Leaving your tailings behind: Environmental bonds, bankruptcy and waste cleanup

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Abstract

The paper studies the impacts of an environmental bond, which fully covers waste cleanup costs, on a mining firm's optimal actions when bankruptcy may shift cleanup costs to the government. A firm's stochastic optimal control problem is described by an HJB equation with the resource price modelled as an Ito process. A theoretical result is derived, showing that when a firm does not have the option to declare bankruptcy, the bond has no impact on the optimal controls. In contrast, if a firm does have a bankruptcy option and if no environmental bond is required, the firm produces too much waste relative to a benchmark case, resulting in an efficiency loss and a cleanup liability imposed on government. In the presence of a bankruptcy option, a bond ensures that the firm acts optimally and no efficiency loss is imposed on society. A numerical solution of the HJB equation is implemented for a hypothetical copper mine and results are analyzed for two different models of bankruptcy risk.

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1 Introduction

Hazardous waste production is a significant consequence of large natural resource extraction projects, such as mines. The resulting waste is often disposed of into local ecosystems and can impose high costs on the environment and society during mining operations and after a mine is abandoned. Without appropriate regulations, profit maximizing firms are likely to generate more waste than is desirable and are unlikely to undertake adequate waste cleanup. It is common in many jurisdictions for firms to be held legally liable for waste cleanup and site remediation at project termination. Waste cleanup costs are project specific and can range from millions to billions of dollars (Boyd, 2002; Lemphers et al., 2010; Ho et al., 2018). An obligation for restoration under a strict liability rule[3] increases the cost of mine abandonment, which may cause some firms to choose to remain inactive as a way to escape restoration costs (Muehlenbachs, 2015). Another cause of inadequate waste cleanup is firm bankruptcy. Large numbers of mining operations in the US and Canada have been abandoned due to bankruptcy resulting in significant environmental damages and cleanup costs, as is documented by Boyd (2002), among others. Likewise, a significant number of oil and gas wells have been abandoned by bankrupt owners with no proper closure or site cleanup.[4] In the event of bankruptcy, the environmental liability may fall to government with restoration costs funded out of general tax revenue, leading to a dead-weight loss.

In practice, the requirement for some sort of financial surety as a complement to the strict liability rule, has been widely used in an attempt to ensure adequate funds are available for end-of-activity restoration.[5] Typically a firm estimates and reports its expected

[3]Strict liability refers to the imposition of liability regardless of whether the firm has adhered to accepted standards of care. In contrast, under the negligence standard, a firm is only liable if it has acted negligently.

[4]This has been been a significant problem in Alberta Canada. See Souza et al. (2018), Lewis & Wang (2018), and Morgan (2017). A similar problem exists in many U.S. states as documented in Ho et al. (2016).

[5]See Sasson (2009) and Miller (2005) for a survey of bonding practices in extractive industries worldwide. The incentives for environmental protection by US hazardous waste managers under different mechanisms are compared in Zhou (2014).

future cleanup costs based on current knowledge and provides a financial surety of an equivalent amount. Possible forms of surety include insurance policies, letters of credit and bank guarantees, cash deposits and trust funds, each of which has advantages and disadvantages (Sasson, 2009). Ideally the value of any surety would be updated over time as the firm's expected cleanup costs are revised. Funds tied up in the surety would be released to the firm upon successful closure and restoration. In this paper we refer to this financial surety as an environmental bond.[6]

Ensuring adequate funds for cleanup is a problematic aspect of environmental bonds. Without a specific template for cost estimations and also in the absence of third-party verification, firms may underestimate their cleanup costs. Moreover, governments are under pressure from firms to keep bonding requirements low. If the bond amount is inadequate and if a firm walks away from its obligation, cleanup costs will be transferred to the government. In 2009, over 10,000 mines operating under an environmental bonding regulation in Canada were classified as abandoned without being cleaned up and with insufficient funds for restoration (Lemphers et al., 2010). The Faro Mine in the Yukon and the Giant Mine in the North West Territories are notable examples where government was left with cleanup liabilities exceeding $350 million. An adequate level of environmental bond increases the likelihood that a firm will meet its obligation to clean up a contaminated site. This fact is confirmed by an empirical study for the US oil and gas sector (Boomhower, 2019).

The objective of this paper is to contrast the effects of mining regulatory policies with and without environmental bond requirements, focusing on impacts on mine value, optimal firm actions and expected cleanup costs imposed on government. In particular, we compare a policy of a bond plus liability for clean up (the bonding policy) with cleanup liability only (the liability policy) under different assumptions about a firm's bankruptcy risk. The goal of the bond is to fully collateralize the government against the possibility that a firm

[6]The terms "financial assurance", a "financial surety" and "bond" are used interchangeably in practice to mean a financial guarantee for mine cleanup.

might go bankrupt and therefore not meet its cleanup obligations. To this end, the required payment to the bond in each time period is determined so that cleanup costs would be fully covered should a mine be closed immediately. The bond is in the form of a cash deposit, which is a common form of environmental bond in practice. In order to focus on the cleanup of mine waste, we assume there are no environmental damages caused by waste creation during the production process. However once the mine is closed, the firm must meet a government regulatory requirement for immediate site clean up. The paper does not address the possibility of environmental accidents associated waste storage facilities.[7]

Our paper contributes to the literature on stock externalities in resource extraction, reviewed in more detail in Section 2. Recent papers related to ours include White et al. (2012), Lappi (2020), and Yang & Davis (2018). Different from these papers, our focus is on the efficiency implications for cleanup costs of an environmental bond policy when bankruptcy is possible over the full life cycle of a mining project and affects decisions regarding project commencement, temporary mothballing and final abandonment. Our paper also contrasts with earlier papers in that we model the mine's output price as stochastic, in order to capture the effect of price uncertainty on optimal operations, as well as on the likelihood of bankruptcy.

The causes and consequences of firm bankruptcy are complex and depend on legal institutions, firm structure and operating practices. Once a firm declares bankruptcy, remaining assets of value are used to fulfill its obligations to creditors and others, but the hierarchy of obligations can be contentious. In this paper we assume in the event of bankruptcy, the bond is used exclusively to fund site cleanup.[8] Bankruptcy may be caused by events be-

[7]Environmental bonds and liability rules have been widely used to control the risk of accidents such as sudden chemical releases. Torsello & Vercelli (1998) provides a critical assessment of these policies for risk control, and Poulin & Jacques (2007), Gerard & Wilson (2009), Smith (2012), and Davis (2015) highlight their practical challenges for different case studies relevant to environmental risks.

[8]Controversy over the use of environmental bonds erupted in the case of the insolvency of Redwater Energy in Canada in 2015. A 2019 decision of Canada's Supreme Court ruled that Redwater must fulfill its environmental obligations before paying back creditors, overturning a lower court decision. See CBC News

yond the control of a firm, such as low product demand and commodity prices. However, bankruptcy may also be viewed as a strategic option by a firm seeking to avoid cleanup costs. To contrast these different possibilities, we consider two different bankruptcy scenarios. In the first, bankruptcy is an exogenous Poisson event, which depends on commodity prices, and is hence beyond the control of the firm operating the mine. In the second, the firm has the option to declare bankruptcy if it is optimal to do so. These scenarios are referred to, respectively, as the exogenous and endogenous bankruptcy scenarios. The two bankruptcy scenarios are contrasted with a case in which the firm operates as a going concern - i.e. bankruptcy is not permitted. The no-bankruptcy case is referred to as the "solvent firm scenario". Given our assumptions that there are no environmental damages prior to mine closure, and further assuming that the government regulation requiring site cleanup upon mine closure is optimal, the solvent firm scenario also represents a social planner case. This case will be used as a benchmark in our analysis.

We develop a stochastic optimal control model of a firm's decisions over the life cycle of the mine. The model draws on the literature of natural resource extraction under uncertainty using a real options approach, such as Brennan & Schwartz (1985), Mason (2001), Slade (2001), and Insley (2017), among others. Resource prices are uncertain, which affects the optimal timing of production and abandonment, and hence waste accumulation and cleanup. The price of the mine's output is modelled as an Ito process. The mine owner chooses the optimal timing to build, operate, mothball, and eventually abandon the project. During operation, the mine owner chooses the optimal rates of production and waste abatement, and the resulting waste that accumulates must be cleaned up when the firm ceases operations permanently (abandonment). Abandonment can be due to low prices, reserve depletion, capacity exhaustion at the waste storage site, or lease expiration, whichever comes first. Once the project is abandoned, it cannot be restarted and waste cleanup is a regulatory

report dated January 31, 2019, "Supreme Court rules energy companies must clean up old wells - even in bankruptcy", by Tracy Johnson.

requirement. The optimal control formulation results in a Hamilton Jacobi Bellman (HJB) partial differential equation (PDE), which is solved using a numerical approach with data for a hypothetical copper mine. The optimal controls as computed by solving the HJB equations are then used in Monte Carlo analysis to describe the distribution of key variables of interest, such as the stock of mining waste, mine production and abatement. The results allow us to contrast the firm's optimal decisions under an environmental bond (plus liability for cleanup) compared to a strict liability rule on its own over the life of a hypothetical mine project for the solvent firm and endogenous and exogenous bankruptcy scenarios. As noted, given our modelling assumptions the solvent firm choices are consistent with a social planner, which allows us to draw conclusions about the efficiency implications of the bond versus liability policies.

To preview our results, we derive a theoretical result which demonstrates the equivalence of the bond and liability policies for the solvent firm, provided the firm receives appropriate interest income on the bond and there are no additional bond service charges.[9] Further we demonstrate that under endogenous bankruptcy, the imposition of a bond yields the same results as for the the solvent firm, fully correcting any market failure caused by the bankruptcy option. Our numerical results also demonstrate that in the face of either endogenous or exogenous bankruptcy risk, an appropriately structured environmental bond not only shields a government from cleanup liability, but also ensures efficient behaviour by a firm in terms of its ongoing waste abatement behaviour. In the absence of a bond, a firm that faces bankruptcy risk may undertake too little abatement, leaving an inefficiently large amount of waste at project termination. This increases total waste cleanup costs, representing a deadweight loss to society. In the presence of bankruptcy risk, a bond does impose costs on firms which may result in fewer mining projects being undertaken, compared to a reliance on strict liability alone. However, under the assumptions of our model, this is the

[9] In practice, certain characteristics of the bond, such as bond service fees and low or no interest received on cash bonds, increase the cost of a bond relative to a strict liability rule alone.

efficient result.

