibrium strategy explicitly, but instead show how it responds to parties' deviations from the convergence path $\{y^i\}$ to show that parties have no more incentive to deviate from the convergence path under $(\sigma'_L, \sigma'_R, \sigma_M)$ than under $(\sigma_L, \sigma_R, \sigma_M^{my})$.

Consider state (R, r) with $2M - r \in [y^i, y^{i+1})$ for y^i such that $\sigma_R(L, y^i) = 2M - y^{i+1}$. The median voter votes against $\ell \in [y^i, z^{i+1}(r))$ since the payoff to voting in favour of ℓ is

$$u_M(\ell) + \delta_M V_M(\sigma'_L, \sigma'_R, \sigma_M; (L, y^i)) < u_M(r) + \delta_M u_M(z^{i+1}(r)) + \delta_M^2 V_M(\sigma'_L, \sigma'_R, \sigma_M; (L, y^i)),$$

by the definition of $z^{i+1}(r)$, where the right-hand side is the payoff to voting in favour of r. The median voter votes against $\ell \in [y^{i-1}, y^i)$ since the payoff to voting in favour of ℓ is

$$\begin{aligned} &u_M(\ell) + \delta_M u_M(z^i(\ell)) + \delta_M^2 V_M(\sigma'_L, \sigma'_R, \sigma_M; (R, 2M - y^{i-1})) \\ &\qquad < u_M(r) + \delta_M u_M(z^{i+1}(r)) + \delta_M^2 V_M(\sigma'_L, \sigma'_R, \sigma_M; (L, y^i)), \end{aligned}$$

since $|M - \ell| > |M - r|$, $|M - z^i(\ell)| > |M - z^{i+1}(r)|$ and $V_M(\sigma'_L, \sigma'_R, \sigma_M; (R, 2M - y^{i-1})) < V_M(\sigma'_L, \sigma'_R, \sigma_M; (L, y^i))$. Similarly, the median voter votes against $\ell \in [y^{k-1}, y^k)$ for y^k such that $\sigma_L(R, 2M - y^{k-1}) = y^k$ and $k \leq i - 2$, and against $\ell \in [y^{k-1}, y^k)$ for y^k such that $\sigma_R(L, y^{k-1}) = 2M - y^k$ and $k \leq i - 1$. That is, in state (R, r), the median voter rejects all policies $\ell \in [0, z^{i+1}(r))$. It may or may not vote for policies $\ell \in (z^{i+1}(r), 1]$. A similar argument shows that in state (R, r) with $2M - r \in [y^i, y^{i+1})$ for y^i such that $\sigma_L(R, 2M - y^i) = y^{i+1}$, the median voter rejects any $\ell \in [0, r]$ and may or may not support $\ell \in (r, 1]$, but always supports $\ell = y^{i+1}$.

Now consider parties' incentives. First, whenever a party's equilibrium policy is being accepted, it never gains by committing to policies that are sure to be rejected, since it faces the same choice in the next election. Consider again state (R, r) with $2M - r \in [y^i, y^{i+1})$ for y^i such that $\sigma_R(L, y^i) = 2M - y^{i+1}$. The payoff to party L from policy $\ell \in [z^{i+1}(r), y^{i+1}]$ that is accepted by the median voter is

$$u_L(\ell) + \delta_L u_L(2M - y^{i+1}) + \delta_L^2 V_L(\sigma'_L, \sigma'_R, \sigma_M; (R, 2M - y^{i+1})),$$

which is decreasing in $\ell \in [y^i, y^{i+1})$. From above, policies $\ell \in [0, z^{i+1}(r))$ cannot be profitably proposed since they are rejected by the median voter, while policies in $(y^{i+1}, M]$, if accepted, yield to party L at most the payoff it obtains from such deviations under $(\sigma_L, \sigma_R, \sigma_M^{my})$. Hence committing to $z^{i+1}(r)$ is optimal for party L.

Now consider policy y^i such that $\sigma_L(R, 2M - y^i) = y^{i+1}$ and state (R, r) with $2M - r \in [y^i, y^{i+1})$. The payoff from $\ell \in [2M - r, y^{i+1})$, if accepted by the median voter, is given by

$$\begin{aligned} &u_L(\ell) + \delta_L u_L(2M - z^{i+1}(\ell)) + \delta_L^2 V_L(\sigma'_L, \sigma'_R, \sigma_M; (R, 2M - y^i)) \\ &\qquad \leq u_L(\ell) + \delta_L u_L(2M - \ell) + \delta_L^2 V_L(\sigma'_L, \sigma'_R, \sigma_M; (R, 2M - y^i)) \\ &\qquad < V_L(\sigma_L, \sigma_R, \sigma_M^{my}; (R, 2M - y^i)). \end{aligned}$$

The first inequality follows from $z^{i+1}(\ell) \le \ell$ and the second since $V_L(\sigma'_L, \sigma'_R, \sigma_M; (R, 2M - y^i)) > \frac{1}{1-\delta_L^2} U_L^+(\ell)$. This shows that y^{i+1} is L' preferred winning policy in $[y^i, y^{i+1})$ given $(\sigma'_L, \sigma'_R, \sigma_M)$. As the median voter rejects any policy $\ell \in [0, 2M - r)$, L cannot profitably deviate to such policies. Finally, deviations to any policies $\ell \in (y^{i+1}, M]$ are never profitable since even if they are accepted by the median voter, L's payoffs are no higher than under $(\sigma_L, \sigma_R, \sigma_M^{my})$.