The next section provides a brief literature review. Section 3 develops the theoretical model. The dynamic programming solution of the model and optimal strategies for extraction and abatement are presented in Section 4. Expected government cleanup costs are specified in Section 5. Theoretical results are presented in Section 6. An application of the model to the copper industry is discussed in Section 7. Discussion of numerical results analysis are given in Section 8. The last section summarizes results and conclusions.

2 Literature

Our paper is related to the literature that addresses negative stock externalities arising from natural resource extraction, whether through the buildup of a stock pollutant or the degradation of land and ecosystems, including Farzin (1996), Roan & Martin (1996), and Keohane et al. (2007). Keohane et al. (2007) is of particular relevance for this paper in their focus on optimal abatement versus site restoration; however in their model government is responsible for site cleanup. A stream of the economics literature, dating back to Solow (1971) and Perrings (1989), explores environmental bonds as a regulatory tool to ensure firms meet their cleanup obligations.[10]

A number of recent papers develop optimal decision models to study different regulatory tools that address stock externalities in resource extraction. Most related to the issues in this paper are White et al. (2012), Lappi (2020), and Yang & Davis (2018).[11] Common elements in their decision models include an initial known stock of a non-renewable resource, costly

[10] Other relevant papers examining environmental bonds and liability include Shogren et al. (1993), Cornwell & Costanza (1994), Kaplow & Shavell (1996), Costanza & Perrings (1990), and Gerard & Wilson (2009), among others.

[11] Other related papers include Igarashi et al. (2010); Lappi & Ollikainen (2019); Lappi (2018); White (2015). White (2015) analyzes a bond and Pigouvian tax in a static model with a focus on bankruptcy risk and the potential for extending liability for cleanup to investors.

resource extraction, build up of a damaging stock externality, costly cleanup/remediation of the stock externality at the end of the project, costly abatement or remediation during operations, and optimal decisions regarding extraction and abatement rates and final cleanup. Distinguishing features of the models include assumptions about abatement, extraction and remediation cost functions, whether remediation is ongoing or only at project end, whether remediation happens immediately on project closure or may be optimally delayed, whether the pollution stock exhibits natural decay, whether bankruptcy is considered, and whether closed form or numerical solutions are presented. White et al. (2012) conclude that with no uncertainty or possibility of bankruptcy, the optimal policy is a tax on damaged land. When bankruptcy occurs with some known probability at the end of the project, an environmental bond is desirable to ensure the government has funds to undertake site cleanup. This avoids the deadweight loss that arises if cleanup is financed from general tax revenue. Yang & Davis (2018) contrast taxes on the flow versus stock of pollution, and emphasize the benefits of the stock tax in terms of providing the correct restoration incentives for pollution creation and site cleanup. When firm bankruptcy is considered a significant risk, they suggest a mixed policy of a bond and stock externality tax would be advantageous. Lappi (2020) extends the Yang & Davis (2018) analysis by including convex rather than linear pollution damage and reclamation cost functions, and by assuming a positive decay rate for the pollution stock. In addition the timing of project closure is a choice variable. These factors mean that it may be optimal to delay site cleanup after the mine has closed, unlike in the Yang & Davis (2018) in which site cleanup is a bang-bang decision. Lappi (2020) demonstrates that a socially optimal policy consists of a pollution tax, a project shut down date when extraction ceases, and a profits tax which collects funds to be used for site cleanup. With no uncertainty in the model, Lappi shows that the timing of the funds deposited for site cleanup is irrelevant.

Our paper has a different focus than existing literature in that we model the efficiency implications in terms of the cost of meeting a regulatory site cleanup requirement, of bond and liability policies when there is a possibility of bankruptcy. We ignore environmental damages that may be caused during waste creation. Compared to the existing literature, our paper also adds several innovations including studying two different models of bankruptcy risk: one in which bankruptcy occurs randomly as a Poisson process (exogenous bankruptcy) and the other when the mine optimally chooses bankruptcy (endogenous bankruptcy). Other papers do not model bankruptcy risk (Igarashi et al., 2010; Lappi, 2020; Yang & Davis, 2018) or include bankruptcy only at the project termination date (White et al., 2012). We provide theoretical and numerical results comparing the bond and liability policies which have not been shown in previous papers. Also new in our paper is the modelling of the firm's operating decisions under price uncertainty over the full life cycle of a mining project, looking at the initial investment decision as well as the option to temporarily shut down the mine, which delays the cleanup obligation. And finally, we calculate the government's expected cleanup liability.

3 Model formulation

This section describes the firm's decision problem as a stochastic optimal control problem which maximizes the value of the mine under the risk neutral or Q-measure.[12] The value of the mine is the expected net present value of cash flows from the mine assuming the firm applies the optimal control over the life of the mine. The government has no control variables in the decision problem, but bears the cost of cleanup if the firm goes bankrupt. Value for the government is the expected present value, in the Q-measure, of cleanup costs of the mine, which we will denote as $F(\cdot)$. Government value is not calculated as part of the stochastic control problem, but rather is computed in the subsequent Monte Carlo analysis

[12] The risk neutral measure (or Q-measure) means that the risky factor (price) is modelled as a risk adjusted process, implying that the risk free rate is the appropriate discount rate to use for the optimal decision model. This approach is standard in finance and common in the real options literature. See Dixit & Pindyck (1994) for an introductory treatment and Bjork (2009) for an advanced treatment.

described in Section 5.[13] Total welfare in this analysis is the sum of mine value to the firm and expected cleanup costs to the government.

3.1 Description of the firm's decision problem

Consider a firm which extracts a non-renewable resource and thereby generates an undesirable waste product stored in a waste facility. In mining, waste facilities consist of waste rock dumps and tailings dams. However, in our formulation the waste generated could also be interpreted as the area of disturbed land. A government regulator requires the waste be cleaned up when the operation is terminated. There is assumed to be a fixed lease end date, denoted by T, and the project must be terminated no later than at time T. This study focuses on two polices intended to ensure waste cleanup: (1) the strict liability rule (liability policy), and (2) an environmental bond combined with liability for cleanup (bond policy). This study considers three scenarios regarding firm behaviour. In the first scenario (referred to as the solvent firm scenario), the firm chooses its optimal actions assuming it is a going concern, ignoring the possibility of bankruptcy. In the second scenario (termed the exogenous bankruptcy scenario), the firm takes into account the possibility of bankruptcy in its optimal decisions and the probability of bankruptcy is modelled as an exogenous risk, with hazard rate to the price of copper. In the third scenario, which we refer to as the endogenous bankruptcy scenario, the firm may choose to go bankrupt if it is optimal to do so. Exogenous bankruptcy risk might be applicable to a mine owned by a large firm which owns a number of similar mines. Even if the value of the mine under consideration is positive, the parent company may go bankrupt due to low commodity prices. Endogenous bankruptcy risk might be applicable to a smaller firm whose main asset is the mine in question.

Waste stocks from mining may also generate negative externalities, which could be ad-

[13]Note that government value could also be computed as part of the firms optimal control problem, but since there is no government control it is sufficient to determine F through Monte Carlo analysis.

dressed by a tax on the waste stock, as is studied in previous literature (Yang & Davis, 2018; Lappi, 2020; White et al., 2012).[14] This study assumes that there are no ongoing damages from the waste stock prior to mine closure, to allow us to focus on the impact of cleanup costs. For simplicity, we have also assumed that there is no risk of accidental release of pollution from the waste facility.

Before being allowed to develop the mine, the firm enters into an environmental contract with the government specifying the firm's cleanup obligations. Once the firm enters into the environmental contract, it can decide the optimal timing of its initial investment to develop the project, which entails significant capital costs. After the project is launched, the firm manages the level of reserve and the stock of waste by choosing the optimal rates of extraction and abatement, respectively. In addition, the firm maximizes its project value by determining the optimal timing of production, mothballing, reopening the operation, and abandoning the facility and site restoration.

The firm's optimal decisions depend on four state variables: the price of the commodity, $P(t)$, the stock of the resources, $S(t)$, the amount of waste in the land fill, $W(t)$, and the stage of operation, $\delta_i,\ i = 1, 2, 3, 4$. Stage 1 ($i = 1$) is pre-construction, Stage 2 ($i = 2$) is active extraction, Stage 3 ($i = 3$) is mothball or temporary shut down, and Stage 4 ($i = 4$) is abandonment and landfill restoration. Note that to avoid notational clutter, we will not show the explicit dependence of state variables on t, when there is no ambiguity. In addition, for concise notation, in some equations we will denote time dependence using t as a subscript, such as $W(t) \equiv W_t$. The firm has three controls: the rate of resource extraction, q, the rate of waste abatement, a, and the decision to move to a new stage of operation, δ^+. In the endogenous bankruptcy case, the firm has an additional control, $T^* < T$, which is the timing of the bankruptcy decision. As is described later, decisions on the rate of resource extraction and waste abatement occur in continuous time, while the firm makes choices at fixed decision

[14]See Tayebi-Khorami et al. (2019) for a discussion of pollution problems causes by mine waste.

dates as to whether to move to a different stage of operation or declare bankruptcy.

The commodity price, $P(t)$, is assumed to be described by a simple one-factor Ito process, which is mean reverting in the drift term. As is discussed in Section 7.1, this model has been used by other researchers to describe commodity prices (Schwartz, 1997). Under the risk neutral measure,

$$dP = \kappa(\hat{\mu} - \ln P)P\ dt + \sigma P(t)dz;\ P(0) = p_0\ given \tag{1}$$
$$P \in [p_{\min}, p_{\max}]$$

where $\kappa, \hat{\mu}, \sigma$ are parameters reflecting the speed of mean reversion, the long run mean of $\ln(P)$, and volatility, respectively. t denotes time where $t \in [0, T]$, and dz is the increment of a Wiener process. The estimation of the parameters is described in Section 7.1. Parameters are estimated for the risk-neutral world (or Q-measure), so that the term $\kappa(\hat{\mu} - \ln P)P$ represents a risk-adjusted drift rate. As the model will be solved numerically, upper and lower limits must be specified for P. p_{max} is chosen to be large enough so as to have no significant effect on the results, while p_{min} is set to zero.

The risk of bankruptcy in the exogenous bankruptcy scenario is modelled as a Poisson process with Q-measure intensity $\lambda(P)$ given by

$$\lambda(P) = \frac{k_0}{P} \tag{2}$$

in which k_0 is a positive constant. The definition of $\lambda(P)$ implies that the firm is always bankrupt at very low prices (if $P \to 0$), and is never bankrupt at significantly high prices (if $P \to \infty$). For the solvent firm and endogenous bankruptcy scenarios, we set $k_0 = 0$, in Equation (2).[15]

The level of resource stock, $S(t)$, falls over time at the extraction rate q. The dynamic

[15] A similar hazard rate is proposed in Ayache et al. (2003) in the context of convertible bonds.

path of resource stock is given as:

$$dS(t) = -qdt; \quad S(0) = S_0 \; given. \tag{3}$$

The waste stock, $W(t)$, as a by-product of the operation, is assumed to be disposed of into a waste facility with a known, maximum capacity denoted by $\bar{w}$. By assumption, $\bar{w}$ is specified by regulation and is optimal from society's point of view. When the mine initially opens at t_s it creates a waste stock, $W(t_s)$, with no associated revenue. During the operation phase, each unit of resource extracted adds to the stock of waste at the constant rate ϕ, and abatement at the rate a reduces the waste flow. Therefore, the rate of change in the volume of waste is given by

$$\begin{aligned} &dW(t) = (\phi q - a)dt, \text{ for } t > t_s, \; W(t_s) = w_s; \;\; W(t) \leq \bar{w}; \\ &dW(t) = 0; \; W(t) = 0; \text{ for } t_0 < t < t_s, \; t_0 = 0 \end{aligned} \tag{4}$$

where t_0 is the time the firm signs a contract with the government for mine development, t_s is the time of mine opening, w_s is the initial level of waste created at t_s, and where $0 \leq w_s \leq \bar{w}$ with $\bar{w}$ set by regulation. The abatement effort is any action, such as recycling the waste, that occurs during the operation phase. Consistent with the model of Keohane et al. (2007), the abatement rate could be higher than the waste generation rate (i.e., $\phi q < a$). It follows that waste abatement could affect the previously generated waste and reduces the waste stock. Abatement is restricted by the installed capital and cannot exceed its maximum value, $\bar{a}$, at each point of time. By assumption, this upper bound does not change over time.

We now specify admissible sets for δ, q, and a. Let Z_δ denote the admissible set for δ where

$$Z_\delta = \{\delta_1, \delta_2, \delta_3, \delta_4\}. \tag{5}$$

We define an admissible set for the extraction rate q, which depends on both the resource stock and stage of operation. Denote this admissible set as $Z_q(S, \delta)$, which is given as follows:

$$\begin{aligned} & q \in Z_q(S, \delta) \\ & Z_q = [0, \bar{q}], \quad \text{if} \ \ S > 0, \ \ \delta = \delta_2. \\ & Z_q = 0, \quad \text{if} \ \ S = 0, \ \ \delta = \delta_2. \\ & Z_q = 0, \quad \text{if} \ \ \delta = \delta_i, \ \ i = 1, 3, 4, \ \ \forall S. \end{aligned} \tag{6}$$

By assumption, the extraction rate cannot exceed its maximum rate $\bar{q}$. This upper bound is known as the capacity constraint and is assumed to remain constant . There is a positive extraction rate only in stage $\delta = \delta_2$, which represents a currently extracting project.

Similarly, we define an admissible set for a, denoted $Z_a(w, q, \delta)$, as follows:

$$\begin{aligned} & a \in Z_a(w, q, \delta) \\ & Z_a = [0, \bar{a}], \quad \text{if} \quad 0 < W < \bar{w}, \ \ \delta = \delta_2 \\ & Z_a = [0, \phi q], \quad \text{if} \quad W = 0, \ \ \delta = \delta_2 \\ & Z_a = [\phi q, \bar{a}], \quad \text{if} \quad W = \bar{w}, \ \ \delta = \delta_2 \\ & Z_a = 0, \quad \text{if} \quad \delta = \delta_i, \ \ i = 1, 3, 4, \ \ \forall W. \end{aligned} \tag{7}$$

It is assumed that $\bar{a} > \phi\bar{q}$, implying that the firm can abate at a rate that exceeds the waste level generated when extraction is at the maximum $\bar{q}$. Note that Equations (3)–(7) imply that

$$\begin{aligned} & 0 \le W \le \bar{w} \\ & 0 \le S \le S_0 \end{aligned} \tag{8}$$

Assumption 3.1. *Extraction cost, $C^q(q)$ is linear in the rate of extraction q and $\frac{\partial C^q(\cdot)}{\partial q} > 0$. In particular, $C^q(q) = \gamma q$ for a constant parameter $\gamma > 0$.*

That the remaining stock of reserves is not included in the cost function implies relatively homogeneous reserves. As noted in (Roan & Martin, 1996, p. 189), this is appropriate for "mining for disseminated ore such as occurs in many gold deposits in the western states, where open pit methods of extracting the ore are used." The non-renewable resource extraction literature often assumes that extraction costs are convex with respect to both extraction and remaining reserves.[16] We note that the theoretical results presented in Section 6 can easily be extended to this more general functional form. The linear form was chosen for simplicity in the numerical example.

Assumption 3.2 gives the cost of abatement as a convex function of abatement, implying that removing each additional unit of pollution is increasingly costly to the firm. Convexity in abatement costs is a common assumption in the literature.[17]

Assumption 3.2. *The abatement cost function, $C^a(a)$, is assumed to be quadratic: $C^a(a) = \alpha a^2$.*

Restoration affects the stock of waste, rather than the flow. To ease the analysis, it is assumed that periodic restoration is not possible, and thus abatement is the only way to maintain the quality of the environment during the active life of the project.

3.1.1 Environmental bond and cleanup costs

As noted, the goal of the bond is to fully collateralize the government against the risk of having to fund any of the cleanup cost. We assume that the firm must deposit an amount with

[16] See the papers by Farzin (1996), Lappi (2020), and White et al. (2012) for example, which assume resource exploitation costs are convex in both extraction and remaining reserves. Yang & Davis (2018) assume extraction costs are convex with respect to extraction, but not related to the level of reserves.

[17] Such as in Farzin (1996), Keohane et al. (2007), Lappi (2020), White et al. (2012), and Yang & Davis (2018).

the government prior to project commencement sufficient to cover the cleanup costs of waste generated during construction. Over the life of the project, the value of the environmental bond in any period must completely cover the closure costs if the firm were to abandon the mine at the end of the current period.[18] At the end of each period, the firm submits a revised cost estimate and the government adjusts the amount of deposited bonds according to these estimates. We assume that the appropriate level of restoration and the associated cost are correctly determined and thus the bond level is appropriate.

This study assumes a convex cost function for cleanup given by Assumption 3.3.[19] There are additional costs for removing a greater volume of waste and, depending on the degree of toxicity, this may require greater safety precautions for workers during restoration. Moreover, the cost of stabilizing the waste to prevent geographical expansion can increase with waste volume (Phillips & Zeckhauser, 1998). As a result, more waste requires more cleanup effort which becomes more costly at the margin.

Assumption 3.3. *The firm's cost of cleaning up the accumulated waste from the state $W(t)$ to zero waste is quadratic and given by $C^f(w) = \beta w^2$, where β is a constant parameter.*

Let $B(t)$ denote the total value of the bond at time t which by definition, in our model, equals the firm's cleanup costs, $C^f(W)$. $B(t)$ changes over time according to:

$$\frac{dB}{dt} = \frac{dC^f(W)}{dt} = \frac{dC^f}{dW}\frac{dW}{dt} = \theta(W)(\phi q - a) \tag{9}$$

where $\frac{dC^f}{dW} \equiv \theta(W)$, and $\frac{dW}{dt} = \phi q - a$ is given by Equation (4). $\theta(W)$ is defined as the

[18]Closure cost estimates may need to cover the costs of a third party undertaking site restoration, which in practice have been found to exceed the firm's cleanup costs by 15% to 30% due to mobilization and other additional costs associated with transferring cleanup responsibility to a third party (Ferreira et al., 2004; White et al., 2012; Peck & Sinding, 2009; Grant et al., 2009; Otto, 2010). In this paper, we ignore these potential additional costs, and assume that cleanup costs will be the same whether undertaken by the firm or a third party.

[19] Lappi (2020) assumes a convex cleanup cost, while White et al. (2012) and Yang & Davis (2018) assume a constant cost per unit of disturbed land.

marginal cleanup cost or the amount that the government collects on the waste flow over a given time interval. The firm's rate of payment on bonds to the government, $\theta(W)(\phi q - a)$, could be positive, negative, or zero. It follows that the value of the bond at time t is given as:

$$B(w) = \bar{B} + \int_{t_s}^{t} \theta(W)(\phi q - a)dt' \,\Big|\, B(w_{t_s}) = \bar{B},\ W(t) = w,\ t > t_s \tag{10}$$

where $\bar{B}$ represents the initial amount deposited to the bond at t_s, the time of mine opening. Note that with our assumed cleanup cost function, $\theta(W)$ increases linearly in W. The next assumption details the cash flows at project closure or bankruptcy.

Assumption 3.4.

- *Under bonding requirements, the solvent firm receives a refund from restoration at the project termination date equivalent equal to the amount of the bond, which is just the firm's cleanup costs* $C^f(W)$.
- *With no environmental bond (strict liability rule), the cash flow at termination is just* $(-C^f(W))$, *reflecting the firm's cost of remediation.*
- *For the bankrupt firm any bond deposit is forfeited.*

Interest received is another cash flow associated with the bond.

Assumption 3.5.

- *The government pays interest on environmental bonds at the risk-free rate,* r.

This assumption is appropriate given that valuation is under the risk neutral measure. In practice not all environmental bonds pay interest and there may also be bond service fees which increase the cost of the bond to the firm. Any added fees could easily be incorporated into the analysis.[20]

[20] The firm may borrow at a rate $\rho > r$ to finance the bond. We do not address financing costs in this paper. See Aghakazemjourabbaf (2019) for a discussion.