It remains to deal with states (R, r) with $r < M$. By construction, in these states σ'_L is optimal. It needs to be shown that in states (R, r) with $r \ge M$, party L does not want to deviate to some $\ell^d > M$. Consider state (R, r) with $r > M$, and suppose party L deviates to $\ell^d > M$ such that $\sigma'_R(L, \ell^d) = \bar{r}(\ell^d)$ and take $\{y^i\}$ to be the convergence path from $(R, \bar{r}(\ell^d))$. It must be that $y^1 \ge 2M - \bar{r}(\ell^d)$. The payoff to party L from ℓ^d is given by

$$u_L(\ell^d) + \delta_L u_L(\bar{r}(\ell^d)) + \sum_{i=1}^{\infty} \delta^{2i}[u_L(y^i) + \delta_L u_L(2M - y^{i+1})] < u_L(\ell^d) + \frac{\delta_L}{1-\delta_L} u_L(M)$$
$$< \frac{1}{1-\delta_L} u_L(M).$$

The first inequality follows by Lemma 1 and the second since $\ell^d > M$. On the equilibrium path, $V_L(\sigma_L, \sigma_R; (R, r)) \ge \frac{1}{1-\delta_L} u_L(M)$, and hence deviation to ℓ^d is not profitable for L. □

A.7 Limited Policy Persistence

A.7.1 Term Limits

Proof of Proposition 6. With term limits, the description of the state includes the length $t \in \{1, T\}$ of the current incumbent's tenure. Consider some equilibrium (σ_L, σ_R) and any state (I, x, T). Proposition 1 implies that $\sigma_L(I, x, T) = \sigma_R(I, x, T) = M$. However, in all states (I, x, t) such that $t \in \{1, ..., T - 1\}$, Proposition 2 holds. To see this, consider state (R, r, t) with $t < T$ and $r < M$. For all $t < T - 1$, choosing *Out* is optimal for party L, as it obtains a payoff of $u_L(r)$ in all such periods. If $t = T - 1$, by choosing *Out* party L obtains a payoff of $u_L(r) + \frac{\delta_L}{1-\delta_L^2} U_L^-(M)$, which is party L's best achievable payoff in that state. Now consider state (R, r, t) with $t < T$ and $r > M$. Party L can still guarantee itself a payoff of at least $\frac{1}{1-\delta_L^2} U_L^+(2M - r)$ by committing to policy $2M - r$. By the previous argument, party L will never commit to a winning policy $\ell > M$. Moreover, party L can benefit from staying *Out* only if $u_L(r) + \frac{\delta_L}{1-\delta_L^2} U_L^+(M) \ge \frac{1}{1-\delta_L^2} U_L^+(2M - r)$, which is false. Hence, nontrivial equilibrium dynamics starting in states (I, x, t) with $t \le T - 1$ converge to symmetric alternations, which guarantees that the necessity argument of Proposition 3 applies to the necessity part of Proposition 6.

To show that the sufficiency argument of Proposition 3 also applies to Proposition 6, consider any equilibrium (σ_L, σ_R) in the model without term limits. In the model with term limit T, define strategies (σ_L^T, σ_R^T) such that

$$\sigma_{-I}^T(I,x,t) = \begin{cases} M & \text{if } t = T, \\ \sigma_{-I}(I,x) & \text{if } t \leq T-1. \end{cases}$$

Strategies (σ_L^T, σ_R^T) constitute an equilibrium in the game with term limit T. To see this, consider party L and state (R, r, t) with $t \leq T-1$. If $r < M$, it was shown above that choosing *Out* until party R reaches its term limit beats any equilibrium payoff following a winning policy by party L. If $r > M$, then by above party L chooses some policy $\ell \in [2M - r, M]$ and the policy prescribed by $\sigma_L^T(R, r)$ is optimal since (σ_L, σ_R) constitutes an equilibrium. Hence, all equilibria constructed in the model without term limits are easily extended to the model with term limits. □

A.7.2 Costly Policy Adjustment

Proof of Proposition 7. With costly policy adjustment, opposition party $-I$'s strategy is conditioned on state (I, x) while the incumbent I's strategy is conditioned on (I, x, y), where y is the opposition party's policy commitment. Define policy ℓ^c as the solution to

$$\frac{1}{1-\delta_L^2}[U_L^+(\max\{\ell^c, \ell^*\}) - U_L^-(\ell^c)] = c, \tag{19}$$

if it exists, and 0 otherwise. It must be that $\ell^c < M$ since $c > 0$. Furthermore, ℓ^c is decreasing in c, $\lim_{c \to 0} = M$ and there exists $\tilde{c}$ such that $\ell^c = 0$ if and only if $c \geq \tilde{c}$. Policy $r^c \in (M, 1]$ can be defined similarly for party R.

For the remainder of the proof, suppose that $\max\{\ell^*, 2M - r^*\} = \ell^* \leq \ell^c = \max\{\ell^c, 2M - r^c\}$. How to deal with other cases will be easily apparent. To show necessity, first note that the corresponding arguments in the proof of Proposition 3 still hold and that any long-run policy outcome $\ell \leq M$ must be such that $\ell \geq \ell^*$. Suppose now that $\ell \in [\ell^*, \ell^c)$ is a long-run policy outcome. Consider state $(R, 2M - \ell)$. By Proposition 2, the equilibrium payoff to party L in this state is $\frac{1}{1-\delta_L^2} U_L^-(\ell)$. If instead, party L deviates to paying c and adjusting its policy to winning policy ℓ, its payoff is $\frac{1}{1-\delta_L} U_L^+(\ell) - c$. This deviation is profitable since $\ell < \ell^c$, yielding the desired contradiction.

To show sufficiency, consider the strategies $(\sigma_L^c, \sigma_R^{my,c})$ defined as

$$\sigma_L^c(R,r) = \begin{cases} \ell^c & \text{if } r \geq 2M - \ell^c, \\ 2M - r & \text{if } r \in [M, 2M - \ell^c), \\ Out & \text{if } r \in [\min\{r^{cc}, M\}, M), \\ \max\{\ell^c, r\} & \text{if } r < \min\{r^{cc}, M\}. \end{cases}$$

$$\sigma_L^c(L,\ell,r) = \begin{cases} 2M - r & \text{if } r > 2M - \ell^c, \\ Out & \text{if either } r \leq 2M - \ell^c \text{ or } r = Out \text{ and } \ell \leq \ell^{cc}, \\ 0 & \text{if } r = Out \text{ and } \ell > \ell^{cc} \end{cases}$$

$$\sigma_R^c(L,\ell) = \begin{cases} 2M - \ell^c & \text{if } \ell \leq \ell^c, \\ 2M - \ell & \text{if } \ell \in (\ell^c, M], \\ Out & \text{if } \ell \in (M, \max\{\ell^{cc}, M\}], \\ \max\{2M - \ell^c, \ell\} & \text{if } \ell > \max\{\ell^{cc}, M\}. \end{cases}$$

$$\sigma_R^c(R,r,\ell) = \begin{cases} 2M - \ell & \text{if } \ell < \ell^c, \\ Out & \text{if either } \ell \geq \ell^c \text{ or } \ell = Out \text{ and } r \geq r^{cc}, \\ 1 & \text{if } \ell = Out \text{ and } r < r^{cc}, \end{cases}$$

where ℓ^{cc} is defined as the solution to

$$u_L(0)+\delta_L U_L^-(\ell^c)-c = \begin{cases} 1 & \text{if } u_L(0) + \delta_L U_L^-(\ell^c) - c \leq \frac{1}{1-\delta_L} u_L(1), \\ \frac{1}{1-\delta_L} u_L(\ell^{cc}) & \text{if } u_L(0) + \delta_L U_L^-(\ell^c) - c \in (\frac{1}{1-\delta_L} u_L(1), \frac{1}{1-\delta_L} u_L(M)], \\ \frac{1}{1-\delta_L^2} U_L^+(\ell^{cc}) & \text{if } u_L(0) + \delta_L U_L^-(\ell^c) - c \in (\frac{1}{1-\delta_L} u_L(M), \frac{1}{1-\delta_L^2} U_L^+(\ell^c)], \\ u_L(\ell^{cc}) + \frac{1}{1-\delta_L^2} U_L^-(\ell^c) & \text{otherwise.} \end{cases}$$

Define r^{cc} similarly. To simplify the exposition, the strategies have been written in a way that a party's response to action *Out* by an opponent should also be read to describe its response to an opponent choosing a losing policy. Consider the optimality of σ_L^c for party L in state (R, r). Its equilibrium payoff to winning policy $\ell \in [\ell^c, M]$ is $\frac{1}{1-\delta_L^2} U_L^+(\ell)$. Its payoff to winning policy $\ell < \ell^c$ is $u_L(2M - \ell) + \frac{1}{1-\delta_L^2} U_L^+(\ell^c)$, which is strictly less than $\frac{1}{1-\delta_L} u_L(M)$. Its payoff to winning policy $\ell > M$ is $u_L(\ell) + \frac{\delta_L}{1-\delta_L^2} U_L^+(\max\{2M - \ell, \ell^c\})$, which is also strictly less than $\frac{1}{1-\delta_L} u_L(M)$. This verifies the optimality of setting policy $\max\{\ell^c, 2M - r\}$ for those $r \in [0, M]$.

Consider state (R, r) with $r < M$. Party R responds to (R, r, Out) with either a policy of 1 or with *Out*, and *Out* can be a best response for party L only if $\sigma_R^c(R, r, Out) = Out$. When this is the case, the argument that *Out* is optimal for party L is as in the proof for equilibrium $(\sigma_L^{\ell^*}, \sigma_R^{my})$. If instead $\sigma_R^c(R, r, Out) = 1$, the payoff to party L if it stays *Out* is $u_L(1) + \frac{\delta_L}{1-\delta_L} U_L^+(\ell^c)$, which is strictly less than $\frac{1}{1-\delta_L} u_L(M)$. The optimality of the policy prescribed by σ_L^c then follows by the argument of the previous paragraph.

It remains to verify the optimality of σ_L^c in states (L, ℓ, r) for some r. First suppose that $r \neq Out$. If $r > 2M - \ell^c$, then by the arguments from above, if party L decides to pay the adjustment cost it is optimal to commit to policy ℓ^c. Its payoff if it stays Out is $u_L(r) + \frac{\delta_L}{1-\delta_L^2} U_L^+(\ell^c) < \frac{\delta_L}{1-\delta_L^2} U_L^-(\ell^c)$. Hence, by the definition of ℓ^c, party L prefers to commit to policy ℓ^c. If $r \leq 2M - \ell^c$, the worst equilibrium payoff for party L if it stays Out is $\frac{1}{1-\delta_L^2} U_L^-(\ell^c)$. If instead it pays the adjustment cost, the best payoff it can achieve is, by the arguments from above, $\frac{\delta_L}{1-\delta_L^2} U_L^+(\ell^c) - c$. Hence, by the definition of ℓ^c, party L prefers to stay Out.