3.1.2 Instantaneous cash flow

The firm's objective is to choose controls to maximize the discounted sum of expected cash flows under the risk neutral measure. Cash flows at any time t will depend on the firm's stage of operations, δ, rate of abatement, a, and extraction, q. T^* refers to the bankruptcy time, which is an optimal decision in the endogenous bankruptcy scenario or an exogenous event in the exogenous bankruptcy scenario. For convenience, we define $T^* = \infty$ for the solvent firm scenario. Cash flows associated with the bond denoted π^B, are given as:

$$\pi^B(W,t) \;=\; -\overbrace{\theta(W)(\phi q - a)}^{\text{bond payment}} + \overbrace{rB}^{\text{bond interest}} \quad ; \text{ if } \delta = \{\delta_2, \delta_3\},\ t_s < t < T^*,\ t_s < t < T. \quad (11)$$

Recall that t_s refers to the time of mine opening. The firm receives interest on the bond during the extraction and mothball stages (stages 2 and 3, respectively). The firm will make payments to, or receive refunds from, the bond in stage 2. Note that the initial deposit to the bond $\bar{B}$ (Equation (10)) does not appear in this cash flow expression, but is included as a switching cost required for the firm to open the mine, (see Section 4.2, Equation (25)). Similarly, the final refund from the bond when the mine closes is included as a switching cost as shown in Section 4.2, Equation (26). Cash flows associated with operations (revenues and operating costs), denoted π^O are given as:

$$\pi^O(P,t;q,a) = P(t)q - C^q(q) - C^a(a) - C^m_{\delta_i}, \quad \delta = \delta_2,\ t_s < t < T^*,\ t_s < t < T. \quad (12)$$

$C^m_{\delta_i}$ refers to fixed costs under both the bond and strict liability policies in stage δ_i. Operating revenues accrue only in stage 2. Total cash flow is specified as follows:

$$\pi(P,W,t;q,a) = \begin{cases} \pi^O(P,t;q,a) + \mathbf{1}_{b=true}\,\pi^B(W,t), & \text{if } \delta = \{\delta_2,\delta_3\},\ t_s < t < T^*,\ t_s < t < T. \\ 0, \quad \text{if} \quad \delta = \delta_4, & \text{or } t = T^*,\ \text{or } t = T. \end{cases} \tag{13}$$

where $\mathbf{1}$ is the indicator function and $b = true$ under the environmental bonding policy and is false otherwise. Because the tax treatment of bonds varies across jurisdictions, we have chosen to ignore taxes in our model specification.[21]

3.2 State and control variables, and the firm's value function

The resource price, $P(t)$, resource stock $S(t)$, waste stock, $W(t)$, and stage of operation, $\delta(t)$, all represent state variables in the decision problem. The value of the firm's operations is a function of these state variables and time, t, denoted as $V(P,S,W,\delta,t)$.

It is assumed that at fixed decision times, the firm makes a decision about whether to move to another stage of operation. These fixed decision times are given as follows:

$$\mathcal{T}_d \equiv \{t_0 = 0 < ... < t_m < ...t_M < T\} \tag{14}$$

where we assume that the optimal decision to move to another stage of operation occurs instantaneously at $t \in \mathcal{T}_d$ (an impulse control). Note that at the lease end date, T, the firm's only option is to terminate the operations. Therefore, time T is excluded from the firm's optimal decisions dates in the above set. Choices regarding optimal rates of abatement, a,

[21] Tax issues include whether the money paid into the bond is deductible for income taxes, whether interest paid to finance the bond is deductible, and whether the bond refund is taxable. See Sasson (2009).

and extraction, q, are made in continuous time during time intervals given as follows:

$$\mathcal{T}_c \equiv \{(t_0, t_1), ..., (t_{m-1}, t_m), ..., (t_{M-1}, t_M)\}. \tag{15}$$

Since we search for the closed loop control, we assume the controls are in feedback form, i.e., functions of the state variables. Control variables can be specified as: $q(P, S, W, \delta, t)$, $a(P, S, W, \delta, t)$; $t \in \mathcal{T}_c$, and $\delta^+(P, S, W, \delta, t)$; $t \in \mathcal{T}_d$. Admissible sets for q, a and δ are given as Z_q, Z_a and Z_δ, specified in Equations (6) and (7), and (5). We specify a control set which contains the controls for all $t_0 \leq t \leq t_M$.

$$K = \{(\delta^+)_{t \in \mathcal{T}_d} \ ; \ (q, a)_{t \in \mathcal{T}_c}\} \tag{16}$$

In the endogenous bankruptcy scenario, we define an additional control, T^*, which represents the optimal bankruptcy time.

The firm's value function can be written as the expected discounted value of the sum of cash flows under the risk neutral measure, given the state variables, and the optimal controls. In the **solvent and exogenous bankruptcy scenarios**, the value function is given as:

$$\begin{aligned} V(p, s, w, \bar{\delta}, t) \ &= \ \sup_K \mathbb{E}_K \left[\overbrace{\int_{t=t'}^{t=\min(T,T^*)} e^{-r(t-t')} \ \pi(P(t), W(t), \delta, t; q, a) dt}^{\text{cash flow, stages 2 or 3}} \right. \\ &\quad - \overbrace{e^{-r(\min(T,T^*)-t')} \sum_{t' \in \mathcal{T}_d}^{\min(T,T^*)} C^{sw}(\delta_i, \delta_j)}^{\text{switching costs between stages } \delta_i \ \& \ \delta_j} \\ &\quad \left. + \overbrace{e^{-r(\min(T,T^*)-t)} V(\cdot, T)}^{\text{value at lease end date}} \ \middle| \ P(t) = p, S(t) = s, W(t) = w, \delta(t) = \bar{\delta} \right]. \end{aligned} \tag{17}$$

where $(p, s, w, \bar{\delta})$ denote realizations of the random and path dependent stochastic variables

(P, S, W, δ). r is the risk-free interest rate, and $\mathbb{E}_K[\cdot]$ is the expectation operator. For $t > T^*$, $V = 0$. At the lease end date, $V(\cdot, T) = 0$ if the mine has already shut down or gone bankrupt prior to T. If the mine is still operating at $t = T - \epsilon$, then $V(\cdot, T)$ reflects the cleanup costs net of any bond refund. This is specified in the boundary conditions in Appendix A.

The value function for the **endogenous bankruptcy scenario** is the same as Equation (17), except that the date of bankruptcy, T^*, is included as an additional control in the supremum operation. For $t > T^*$, $V = 0$. For $t < T^*$,

$$
\begin{aligned}
V(p, s, w, \bar{\delta}, t) \;=\; \sup_{K, T^*} \mathbb{E}_{K,T^*} \Bigg[& \overbrace{\int_{t=t'}^{t=\min(T,T^*)} e^{-r(t-t')}\, \pi(P(t), W(t), \delta, t; q, a) dt}^{\text{cash flow, stages 2 or 3}} \\
& - \overbrace{e^{-r(\min(T,T^*)-t')} \sum_{t' \in \mathcal{T}_d}^{\min(T,T^*)} C^{sw}(\delta_i, \delta_j)}^{\text{switching costs between stages } \delta_i \text{ \& } \delta_j} \\
& + \overbrace{e^{-r(\min(T,T^*)-t)} V(\cdot, T)}^{\text{value at lease end date}} \;\Bigg|\; P(t) = p, S(t) = s, W(t) = w, \delta(t) = \bar{\delta} \Bigg].
\end{aligned} \tag{18}
$$

4 The Firm's HJB Equation and Optimal Controls

Equations (17) and (18) are solved backwards in time using dynamic programming. For a particular decision time, $t_m \in \mathcal{T}_d$, we define t_m^- and t_m^+ to represent the moments just before and after t_m. Specifically $t_m^- = t_m - \epsilon$ and $t_m^+ = t_m + \epsilon$, $\epsilon \to 0^+$. [22] For example, at t_{m+1} we determine the optimal control δ^+, which is the stage of operation. Then given that stage of operation, going backward in time, we solve for the optimal controls q and a in continuous time between $t_{m+1}^- \to t_m^+$.

[22] As a visual aid, the times around t_m and t_{m+1} are depicted below, going forward in time: $t_m^- \to t_m \to t_m^+ \to t_{m+1}^- \to t_{m+1} \to t_{m+1}^+$.

4.1 Optimal abatement, a, and extraction, q, from $t^-_{m+1} \to t^+_m$

We assume a positive hazard rate for exogenous bankruptcy, $\lambda(P) > 0$ in deriving the HJB equation, which we model as a Poisson jump process. For the solvent and endogenous bankruptcy scenarios, set $\lambda(P) = 0$. Consider a time interval $h < (t_{m+1} - t_m)$. For $t \in (t^+_m, t^-_{m+1} - h)$, the dynamic programming principle states that (for small h), $t < T^*$,

$$
\begin{aligned}
V(p,s,w,\bar{\delta},t) \quad &= e^{-rh}\mathbb{E}_{K,T^*}\Big[V(P(t+h),S(t+h),W(t+h),\delta(t),(t+h))\Big| \\
&\qquad P(t)=p,S(t)=s,W(t)=w,\delta(t)=\bar{\delta}\Big] + \pi(p,s,w,\bar{\delta})h.
\end{aligned} \tag{19}
$$

Note that for $t \in (t^+_m, t^-_m)$, the operating stage is fixed and hence there are no switching costs. Letting $h \to 0$ and using Ito's Lemma for jump (bankruptcy) and diffusion process,[23] the equation satisfied by the value, V, for all operating states except for abandonment is given as:

$$
\frac{\partial V}{\partial t} + \mathcal{L}V + \max_{q,a}\Big\{-q\,\frac{\partial V}{\partial s} + (\phi q - a)\,\frac{\partial V}{\partial w} + \pi(t)\Big\} = 0, \quad \text{for } \delta = \delta_i, \quad i = 1,2,3; \quad t < T^*. \tag{20}
$$

in which $\mathcal{L}V$ is the differential operator as follows

$$
\mathcal{L}V = \frac{1}{2}\sigma^2 p^2\,\frac{\partial^2 V}{\partial p^2} + \kappa(\hat{\mu} - \ln p)p\,\frac{\partial V}{\partial p} + \big(r + \lambda(p)\big)V. \tag{21}
$$

We maximize with respect to the control variables a and q, and $\pi(t)$ refers to net cash flows as defined Equation (12). Note that since we are valuing the project in the risk neutral world, $\lambda(p)$ represents the market price of bankruptcy risk rather than the actual risk that

[23] Dixit & Pindyck (1994) provide an introductory treatment of optimal decisions under uncertainty characterized by an Ito process such as Equation (1). A more advanced treatment is given by Bjork (2009) and Oksendal & Sulèm (2005). Cont & Tankov (2009) is the standard reference for financial modelling with jump processes. Note that we are applying Ito's Lemma to infinitely smooth test functions, as required by viscosity solution theory. This does not require that the value function be smooth. See Barles & Souganidis (1991).

would be calculated using historical bankruptcy data. This is discussed further in Section 7.1.

Once the project is in Stage 4, the project value goes to zero.

$$V(p, s, w, \delta = \delta_4, t) = 0. \tag{22}$$

4.2 Choice of operating stage, δ, and bankruptcy time, T^*, at t_m

For $t_m \in \mathcal{T}_d$, the firm checks to determine whether it is optimal to switch to a different operating stage. In the numerical example, decision dates occur annually. The firm will choose the operating stage which yields the highest value net of any costs of switching. Let $C^{sw}(\delta^-, \delta')$ denote the cost of switching from stage δ^- to δ'. Recall that $t = t^-$ represents the moment before t_m and $t = t^+$ denote the instant after t_m. Solving going backward in time, and noting the optimal stage is denoted as δ^+, the value at t_m^- is given by the following expression when endogenous bankruptcy is not permitted:

$$\begin{aligned} V(p, s, w, \delta^-, t_m^-) &= V(p, s, w, \delta^+, t_m^+) - C^{sw}(\delta^-, \delta^+) \\ \delta^+ &= \arg\max_{\delta'} \, [V(p, s, w, \delta', t_m^+) - C^{sw}(\delta^-, \delta')]. \end{aligned} \tag{23}$$

In the event of a tie, it is assumed that the firm remains in the current stage, δ^-. In the endogenous bankruptcy scenario, Equation (23) is adjusted as follows:

$$\begin{aligned} V(p, s, w, \delta^-, t_m^-) &= \max \left[[V(p, s, w, \delta^+, t_m^+) - C^{sw}(\delta^-, \delta^+],\ 0) \right] \\ \delta^+ &= \arg\max_{\delta'} \, [V(p, s, w, \delta', t_m^+) - C^{sw}(\delta^-, \delta')]. \end{aligned} \tag{24}$$

Switching costs differ under the bond and strict liability policies for project commencement as well as for mine abandonment. Opening the mine under the bond requires the investment cost and initial bond payment $\bar{B}$, whereas the latter is absent under the liability rule. Denoting the investment cost with I, the cost of opening the mine is given as

$$C^{sw}(\delta_1, \delta_2) = I + \mathbf{1}_{b=true}\bar{B} \tag{25}$$

The cost to switch to Stage 4 (abandonment) from either Stage 2 (operating) or Stage 3 (mothballed) is given by

$$C^{sw}(\delta_i, \delta_4) = -[\mathbf{1}_{b=true}B(w) - C^f(w)] \qquad i = 2, 3. \tag{26}$$

Under strict liability this is just the firm's own cleanup cost $C^{sw}(\delta_i, \delta_4) = C^f(w) > 0,\ i = 2, 3.$ Under the bonding policy, $B(w) = C^f(w)$, so there is no net cost to the firm of closing the mine. The firm receives the refund on the bond and uses it to undertake cleanup. Switching costs between other stages are set as constants, as detailed in Section 7.

The stochastic optimal control problem given by Equations (20)–(24) is solved using a standard numerical approach described in Appendix B.

4.3 Characterizing optimal extraction and abatement policies

In this section, we examine the first order conditions for extraction and abatement which hold during in Stage 2, $\delta = \delta_2$, when the firm is actively producing the ore. These first order conditions provide intuition about the optimal extraction and abatement rates, denoted a^* and q^*, and in particular whether the solutions are bang-bang.

The optimal extraction rate, q^*, and the optimal abatement rate, a^*, are obtained by maximizing Equation (20) with respect to the terms that contain q and a. Specifically we find q and a such that:

$$\max_{q\in Z_q,\, a\in Z_a} \Big\{ - q\, \frac{\partial V}{\partial s} + (\phi q - a)\, \frac{\partial V}{\partial w} - \overbrace{\mathbf{1}_{b=true}\theta(W)(\phi q - a) + P(t)q - C^q(q) - C^a(a)}^{\text{components of } \pi(\cdot;q,a)} \Big\} = 0 \tag{27}$$

Recall that the extraction cost is assumed to be liner in q, so that $\partial C^q / \partial q = \gamma$ for some constant γ. The optimal extraction rate for a firm satisfies:

$$P - \frac{\partial C^q}{\partial q} - \frac{\partial V}{\partial s} + \overbrace{\phi\Big[\frac{\partial V}{\partial w} - \mathbf{1}_{b=true}\ \theta(w)\Big]}^{\text{marginal cost of waste buildup}} \begin{cases} \geq 0 & \Rightarrow q^* = \bar{q} \\ < 0 & \Rightarrow q^* = 0 \end{cases} \tag{28}$$

which is a bang-bang control. The first three terms in Equation (28) are the marginal revenue from extraction, marginal cost of extraction, and marginal value of the reserve to the firm. We have called the term in square brackets the firm's *marginal cost of waste buildup* which has two components: 1) the marginal value of the waste stock to the firm, $\frac{\partial V}{\partial w}$, and 2) the marginal cleanup cost, $\mathbf{1}_{b=true}\ \theta(w)$. It follows that given an optimal abatement rate, the firm extracts at capacity as long as the marginal revenue less marginal cost of extracting a reserve is non-negative.

Given q^*, Equation 28 is quadratic in a, hence the optimal abatement rate, a^*, is the root of:

$$C^{a\prime}(a^*) + \left[\frac{\partial V}{\partial w} - \mathbf{1}_{b=true}\ \theta(w)\right] = 0,\ \text{if}\ \min(Z_a) < a^* < \max(Z_a), \tag{29}$$

otherwise $a^* \in \{\min(Z_a), \max(Z_a)\}$, where Z_a is defined in Equation (7). Along the optimal abatement path if $0 < a < \bar{a}$, the marginal cost of waste buildup, $\left[\frac{\partial V}{\partial w} - \mathbf{1}_{\mathbf{b=true}}\theta(w)\right]$, is equal to the marginal abatement cost. If the marginal cost of abatement exceeds the marginal cost of waste buildup, the firm will reduce abatement until the equality is restored or until abatement is at its minimum value as specified in Z_a. Similarly if the marginal cost of abatement is less than the marginal cost of waste buildup, the firm will increase its abatement level until the equality is restored or the maximum a is reached.

5 Government expected cleanup costs

Let $\pi^G(W,t)$ denote the government's cash flow which equals the cleanup cost in the event the firm goes bankrupt. For the endogenous bankruptcy scenario, the firm declares bankruptcy when it is optimal to do so, i.e. when $V(\cdot) = 0$. For the exogenous bankruptcy scenario, bankruptcy occurs according to a Poisson process, with hazard rate $\lambda(P)$, defined in Equation (2). The government's value function, $F(\cdot, t)$, can be expressed as:

$$F(p,s,w,\bar{\delta},t) = \mathbb{E}\left[\pi^G(W,T^*)\right], \quad \text{where} \quad \pi^G(W,t) = C^f(W,T^*); \tag{30}$$
$$T^* = \inf\{0 \le t \le T; V(p,s,w,\bar{\delta},t) = 0\}, \quad \text{Endogenous bankruptcy,} \quad \text{or}$$
$$T^* = \text{Time of bankruptcy occurs with hazard rate } \lambda(P), \quad \text{Exogenous bankruptcy.}$$

These expectations are computed using Monte Carlo analysis based on the stored optimal controls from the numerical solution of the firm's HJB equation. This is described in Appendix B

6 Theoretical results: equivalence of solvent firm (bond and liability) and endogenous bankruptcy (bond) scenarios

In this section we demonstrate that the solvent firm scenario (liability and bond cases) and the endogenous bankruptcy scenario (bond) are equivalent. This allows the paper to focus on the four scenarios identified earlier.[24] For ease of exposition, we will make the following assumptions.

Assumption 6.1. *No switching costs between operating stages. This implies* $C^{sw}(\delta^-, \delta^+) = 0$ *in Equation* (23).

Assumption 6.2. *Initial waste stock at mine opening is zero, which implies* $\bar{B} = 0$ *in Equation* (10).

Assumption 6.3. *No abandonment stage. This implies that the admissible set for* δ *(originally specified in Equation* (5)*) is changed to* $Z'_\delta = [\delta_1, \delta_2, \delta_3]$ *The revised control set* K' *excludes the abandonment option,* δ_4.

$$\begin{aligned} K' &= \{(\delta^+)_{t \in \mathcal{T}_d}\ ;\ (q, a)_{t \in \mathcal{T}_c}\} \\ \delta^+ &\in Z'_\delta; \quad q \in Z_q; \quad a \in Z_a \end{aligned} \tag{31}$$

where Z_q *and* Z_a *were previously defined in Equations* (6) *and* (7)*; and* $\mathcal{T}_d$ *and* $\mathcal{T}_c$ *were defined in Equations* (14) *and* (15).

We emphasize these assumptions are only used for the theoretical analysis in this section and not for the subsequent numerical analysis. We will later argue that the theoretical results continue to hold even if these assumptions are relaxed.

Remark 6.1. *Note that although we have removed the abandonment control, the operator can still mothball the operation until terminal time* T.

[24] The paper analyzes the following scenarios: (1) solvent firm, (2) endogenous bankruptcy, liability, (3) exogenous bankruptcy, bond, and (4) exogenous bankruptcy, liability.

6.1 Comparing mine values and optimal controls for the solvent firm under bond and liability policies

6.1.1 Mine values

The bond value at time t was defined in Equation (10) as the integral of cash flows associated with the bond from time $t_0 = 0$ to t, plus the initial payment to the bond when the mine opens (moves to stage 2), $\bar{B}$. Given Assumption 6.2, $\bar{B} = 0$. The total discounted cash flow associated with the bond, π^B, from t to T is given by:

$$\begin{aligned} \text{total bond cash flow} &= \int_t^T e^{-r(t'-t)} \pi^B(W_{t'}) dt' \\ \text{where } \ \pi^B &= -\theta(W_t)(\phi q - a) + rB_t \end{aligned} \tag{32}$$

The following lemma will prove useful in one of our main theorems.

Lemma 6.1 (Discounted bond cash flows)**.**

$$\int_{t'=t}^{t'=T} e^{-r(t'-t)} \pi^B(W_{t'}) dt' = -e^{-r(T-t)} B(W_T) + B(w) \,\Bigg|\, W(t) = w \tag{33}$$

Proof.

$$\int_t^T e^{-r(t'-t)} \pi^B(W_{t'}) dt' = \int_t^T e^{-r(t'-t)} [-\theta(W_{t'})(\phi q - a) + rB(W_{t'})] dt' \,\Bigg|\, W(t) = w \tag{34a}$$

$$= \int_t^T e^{-r(t'-t)} [-\theta(W_{t'})(\phi q - a)] dt' + \int_t^T e^{-r(t'-t)} rB(W_{t'}) dt' \,\Bigg|\, W(t) = w \tag{34b}$$

Use integration by parts to evaluate the last integral term in Equation (34b). This results in:

$$\int_t^T e^{-r(t'-t)} rB(W_{t'}) dt' = -e^{-r(t'-t)} B(W_{t'}) \Big|_t^T + \int_t^T e^{-r(t'-t)} \theta(W_{t'})(\phi q - a) dt' \,\Bigg|\, W(t) = w \tag{35}$$

Substituting Equation (35) for the last integral term in Equation (34b) gives Lemma 6.1

□

Next we will compare the value of the mine under the bond and liability policies at any time t and will show that the values differ only by the value of the bond. By definition the value of the bond at any time is equal to the cost of cleaning up the current waste stock: $B(W_t) = C^f(W_t)$.