Now suppose that $r = \{Out\}$. If party L decides to pay the adjustment cost, it will set it preferred policy 0. When it is optimal to do this as opposed to staying Out is precisely what is resolved by the definition of ℓ^{cc} above. □

Proof of Corollary 3. The results of the corollary follow from the properties of ℓ^c and r^c. □

A.8 Office-Motivated Parties

Proof of Proposition 8. Define policy $\ell^{out} \in [0, M)$ as the solution to

$$\frac{1}{1-\delta_L} u_L(\ell^{out}) = \frac{1}{1-\delta_L^2} [U_L^+(\max\{\ell^{out}, \ell^*\}) + b], \tag{20}$$

if it exists or as $\ell^{out} = 0$ otherwise. If $\ell^{out} > 0$, then party L is indifferent between never holding office and having policy ℓ^{out} implemented forever and gaining office every second election and having policies alternate at $(\ell^{out}, 2M - \ell^{out})$. Further define policy $\ell^{in} \in [0, M)$ as the solution to

$$\frac{1}{1-\delta_L} [u_L(2M - \ell^{in}) + b] = \frac{1}{1-\delta_L^2} [U_L^+(\max\{\ell^{in}, \ell^*\}) + b] \tag{21}$$

if it exits or as $\ell^{in} = 0$ otherwise. If $\ell^{in} > 0$, then party L is indifferent between holding office forever and implementing policy $2M - \ell^{in}$ and holding gaining office every second election and having policies alternate at $(\ell^{in}, 2M - \ell^{in})$. Policies r^{out} and r^{in} can be defined similarly for party R, where r^* plays the role of ℓ^*. Suppose that $\ell^{out} \in [\ell^*, M)$. Then, (20) yields that $u_L(\ell^{out}) - u_L(2M - \ell^{out}) = \frac{b}{\delta_L}$. Substituting into (21) yields that

$$\begin{aligned} \frac{1}{1-\delta_L} u_L(\ell^{out}) - & \frac{1}{1-\delta_L^2} [U_L^+(\ell^{out}) + b] \\ & = \frac{\delta_L}{1-\delta_L} [u_L(\ell^{out}) - u_L(2M - \ell^{out}) - \delta_L b] \\ & > 0, \end{aligned}$$

and hence $\ell^{in} \in (\ell^{out}, M)$. The same can be shown in the cases in which one or both of ℓ^{out} and ℓ^{in} are smaller than ℓ^*.

Proposition 2, which characterises equilibrium policy paths, no longer obtains if parties care about holding office, since there can be non-trivial long-run policy outcomes in which some party is maintained in office forever.

Proposition 10. *Consider some equilibrium (σ_L, σ_R) and some state (I, x) along with the policy path $\{y^i\}$ induced by (σ_L, σ_R) starting from (I, x). Then either*

i. $\{y^i\}$ has limit points $(\hat{\ell}, 2M - \ell)$ for some $\ell \leq M$, and both $\sigma_L(R, 2M - \hat{ell}) = \hat{\ell}$ and $\sigma_R(L, \hat{\ell}) = 2M - \hat{\ell}$, or

ii. $\{y^i\}$ has a unique limit point $x \neq M$, and whenever $x < M$ either $(I^0, y^0) = (R, x)$ or there exists $N > 0$ such that $\sigma_R(L, y^N) = x$. Furthermore, $\sigma_L(R, x) \in \{Out\} \cup [x, 2M - x]^c$. The statement for $x > M$ is symmetric.

Proof of Proposition 10. Equilibrium policy path $\{y^i\}$ can have no more than two limit points since, as shown for Proposition 2, all its limit points must be equidistant from the median. First consider part i. By Markov strategies it follows that parties choose winning policies in each period. By Lemma 2, in the limit the payoff to party L can be no less than $\frac{1}{1-\delta_L^2}[U_L^+(\ell) + b]$. As shown for Proposition 2, since $(\hat{\ell}, 2M - \ell)$ are limit points of equilibrium policy sequence $\{y^i\}$, in the limit the payoff to party L can be no more than $\frac{1}{1-\delta_L^2}[U_L^+(\ell) + b]$. Hence, in the limit, party L's payoff is exactly $\frac{1}{1-\delta_L^2}[U_L^+(\ell) + b]$, which implies that, given Markov strategies, $\sigma_L(R, 2M - \hat{\ell}) = \hat{\ell}$ and $\sigma_R(L, \hat{\ell}) = 2M - \hat{\ell}$.

For part ii, suppose that $x < M$ is the unique limit point of $\{y^i\}$. Suppose that $y^i \neq x$ for all i. By Markov strategies, parties must choose winning policies in each period. In the limit, party R's payoff in state (L, y^i) converges to $\frac{u_R(x)}{1-\delta_R} + \frac{b}{1-\delta_R^2}$. Consider a deviation for party R in state (L, y^n) to $2M - y^n > M$ for n sufficiently large. By Lemma 2, party R's payoff would be at least $U_R^+(y^n) + \frac{b}{1-\delta_R^2}$, a contradiction. Hence, there must exist some $N \geq 0$ such that $y^n = x$ for all $n \geq N$. By an argument similar to that above, it must be that for all $n \geq N$, $(I, y^n) = (R, x)$, yielding the rest of part ii. □

Returning to the proof of Proposition 8, suppose that $\ell^{out} \geq 2M - r^{in}$. I suppose first that $2M - r^{in} \geq \ell^*$ and show that policies in alternation $(\ell, 2M - \ell)$ with $\ell \in (2M - r^{in}, \ell^{out})$ cannot be long-run policy outcomes. Extending the argument to the case in which only $\ell^{out} > \ell^*$ is straightforward. Towards a contradiction, suppose they were. Consider a deviation by party R in state $(R, 2M - \ell)$ to policy ℓ. In state (R, ℓ), the payoff to party L is $\frac{1}{1-\delta_L^2}[U_L^+(\ell) + b]$. Since $\ell < \ell^{out}$, staying Out forever yields party L a strictly higher payoff and hence it must be that $\sigma_L(R, \ell) = Out$. Since $\ell > 2M - r^{in}$, then the deviation to ℓ is strictly profitable for party R.

Second, I show that all policies $\ell \notin [2M - r^{in}, \ell^{out}]$ can never be non-trivial long-run policy outcomes. The argument above has shown that such policies are not observed in the long-run as symmetric alternations. By Proposition 10, if some such policy $\ell > M$ is a non-trivial long-run policy outcome, then there exists an equilibrium (σ_L, σ_R), an initial state $(I, x) \neq (R, \ell)$ and an induced sequence of policies $\{y^i\}$ such that for some $N > 0$ $\sigma_R(L, y^{N-1}) = \ell$ and $\sigma_L(R, \ell) \in \{Out\} \cup [\ell, 2M - \ell]^c$. If $\ell > \ell^{out}$, then Out (or any losing policy) is not a best-response for party L in state (R, ℓ). In particular, a deviation to ℓ yields payoff of at least $\frac{1}{1-\delta_L^2}[U_L^+(\ell) + b]$, higher than its equilibrium payoff of $\frac{u_L(\ell)}{1-\delta_L}$ by (20). If $\ell < 2M - r^{in}$, then consider the deviation by R in state (L, y^{N-1}) to policy $2M - \ell$. The payoff to this deviation is at least $\frac{1}{1-\delta_R^2}[U_R^+(\ell) + b]$, higher than its equilibrium payoff of $\frac{1}{1-\delta_R}[u_R(\ell) + b]$ by R's version of (21). A similar argument yields the result for those remaining $\ell < M$.

The final step in the proof is relevant only for cases in which $2M - r^{in} > \ell^{out}$. In that case, some alternations at policies more extreme than $2M - r^{in}$ but within ℓ^* can be ruled out. Consider map $G : [2M - r^{in}, \ell^{out}] \to [0, 2M - r^{in}]$ defined as the solution to

$$\frac{1}{1 - \delta_R}[u_R(\ell) + b] = \frac{1}{1 - \delta_R^2}[U_R^+(G(\ell)) + b],$$

if it exists and 0 otherwise. Note that a discontinuity in G can only occur at $G(\ell) = 2M - r^*$ By the definition of r^{in}, we have that $G(2M - r^{in}) = 2M - r^{in}$, $G(\ell) < \ell$ for all $\ell > 2M - r^{in}$ and G is strictly decreasing on $[2M - r^{in}, \ell^{out}]$ when its value is positive. Define mapping $H : [2M - r^{in}, \ell^{out}] \to [0, \ell^{out}]$ as the solution to

$$\frac{1}{1 - \delta_L}u_L(\ell) = \frac{1}{1 - \delta_L^2}[U_L^+(H(\ell)) + b],$$

if it exists and 0 otherwise. Note that a discontinuity in H can only occur at $H(\ell) = \ell^*$ By the definition of ℓ^{out}, we have that $H(\ell^{out}) = \ell^{out}$, $H(\ell) < \ell$ for all $\ell < \ell^{out}$ and H is strictly increasing on $[2M - r^{in}, \ell^{out}]$ when its value is positive. Since $G(2M - r^{in}) > H(2M - r^{in})$ and $G(\ell^{out}) > H(\ell^{out})$, if there can exist at most one value $\ell^b \in (\ell^*, 2M - r^{in})$ satisfying $G(\ell^b) = H(\ell^b)$. In all other cases, set $\ell^b = \ell^*$.

For those cases in which $\ell^b > \ell^*$, it remains to be shown that all policies $\ell \in (\ell^b, 2M - r^{in})$ can never be long-run policy outcomes supported by alternation. Consider some long-run policy outcome $\ell \in [\ell^*, 2M - r^{in})$ supported by alternation. By Proposition 10, it must be that either (i) $\sigma_L(R, \ell) = Out$ for all $\ell \in [2M - r^{in}, \ell^{out}]$, or (ii) $\sigma_L(R, \ell) \in (\ell, M]$ for all $\ell \in [2M - r^{in}, \ell^{out}]$, or (iii) there exists some $\tilde{\ell}$ such that $\sigma_L(R, \tilde{\ell}) = Out$ and for any $\epsilon > 0$, there exists $\hat{\ell}^\epsilon$ such that $\sigma_L(R, \hat{\ell}^\epsilon) \in (\hat{\ell}^\epsilon, M]$ and $|\hat{\ell}^\epsilon - \tilde{\ell}| < \epsilon$. In case (i), consider a deviation by party R in state (L, ℓ) to ℓ^{out}. Party R's payoff from this deviation is $\frac{1}{1-\delta_R}[u_R(\ell^{out}) + b]$, and hence it is not profitable only if $\ell \leq G(\ell^{out}) < H(\ell^{out})$. In case (ii), it must be that $V_L(\sigma_L, \sigma_R; (R, 2M - r^{in})) \geq \frac{1}{1-\delta_L}u_L(2M - r^{in})$. Consider a deviation by party L in state $(R, 2M - \ell)$ to $\sigma_L(R, 2M - r^{in})$. This deviation is not profitable only if