Theorem 6.1 (Difference in mine values under the bond and liability policies for the solvent firm)**.** *Under Assumptions 6.1, 6.2, and 6.3, we denote the value of the mine under the liability policy as $V(\cdot,t)$ and the value of the mine under the bonding policy as $\hat{V}(\cdot,t)$. Then,*

$$\hat{V}(\cdot,t) = V(\cdot,t) + B(w) \tag{36}$$

Proof. We rewrite Equation (17), for the solvent firm to include the new control set K'.

Solvent firm value, liability:

$$V(p,s,w,\bar{\delta},t) = \sup_{K'} \mathbb{E}_{K'} \left[\int_{t'=t}^{t'=T} e^{-r(t'-t)} \, \pi^O(P(t'),S(t'),W(t'),\delta(t')) \, dt' \right. \tag{37}$$
$$\left. -e^{-r(T-t)} B(W_T) \;\middle|\; P(t') = p, S(t') = s, W(t') = w, \delta(t') = \bar{\delta} \right].$$

In the above equation, the terminal value for the solvent firm at time T just equals the cleanup costs, $B(W_T) = C^f(W_T)$. Since there is no abandonment state, cleanup will only occur at time T.

The value of the mine under the bond policy, $\hat{V}(\cdot, t)$, is given by:

Solvent firm value, bond:

$$\hat{V}(p,s,w,\bar{\delta},t) = \sup_{K'} \mathbb{E}_{K'} \left[\int_{t'=t}^{t'=T} e^{-r(t'-t)} \left[\pi^O(P(t'),S(t'),W(t'),\delta(t'))\right] dt' \right.$$
$$\left. - \int_{t'=t}^{t'=T} e^{-r(t'-t)} [\theta(W_t)(\phi q - a) - rB(w)] dt' \;\middle|\; P(t') = p, S(t') = s, W(t') = w, \delta(t') = \bar{\delta} \right]. \tag{38}$$

Using the result from Lemma 6.1 given by Equation (33), Equation (38) can be rewritten as:

$$\begin{aligned} \hat{V}(p,s,w,\bar{\delta},t) = \sup_{K'} \mathbb{E}_{K'} &\left[\int_{t'=t}^{T} e^{-r(t'-t)} \left[\pi^O(P(t'),S(t'),W(t'),\delta(t'))\right] dt' \right. \\ &\left. (-e^{-r(T-t)} B(W_T) + B(w)) \;\middle|\; P(t') = p, S(t') = s, W(t') = w, \delta(t') = \bar{\delta} \right]. \end{aligned} \tag{39}$$

We now employ the following fact, which can usefully be applied in optimal control problems (Forsyth & Labahn, 2007). Given two functions $h(x), g(x),\ x \in Q$, then

$$\sup_x h(x) - \sup_{x'} g(x') \leq \sup_x (h(x) - g(x)) \tag{40a}$$

$$\sup_x h(x) - \sup_{x'} g(x') \geq \inf_x (h(x) - g(x)) \tag{40b}$$

Using Equation (40a), we subtract $\hat{V}(\cdot,t)$ from $V(\cdot,t)$ to get:

$$
\begin{aligned}
V(\cdot,t)-\hat{V}(\cdot,t) &= \sup_{K'}\left[\int_{t'=t}^{t'=T} e^{-r(t-t')}\ \pi^{O}(P(t),S(t),W(t),\delta)\ dt' - e^{-r(T-t)}B(W_T)\right] \\
&- \sup_{K'}\mathbb{E}_K\ \left[\int_{t'=t}^{t'=T} e^{-r(t'-t)}\ [\pi^{O}(P(t'),S(t'),W(t'),\delta(t'))]\ dt' + (-e^{-r(T-t)}B(W_T)+B(w))\right] \\
&\leq -\ \sup_{K'}\mathbb{E}_K\ \left[B(w)\right] =\ -B(w)
\end{aligned}
\tag{41}
$$

The final line in Equation (41) follows because the B depends only on the waste stock w. Similarly, using Equation (40b),

$$
\begin{aligned}
V(\cdot,t)-\hat{V}(\cdot,t) &\geq \inf_{K'}\Big(-e^{-r(T-t)}B(W_T)\ +\ (e^{-r(T-t)}B(W_T))\ -\ B(w))\Big) \\
&=\ -B(w)
\end{aligned}
\tag{42}
$$

Theorem 6.1 and Equation (36) follow from Equations (41) and (42). □

Corollary 6.1.1. *Assuming the firm begins in the pre-construction phase (stage 1), it follows from Theorem 6.1 that at time $t=t_0$, the values of the mine under the bond and liability policies will be the same.*

6.1.2 Optimal controls

Lemma 6.2 (Equivalence of optimal controls for q and a for the bond and liability policies for the solvent firm for $t \notin \mathcal{T}_d$)**.** *Assuming the same tie breaking rules are used in the event of a tie between one or more choices of the controls, then the optimal controls for a and q will be the same for Equation (37) (liability policy) and Equation (38) (bond policy).*

Proof. Denote the mine value under the bond policy as $\hat{V}(\cdot,t)$ and under the liability policy as $V(\cdot, t)$. For the bond policy, the optimal q and a are chosen by maximizing:

$$\max_{q \in Z_q,\, a \in Z_a} \left\{ -q \frac{\partial \hat{V}}{\partial s} + (\phi q - a) \frac{\partial \hat{V}}{\partial w} - \overbrace{\theta(W)(\phi q - a) + P(t)q - C^q(q) - C^a(a)}^{\text{components of } \pi(\cdot; q, a)} \right\} = 0 \quad (43)$$

For the liability policy, optimal q and a are chosen by maximizing:

$$\max_{q \in Z_q,\, a \in Z_a} \left\{ -q \frac{\partial V}{\partial s} + (\phi q - a) \frac{\partial V}{\partial w} - \overbrace{P(t)q - C^q(q) - C^a(a)}^{\text{components of } \pi(\cdot; q, a)} \right\} = 0 \quad (44)$$

From Theorem 6.1, we can substitute $\hat{V}(\cdot, t) = V(\cdot, t) + B(w)$ into Equation (43) to get:

$$\max_{q \in Z_q,\, a \in Z_a} \left\{ -q \frac{\partial V}{\partial s} + (\phi q - a) \left(\frac{\partial V}{\partial w} + \frac{\partial B}{\partial w} \right) - \overbrace{\theta(W)(\phi q - a) + P(t)q - C^q(q) - C^a(a)}^{\text{components of } \pi(\cdot; q, a)} \right\} = 0 \quad (45)$$

Noting that $\partial B / \partial W = \theta(W)$ this reduces to

$$\max_{q \in Z_q,\, a \in Z_a} \left\{ -q \frac{\partial V}{\partial s} + (\phi q - a) \frac{\partial V}{\partial w} - \overbrace{P(t)q - C^q(q) - C^a(a)}^{\text{components of } \pi(\cdot; q, a)} \right\} = 0 \quad (46)$$

which is exactly the same as Equation (44). Therefore the pair (a^*, q^*) which maximizes Equation (44) is the same (a^*, q^*) that maximizes (46), assuming the same tie breaking rule is applied. Hence the bond and liability policies have the same optimal controls for $t \notin \mathcal{T}_d$. □

Remark 6.2. *It should be noted that Lemma 6.2 does not depend on the particular functional forms for $C^q(q)$ or $C^a(a)$.*

Lemma 6.3 (Equivalence of optimal control for project stages $(\delta_1, \delta_2, \delta_3)$ at $t \in \mathcal{T}_d$)**.** *Assuming the same tie breaking rule is used, the firm will choose the same optimal control for the operating stage under the bond and liability policies.*

Proof. From Equation (23), we can observe that the optimal operating stage is chosen for fixed $W(t) = w$. From Theorem 6.1, we know that the mine values under the bond and liability policies differ only by $B(w)$. Hence with a fixed w, the optimal control for the operations stage will be the same for both policies. □

The next theorem follows immediately from Lemmas 6.2 and 6.3

Theorem 6.2. *The controls for the bond and liability polices are the same for all* $t \in [0, T]$.

6.2 Equivalence of endogenous bankruptcy under a bond with the solvent firm case

We will now incorporate two additional controls in our problem: an optimal shut down time T' for the solvent firm scenario (i.e. no bankruptcy) with a bond and an optimal bankruptcy time T^* for the endogenous bankruptcy scenario with a bond. At T' the mine is abandoned and the site is cleaned up.[25] At T^* the firm decides to declare bankruptcy, ceasing operations and taking on no responsibility for site cleanup. In this section, we denote the value of the mine with the shutdown option as $V^s(\cdot, t)$, and the value of the mine with the endogenous bankruptcy option as $V^{eb}(\cdot, t)$.

Theorem 6.3 (Optimal abandonment time for the solvent firm versus optimal bankruptcy time, both under the bond policy)**.** *Assuming the same tie breaking rules apply, and given assumptions 6.1 and 6.2, the mine abandonment time for the solvent firm,* T'*, will be the same as the optimal bankruptcy time,* T^**, both under the bond policy.*

Proof. We write the value function, $V^s(p, s, w, \bar{\delta}, t)$ under the bond policy for the solvent firm including the shut down time T' as an optimal control. This is just Equation (39) altered to

[25] This is equivalent to having a shut down stage of operations, δ_4 as defined in Equation (5).

include T'. To avoid trivial situations, we assume that the starting time $t < T'$ and $t < T^*$.

$$V^s(p, s, w, \bar{\delta}, t) = \sup_{K', T'} \mathbb{E}_{K',T'} \left[\int_{t'=t}^{\min(T,T')} e^{-r(t'-t)} \left[\pi^O(P(t'), S(t'), W(t'), \delta(t'))\right] dt' \right.$$
$$\left. \big(- e^{-r(\min(T,T')-t)} B(W_{\min(T,T')}) + B(w)\big) \,\middle|\, P(t') = p, S(t') = s, W(t') = w, \delta(t') = \bar{\delta} \right]. \tag{47}$$

The value of the mine with the endogenous bankruptcy option with the control bankruptcy time T^* is give as:

$$\hat{V}^{eb}(p, s, w, \bar{\delta}, t) = \sup_{K', T^*} \mathbb{E}_{K',T^*} \left[\int_{t'=t}^{\min(T,T^*)} e^{-r(t'-t)} \left[\pi^O(P(t'), S(t'), W(t'), \delta(t'))\right] dt' \right.$$
$$\left. \big(- e^{-r(\min(T,T^*)-t)} B(W_{\min(T,T^*)}) + B(w)\big) \,\middle|\, P(t') = p, S(t') = s, W(t') = w, \delta(t') = \bar{\delta} \right]. \tag{48}$$

Comparing Equations (47) and (48), we observe that the control T' in Equation (47) plays the same role as in the control T^* in Equation (48). Hence the two value functions are exactly the same and the stopping times, i.e. bankruptcy and shut down times, are identical, if they exist. □

6.3 Relaxing Assumptions 6.1 - 6.3

In Section 6.2, we explicitly allowed for optimal abandonment thereby relaxing Assumption 6.3. Assumption 6.1 can be removed at the expense of tedious notational complexity in Equations (37) and (38). Assumption 6.2 can be removed by requiring a large production of waste during an infinitesimal time interval upon entering the production stage with no associated revenue or abatement. Hence the initial bond payment required when the mine opens ($\bar{B}$ in Equation (10)) can be incorporated as part of π in Equation (38) in the proof of Theorem 6.1. It follows that Theorems 6.1, 6.2 and 6.3 will apply even if assumptions 6.1 - 6.3 are relaxed.