$\ell \le H(2M - r^{in}) < G(2M - r^{in})$. In case (iii), an argument similar to the case (i) above yields that party R cannot profitably deviate to $\tilde{\ell}$ in state (L, ℓ) only if $\ell \le G(\tilde{\ell})$. Again, an argument similar to the case (ii) above yields that party L cannot profitably deviate to $\sigma(R, \tilde{\ell}^{\epsilon})$ for ϵ sufficiently small only if $\ell \le H(\tilde{\ell})$. Given the properties of functions G and H derived above, it follows that $\min\{G(\tilde{\ell}), H(\tilde{\ell})\} \le \ell^b$. □

Proof of Corollary 4. Verifying the claim of Corollary 4 requires at least a partial answer to sufficiency in Proposition 8. First, for the case in which $\ell^{out} > 2M - r^{in} > G(\ell^{out}) \ge \ell^* \ge 2M - r^*$, I construct an equilibrium that show that the set of long-run policy outcomes supported by alternation contains the set $[\ell^*, G(\ell^{out})] \cup [\ell^{out}, M]$. Similar constructions apply to other cases. Consider strategies (σ^b_L, σ^b_R) defined as follows.

$$\sigma^b_L(R, r) = \begin{cases} \ell^* & \text{for } r \ge 2M - \ell^* \\ 2M - r & \text{for } r \in [M, 2M - \ell^*) \\ r & \text{for } r \in (\ell^{out}, M] \\ Out & \text{for } r \in [0, G(\ell^{out})] \text{ or } r = \ell^{out} \\ \text{Best of } Out \text{ or } r & \text{otherwise} \end{cases}$$

$$\sigma^b_R(L, \ell) = \begin{cases} 2M - \ell & \text{for } \ell \le G(\ell^{out}) \\ \ell^{out} & \text{for } \ell \in [G(\ell^{out}), \ell^{out}) \\ 2M - \ell & \text{for } \ell \in [\ell^{out}, M] \\ \ell & \text{for } \ell \in (M, r^{out}] \\ Out & \text{for } \ell \in [\max\{r^{out}, 2M - G(\ell^{out})\}, \max\{r^{out}, 2M - \ell^*\}] \\ \text{Best of } Out \text{ or } \ell & \text{otherwise} \end{cases}$$

Consider the optimality of σ^b_L for party L facing σ^b_R. For states (R, r) with $r \in [M, 2M - \ell^{out}] \cup (2M - G(\ell^{out}), 1]$, the argument follows as in the case of equilibrium $(\sigma^{\ell^*}_L, \sigma^{my}_R)$. For states (R, r) with $r \in [2M - \ell^{out}, 2M - G(\ell^{out})]$, the best response of party L must either be $2M - r$ or some policy $\ell \in (\ell^[out], r]$. Party L's payoff to $2M - r$ is $u_L(2M - r) + b + \frac{\delta_L}{1-\delta_L^2}[U^+_L(\ell^{out}) + b]$, which is higher than $\frac{\delta_L}{1-\delta_L^2}[U^+_L(\ell) + b]$, the payoff to $\ell \in (\ell^{out}, M]$. Since $2M - \ell^{in} < 2M - \ell^{out} \le r^{in} < r^{out}$, party R responds to any $\ell \in (M, 2M - \ell^{in}]$ with policy ℓ, and hence party L has no incentive to choose such a policy. Similarly, party L has no incentive to choose any policy $\ell \in (2M - \ell^{in}, r]$.

Consider state (R, r) with $r \in [0, G(\ell^{out})]$. The equilibrium payoff to party L if it chooses a winning policy is $\frac{1}{1-\delta_L^2}[U^+_L(r) + b]$ if $r \ge \ell^*$ and $\frac{1}{1-\delta_L^2}[U^+_L(\ell^*) + b]\}$ otherwise. Since $r < \ell^{out}$, staying Out is optimal. Similarly, for state (R, r) with $r \in (\ell^{out}, M]$, the equilibrium payoff to party L is $\frac{1}{1-\delta_L^2}[U^+_L(r) + b]$, and hence policy r is optimal. For those $r \in [G(\ell^{out}), \ell^{out})$, it may be optimal for party L to choose winning policy r even if $r < \ell^{out}$ since its equilibrium payoff to policy r is given by $u_L(r) + b + \frac{\delta_L}{1-\delta_L^2}[U^+_L(G(\ell^{out})) + b]$, which is strictly larger than

$\frac{1}{1-\delta_L^2}[U_L^+(r)+b]$. Which of Out or r is optimal is simple, if tedious, to verify. Note that if $r = \ell^{out}$, party L is indifferent between staying Out and choosing winning policy ℓ^{out}, which yields payoff $\frac{\delta_L}{1-\delta_L^2}[U_L^+(\ell^{out})+b]$.

Now consider the optimality of σ_R^b for party R facing σ_L^b. Again, for states (L,ℓ) with $\ell \in [0, G(\ell^{out})) \cup (\ell^{out}, M]$, the argument follows as in the case of equilibrium $(\sigma_L^{\ell^*}, \sigma_R^{my})$. For states (L,ℓ) with $\ell \in [G(\ell^{out}), \ell^{out})$, party R's equilibrium payoff is $\frac{1}{1-\delta_R}[u_R(\ell^{out})+b]$, which since $\ell^{out} > 2M - r^{in}$ is strictly greater than $\frac{1}{1-\delta_R^2}[U_R^+(\ell^{out})+b]$, the best payoff it can achieve by choosing any winning policy r for which $\sigma_L^b(R,r) \neq Out$. Furthermore, party R's preferred winning policy r for which $\sigma_L^b(R,r) = Out$ is ℓ^{out}, its equilibrium choice.

For those states (L,ℓ) with $\ell \in (M, r^{out}] \cup [\max\{r^{out}, 2M - G(\ell^{out})\}, \max\{r^{out}, 2M - \ell^*\}]$, the argument is similar to that for party L. That is, party R's equilibrium payoff to winning strategy ℓ is $\frac{1}{1-\delta_R^2}[U_R^+(\ell)+b]$ and the definition of policy r^{out} can be applied directly to find which of ℓ or Out is optimal. Again, for those states (L,ℓ) with $\ell > r^{out}$ for which party R's payoff to winning policy ℓ exceeds $\frac{1}{1-\delta_R^2}[U_R^+(\ell)+b]$, a simple verification determines which of ℓ or Out is optimal.

Second, suppose that both $2M - r^{in} \geq \ell^{out}$ and $\ell^{in} \geq 2M - r^{out}$. Then a simple modification of equilibrium $(\sigma_L^{\ell^*}, \sigma_R^{my})$ shows that the bound $\max\{\ell^*, 2M - r^*\}$ on long-run policy outcomes is tight even with office benefits. Consider strategies $(\sigma_L^{\ell^*,b}, \sigma_R^{my,b})$, defined as follows.

$$\sigma_L^{\ell^*,b}(R,r) = \begin{cases} \ell^* & \text{for } r \geq 2M - \ell^* \\ 2M - r & \text{for } r \in [M, 2M - \ell^*) \\ r & \text{for } r \in (\ell^{out}, M] \\ Out & \text{for } r \in [0, \ell^{out}] \end{cases}$$

$$\sigma_R^{my,b}(L,\ell) = \begin{cases} 2M - \ell & \text{for } \ell \in [0, M] \\ \ell & \text{for } \ell \in (M, r^{out}) \\ Out & \text{for } \ell \in [r^{out}, 1] \end{cases}$$

The verification that $(\sigma_L^{\ell^*,b}, \sigma_R^{my,b})$ constitutes an equilibrium mostly follows from the arguments showing that $(\sigma_L^{\ell^*}, \sigma_R^{my})$ constitutes an equilibrium in the absence of office benefits. It remains only to verify that (i) staying Out is optimal for the parties when their strategies call for it and that (ii) no party has an incentive to commit to a policy to which its opponent responds to by staying Out. It is straightforward to see that (i) and (ii) follow from the definitions of (ℓ^{out}, r^{out}) and (ℓ^{in}, r^{in}), respectively.

The two equilibrium constructions from above show that for any $b > 0$, either $[\ell^*, M] = \mathcal{L}^b$ or $\ell^* < \ell^{out}$ and $[\ell^{out}, M] \subseteq \mathcal{L}^b$. Results i and ii Corollary 4 then follow from the properties of ℓ^{out} (or r^{out} in comparable cases). Result iii of Corollary 4 follows since in the symmetric case $\ell^{in} = 2M - r^{in} > \ell^{out} = 2M - r^{out}$. □

A.9 Legislative Bargaining

Proof of Proposition 9. Consider consistent proposal strategies (σ_L, σ_R) that generate convergence path $\{y^i\} \to \hat{\ell}$ when the median legislator is decisive and $\sigma_M = \sigma_M^{my}$. It will be shown that σ_M^{my} is indeed a best response for the median legislator. It is straightforward to establish results equivalent to Lemmma 3 that characterises consistent proposal strategies on convergence paths.

Consider a convergence path $\{y^i\} \to \hat{\ell}$ with policy y^i such that $\sigma_L(L, 2M - y^i) = y^{i+1}$. Since each legislator is recognised with equal probability in each period, legislator L's equilibrium payoff is given by

$$\begin{aligned} V_L(\sigma_L, \sigma_R, \sigma_M^{my}; (L, 2M - y^i)) &= u_L(y^{i+1}) + \frac{1}{2}\delta_L V_L(\sigma_L, \sigma_R, \sigma_M^{my}; (L, y^{i+1})) \\ &\quad + \frac{1}{2}\delta_L V_L(\sigma_L, \sigma_R, \sigma_M^{my}; (R, y^{i+1})) \\ &= \frac{2}{2-\delta_L}\left[u_L(y^{i+1}) + \frac{1}{2}\delta_L V_L(\sigma_L, \sigma_R, \sigma_M^{my}; (R, y^{i+1}))\right], \end{aligned} \tag{22}$$

where the second equality is due to consistent proposal strategies. A lower bound on $V_L(\sigma_L, \sigma_R, \sigma_M^{my}; (L, 2M - y^i))$ can be determined as in Section 4.2 by considering a deviation to y^i by L in state $(L, 2M - y^i)$. Hence

$$\begin{aligned} V_L(\sigma_L, \sigma_R, \sigma_M^{my}; (L, 2M - y^i)) \geq u_L(y^i) &+ \frac{1}{2}\delta_L V_L(\sigma_L, \sigma_R, \sigma_M^{my}; (L, 2M - y^i)) \\ &+ \frac{1}{2}\delta_L V_L(\sigma_L, \sigma_R, \sigma_M^{my}; (R, y^i)). \end{aligned} \tag{23}$$