Remark 6.3 (Significance of Theorems)**.** *These three theorems imply that our numerical computations can be restricted to four scenarios: (1)solvent firm, liability, (2) endogenous bankruptcy, liability, (3) exogenous bankruptcy, liability, (4) exogenous bankruptcy, bond. We do not need to consider the solvent firm bond scenario nor the endogenous bankruptcy bond scenario as these are equivalent to scenario (1).*

7 An application to copper industry

A numerical example of optimal decisions over the life of a copper mine is developed based on available data from an open-pit copper mine in British Columbia, supplemented by researcher assumptions when data is lacking. The parameters of the stochastic model assumed for copper prices are estimated using copper futures contracts. The numerical approach to solving the firm's optimal control problem represented by Equations (20)–(24) is described in Appendix B. Monte Carlo analysis to determine the distribution over time of optimal controls as well as expected value for the firm and government is also described in Appendix B.

7.1 Price process parameters and the bankruptcy hazard function

The parameters of the stochastic differential equation describing copper prices (Equation (1)) are estimated in the risk-neutral world using copper futures price data. We define the parameter $\hat{\mu} = \mu - \eta$ where η is the market price of price risk.[26] Estimation results are

[26] As a note of clarification, this definition of the market price of risk is consistent with Schwartz (1997), but is different than the more commonly used definition as specified in Insley & Lei (2007). The more common definition defines market price of risk as $\hat{\lambda} = (\hat{\mu} - r)/\hat{s}$ where $\hat{\mu}$ and $\hat{s}$ are the return and volatility of a hedging asset. Using this definition the risk adjusted drift rate would be $\kappa(\mu - ln(P) - \hat{\lambda}\sigma/\kappa)P$.

κ	0.0264 (0.001)	Root Mean Square Error	0.07
μ	2.7051 (0.079)	Mean Absolute Error	0.05
η	2.7845 (0.026)	Log-likelihood function	9652
σ^2	0.0458 (0.002)	Number of observations	937

Table 1: *Estimation results for the one-factor copper price model using Kalman Filter. RMSE, MAE, μ, and η are in terms of US \$/lb. Standard errors are in parenthesis. Weekly futures data from Aug 1st, 1997 to Jul 13th, 2015.*

provided in Table 1. To obtain estimates, we have used a Discrete Kalman Filtering approach and a Maximum Likelihood Function.[27] This study uses weekly data for copper futures contracts traded on the London Metal Exchange (LME).[28] The estimation was done for six futures contracts dated from August 1997 to July 2015, with 1, 6, 11, 16, 21, and 24 months to maturity.[29] To find real copper prices, futures prices are deflated by the US Consumer Price Index. Consistent with Schwartz (1997), futures contracts closest to maturity proxy the market spot prices. The Root Mean Square Error (RMSE) and Mean Absolute Error (MAE) of the estimates of log futures prices are 7 cents per pound and 5 cents per pound, respectively. All parameter estimates are significant. These findings suggest that the one-factor model provides a good tracking of the copper market prices as shown in Figure (1). Further details on parameter estimations are provided in Aghakazemjourabbaf (2019).

Figure (2) displays the mean and percentiles from 64,000 Monte Carlo simulations of the estimated price process for 15 years into the future, which is the assumed life of the copper mine. We observe a wide spread between the 5th and 95th percentiles, and a general downward trend. This implies that under the risk neutral measure, the mine owner assumes a fairly negative price trend but with large variability when making investment decisions. For the numerical computations, we must choose a maximum value for the price of copper. The numerical value chosen for p_{max} is \$30 per pound which is several times larger than the

[27]These methods are explained in Schwartz (1997).

[28]Data for this study were collected from Datastream.

[29]Long maturity contracts are of most interest as the goal of this study is to value a long-term investment project.

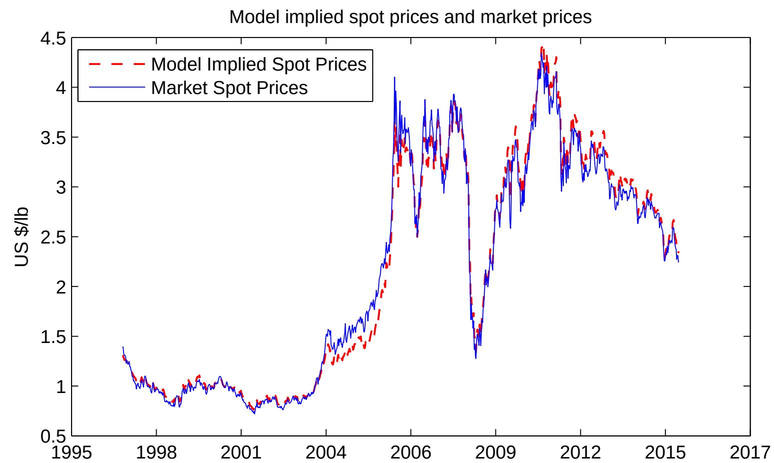


Figure 1: *Model implied copper spot prices and market copper prices. Weekly data from Aug 1st, 1997 to Jul 13th, 2015. Nominal prices are deflated by the US Consumer Price Index, base year=2007*

95th percentile shown in Figure (2).

For asset valuation under the $\mathbb{Q}$ measure, the risk of bankruptcy should reflect the market price of bankruptcy risk, which can be higher or lower than the historical bankruptcy risk.[30] Estimating the market price of bankruptcy risk is beyond the scope of this paper. As described in Section 3, we model the risk of exogenous bankruptcy as a Poisson process with hazard rate $\lambda(P) = k_0/P$ with $k_0 = 0.1$.[31] Looking at Figure 2, copper prices generally fall within the range of \$0.50 to \$5.00 which implies a range for λ of 0.02 to 0.2.

7.2 Project specification

Assumptions regarding the project specification are provided in Table 2. The numerical example is based on data from Copper Mountain, an open-pit mine located in south-western

[30] It has been observed that the corporate bond yields exceed the risk-free rate by an amount greater than what is justified by historical default rates (Amato & Remolona, December, 2003). The spread between corporate bond yields and the risk free rate can be used as an indication of bankruptcy risk, in the Q-measure.

[31] This functional form has been used in the literature to relate the probability of bankruptcy with a firm's stock price (Ayache et al., 2003; Muromachi, 1999).

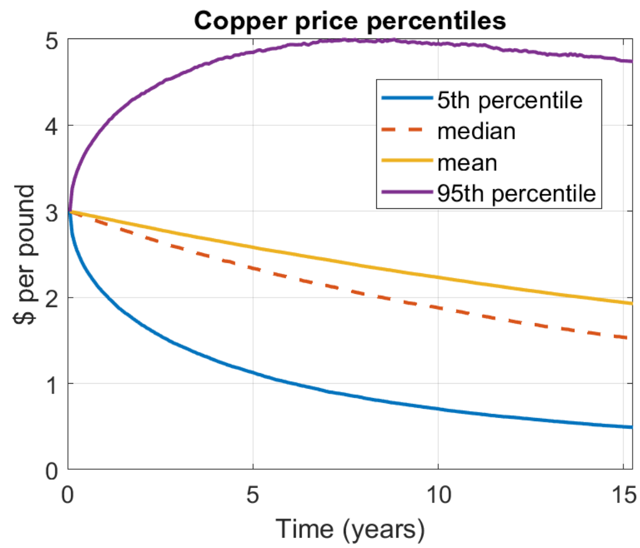


Figure 2: *Monte Carlo simulation of copper price process, 64000 simulations with a time step of 1/16 of a year*

British Columbia with an expected life of 15 years when it was first proposed.[32] In 2007, the Copper Mountain project proceeded to a feasibility study to construct an open-pit mine at the estimated cost of US $380 million. An additional US $5 million for the feasibility study, environmental testing and geological consulting increased the construction cost to US $385 million. This mine had a production target of 78.2 million pounds of copper per year, starting from June 2011, with an estimated average production cost of US $1.35 per pound of copper. The fixed cost of sustaining capital was estimated to be US $1.66 million per year. The mine's strip ratio (i.e., waste/ore) varies from 1 to 2 pounds of waste per each pound of ore extracted. We have assumed a strip ratio of 2 in the numerical example.

Cleanup costs are site specific, depending not just on the mine characteristics, but also on the stringency of cleanup regulations. In 2016 the Auditor General in B.C. Canada released a report on the total potential environmental liabilities faced by tax payers from the

[32] The project was described by the Minerals Resources Education Program of BC. Copper Mountain in an on-line report http://www.mineralsed.ca/s/MineProfile.asp?ReportID=534356.

B.C. mining industry (Bellringer, 2016). Subsequently information was released providing estimates of outstanding remediation costs by mine site (Shen, 2016). For metal mines the cleanup liability for individual mines ranged from negligible to over $200 million for the Teck Highland Valley Copper Mine. In our analysis, the cleanup cost is based on our assumed cleanup cost function which depends on the waste stock, w, according to $C^f(w) = \beta w^2$. We set the parameter $\beta = 0.00003$ which provides a reasonable range of cleanup costs. At the assumed upper limit on the waste stock of 2200 pounds, the cleanup cost would be $145 million. The maximum feasible rate of abatement is assumed to be twice as high as the waste buildup rate, i.e., $\bar{a} = 2\phi\bar{q}$.[33] This assumption allows for the possibility that the abatement rate may exceed the rate of waste buildup.