By convergence and consistent strategies, $\sigma_R(R, y^i) = \sigma_R(R, 2M - y^i) = 2M - y^i$, and hence, as for (22) above,

$$V_L(\sigma_L, \sigma_R, \sigma_M^{my}; (R, y^i)) = \frac{2}{2-\delta_L}\left[u_L(2M - y^i) + \frac{1}{2}\delta_L V_L(\sigma_L, \sigma_R, \sigma_M^{my}; (L, 2M - y^i))\right]. \tag{24}$$

Under consistent strategies, an upper bound on $V_L(\sigma_L, \sigma_R, \sigma_M^{my}; (L, 2M - y^i))$ can be obtained as in Section A.4 by considering a deviation to y^{i+1} in state $(R, 2M - y^i + \epsilon)$ for small ϵ. That is

$$\begin{aligned} V_L(\sigma_L, \sigma_R, \sigma_M^{my}; (L, 2M - y^i)) \geq u_L(y^i) &+ \frac{1}{2}\delta_L V_L(\sigma_L, \sigma_R, \sigma_M^{my}; (L, 2M - y^i)) \\ &+ \frac{1}{2}\delta_L V_L(\sigma_L, \sigma_R, \sigma_M^{my}; (R, y^i)). \end{aligned} \tag{25}$$

Finally, (23), (25) and (24) yield

$$V_L(\sigma_L, \sigma_R, \sigma_M^{my}; (L, 2M - y^i)) = \frac{2-\delta_L}{2(1-\delta_L)}\left[u_L(y^i) + \frac{\delta_L}{2-\delta_L} u_L(2M - y^i)\right]. \tag{26}$$

This is the equivalent of (8) which states that L's equilibrium payoff at $(R, 2M - y^i)$ is the payoff to alternation at $(y^i, 2M - y^i)$. Expression (26) incorporates the fact that the future sequence of proposers is random and that convergence is staggered. A calculation like the one in (24) yields $V_L(\sigma_L, \sigma_R, \sigma_M^{my}; (R, y^{i+1}))$, and (22) can be rewritten, after substituting (26), as

$$(2 - \delta_L)\left[u_L(y^i) - u_L(y^{i+1})\right] + \frac{\delta_L^2}{2 - \delta_L}\left[u_L(y^{i+1}) - u_L(y^{i+2})\right] \\ = \delta_L\left[u_L(2M - y^{i+2}) - u_L(2M - y^i)\right]. \quad (27)$$

Equation (27) is the equivalent of (9), the second-order differential equation that determines consistent equilibrium convergence path policies, in the legislative bargaining model. Conditions for existence of convergence paths in this model would hinge on the properties of the payoffs of legislators L and R relative to (27). However, for the purposes of Proposition 9, all that is required is that (27) must hold along any convergence path in consistent proposal strategies.

As in A.5, a bound on the moderation of convergence outcomes can be derived by constructing 'compromise' functions α_L and α_R. An argument as in A.5 shows that given some $y < M$, $\alpha_L(y)$ can be defined as

$$\frac{u_L'(y)}{u_L'(2M - y)} = \frac{\delta_L}{\alpha_L(y)(2 - \delta_L) + (1 - \alpha_L(y))\frac{\delta_L^2}{2 - \delta_L}}.$$

In particular, $\alpha_L(M) = \frac{\delta_L}{2} < \frac{1}{2}$, and a similar argument shows that $\alpha_R(M) < \frac{1}{2}$. Hence, as in Section A.5, as convergence paths approach the median, both legislators require that their opponent's next moderate move be larger than their own current moderate move, which contradicts convergence.

I have assumed that median voter behaves myopically. In fact, it can shown that this voting strategy is optimal. Consider policy y^i such that $\sigma_L(L, 2M - y^i) = y^{i+1}$. Suppose that in state $(L, 2M - y^i)$ legislator L proposes $z \in [y^i, y^{i+1})$. If the median voter votes in favour of z its payoff is given by

$$u_M(z) + \frac{1}{2}\delta_M V_M(\sigma_L, \sigma_R, \sigma_M^{my}; (L, z)) + \frac{1}{2}\delta_M V_M(\sigma_L, \sigma_R, \sigma_M^{my}; (R, z)) \\ > u_M(2M - y^i) + \frac{1}{2}\delta_M V_M(\sigma_L, \sigma_R, \sigma_M^{my}; (L, 2M - y^i)) + \frac{1}{2}\delta_M V_M(\sigma_L, \sigma_R, \sigma_M^{my}; (R, 2M - y^i)),$$

where the right-hand side is the payoff to supporting the status quo. This follows since $u_M(z) > u_M(2M - y^i)$, $V_M(\sigma_L, \sigma_R, \sigma_M^{my}; (L, z)) = V_M(\sigma_L, \sigma_R, \sigma_M^{my}; (L, 2M - y^i))$ since $\sigma_L(L, z) = \sigma_L(2M - y^i) = y^{i+1}$ and $V_M(\sigma_L, \sigma_R, \sigma_M^{my}; (R, z)) > V_M(\sigma_L, \sigma_R, \sigma_M^{my}; (R, 2M - y^i))$ since $\sigma_R(R, \ell) = 2M - \ell$ for $\ell \in [y^i, y^{i+1})$. Similar arguments show that the median legislator accepts any policy $z \in [y^{i+1}, 2M - y^{i+1}]$ and rejects any policy $z \in [y^i, 2M - y^i]$. Furthermore, these arguments do not depend on which legislator makes the proposal, since future periods' draws of proposers are not affected by the identity of the legislator responsible for the status quo policy. □