Either mothballing the mine or resuming operations after mothballing are assumed to entail an up-front cost of $5 million. It is further assumed that remaining in the mothballed stage costs $1 million per year for environmental monitoring and maintenance. Note that in Equation (12) in the production phase, $C^m_{\delta_2}$ equals the fixed costs of sustaining capital ($1.66 million), while at the mothballed stage $C^m_{\delta_3}$ is the summation of costs for sustaining capital $1.66 million and environmental monitoring and maintenance costs of $1 million.

8 Numerical results analysis

This sections presents results of the numerical solution of the HJB equation as well as the Monte Carlo analysis. The solution of the HJB equation determines the optimal controls and mine value which are dependent on the state variables – stage of operations, the resource stock, resource price, and level of the waste stock. To depict the results graphically, we select representative values for at least two of the four state variables; however the numerical

[33]This study sets the abatement ceiling high enough so that the likelihood it binds is small, because abating at high rates is prohibitively expensive.

Life of project		$T = 15$	years
Risk-free rate*		$r = 0.02$	per year
Max. price in computational domain*		$p_{max} = 30$	\$/lb
Initial reserve		$s_0 = 1173$	million lb
Strip ratio (waste:ore)		$\phi = 2:1$	
Production capacity		$\bar{q} = 78.2$	million lb/year
Abatement ceiling*		$\bar{a} = 2\phi\bar{q}$	million lb/year
Landfill capacity*		$\bar{w} = 2200$	million lb
Extraction cost parameter	$C^q(q) = \gamma q$	$\gamma = 1.35$	\$/lb
Abatement cost parameter*	$C^a(a) = \alpha a^2$	$\alpha = 10^{-3}$	
Firm's cleanup cost parameter**	$C^f(w) = \beta w^2$	$\beta = 3 \times 10^{-5}$	
Hazard function*	$\lambda(p, w) = k_0/p$	$k_0 = 10^{-1}$	
Project stages		$\delta_1, \delta_2, \delta_3, \delta_4$	
Fixed decision time*		τ_d	every year
Construction cost	I	\$385	million
Cost to mothball and reactivate*	$C^{sw}(\delta_2, \delta_3), C^{sw}(\delta_3, \delta_2)$	\$5	million
Fixed costs, operating	C_2^m	\$1.66	million/year
Fixed costs, mothballed*	$C_3^{m_2}$	\$2.66	million/year

Table 2: *Parameter values and functional forms for the prototype open-pit copper mine. All dollar values are based on 2007 US dollars. *Assumed by the authors. **β is calibrated based on representative mine cleanup costs in B.C. (Shen, 2016). Other values are from a 2007 feasibility study conducted by the Copper Mountain Mining Corporation.*

solution is available over the full ranges of the state variables. The results presented in this manner describe the firm's optimal strategy under each policy.

The results of the Monte Carlo analysis are used to depict the mean and median of variables of interest evolving over time under each policy. The Monte Carlo analysis is also used to determine the expected clean up cost burden imposed on the government. The sum of the firm value from the mine and the expected cost to the government is a measure of total social welfare under each policy.

As a reminder, the results for four different cases to be analyzed in the numerical results are summarized in Table 3. It has already been demonstrated in Section 6 that the optimal controls for the solvent firm scenarios with or without a bond are the same, and mine value

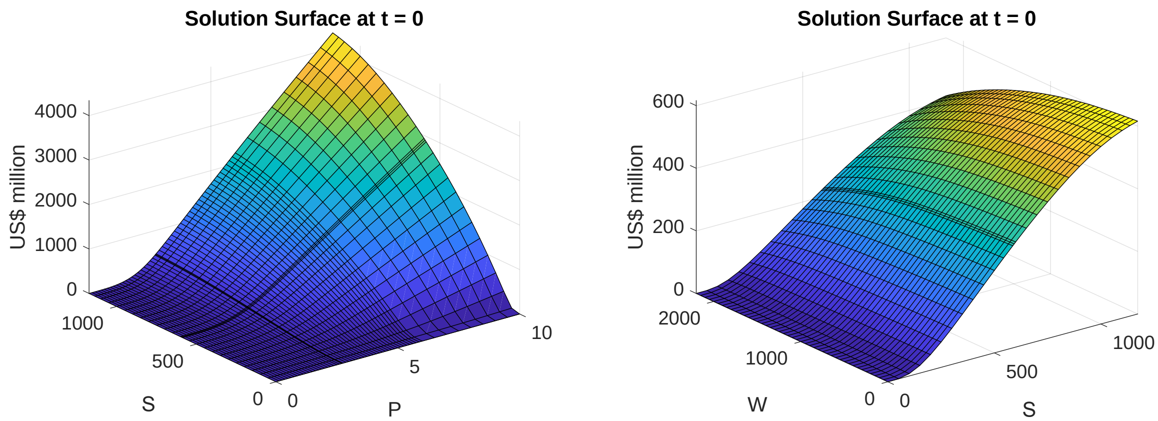
Scenario	**Description**
Solvent firm bond or liability policy	- Either bond required or liability only, No bankruptcy permitted. - Firm must cleanup waste upon mine closure. - Closure can happen anytime up to and including T=15. - Closure must happen by T=15.
Endogenous bankruptcy, liability policy	- No bond. Firm declares bankruptcy if optimal. - If not bankrupt, firm must cleanup upon mine closure. - No cleanup costs if bankrupt.
Exogenous bankruptcy, liability policy	- No bond. Exogenous risk of bankruptcy (Equation (2)). - If not bankrupt, firm must cleanup upon mine closure. - No cleanup costs if bankrupt.
Exogenous bankruptcy, bond policy	- Bond required. Exogenous risk of bankruptcy (Equation (2)). - No cleanup costs if bankrupt.

Table 3: Summary of scenarios analyzed in numerical example.

differs only by the amount of the bond. In our numerical results we will use the term 'solvent firm' to refer to this benchmark case, whether under the bond or liability policies. Note that if interest is not paid on the bond or if there are other costs to the firm in setting up the bond, then the bond and liability policies will no longer be equivalent for the solvent firm (Aghakazemjourabbaf, 2019). Given our assumption that the mine waste causes no environmental damages prior to mine closure, the solvent firm scenario, yields the same optimal control as a social planner.

8.1 Valuation results

We begin by showing how the value of the mining project varies with the price of copper, the starting stock of waste, and the level of copper reserves. Figure 3 depicts the mine value for the solvent firm scenario, in stage δ_1 (prior to construction) at the initial time, $t = t_0$. Since this is prior to construction, the waste level referred to in this figure reflects the starting waste level, w_s that will be created once the mine opens at $t = t_s,\ t_s > t_0$. Figure 3(a) shows the value of the project for different starting prices and different levels of reserve,



(a) Value versus resource stock and price (b) Value versus waste and resource stocks

Figure 3: Project value prior to construction under for the solvent firm at starting time of $t = t_0$, strict liability rule. In the left-hand panel, the level of waste created upon mine opening at $t_s > t_0$ is $w_s = 500$ million pounds, and in the right-hand panel, the price is fixed at \$3/pounds. S : million pounds, P : US\$/pound, W : million pounds.

with $w_s = 500$ million pounds.[34] We observe, as expected, that there is an increasing trend in the value of the project with respect to copper prices and reserve levels. Figure 3(b) represents the value of the project across different resource stock levels and different levels for the starting waste stock, when the price of copper is \$3/pound. At a given level of the initial reserve, generating a larger amount of waste during the construction phase reduces the project value because the upper limit of the waste stock will bind faster during operations, and thus once the construction is completed, the firm will have to exercise more abatement to maintain space in the waste facility.

Figure (4) compares project values at time $t = t_0$ for the different cases of interest at a price of \$3 per pound for copper, full copper reserves and across a range of possible starting waste stocks. In the figure, we observe that the firm's value under the endogenous bankruptcy liability scenario is higher than for the solvent firm. With no bond required, it

[34]The 500 million pounds of waste is chosen for the purpose of illustration only. Changing the initial level of waste changes the project value but the shapes of the general surfaces remain the same.

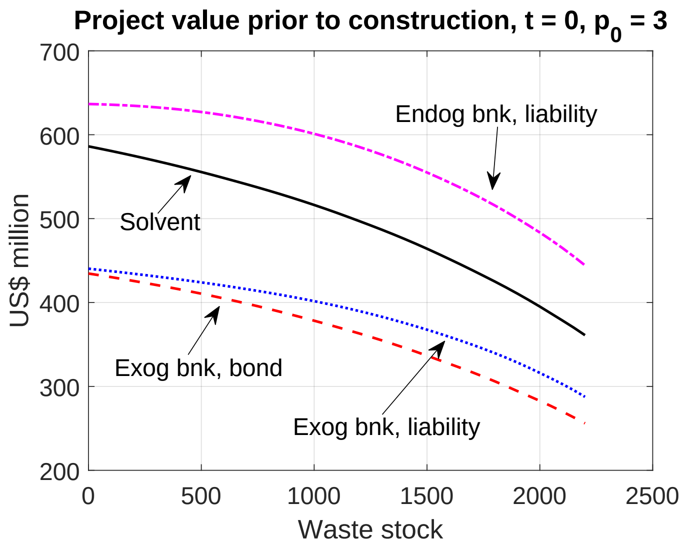


Figure 4: *A comparison of project values prior to construction (stage 1) for* p_0 *=\$3/pound and* $s_0 = 1173$ *million pounds. Waste stock refers to the level of waste created upon mine opening at* $t_s > t_0$ *in million pounds.*

makes sense that the ability to declare bankruptcy and avoid cleanup costs provides value to the firm. But this is costly to society, as will be seen below in the Monte Carlo results. The exogenous bankruptcy cases are both significantly lower in value, a result that depends on the assumed risk of bankruptcy. From Equation (20) and (21), we see that the probability of exogenous bankruptcy acts like an increase in the discount rate. Over all levels of starting waste, exogenous bankruptcy with a bond provides lower value to the firm than under strict liability alone. This follows because without a bond, the firm avoids cleanup costs in the event of bankruptcy, whereas with a bond, the firm has to prepay for cleanup costs.

Using the optimal controls for the firm as determined by the solution of the HJB equation, the Monte Carlo analysis computes the expected costs that accrue to the government if the firm follows its optimal strategy under these four different cases. Starting values chosen for the Monte Carlo analysis are $p = \$3$ per pound for copper, an initial waste quantity of 500 pounds, and copper reserves at the maximum value of 1173 million pounds. Expected government, firm and total values are given in Table 4. The firm's values shown in the table are consistent with individual points (at $w = 500$) in Figure 4. Table 4 demonstrates that although the endogenous bankruptcy case under liability provides greater value to a firm than the solvent case, the former results in a considerable expected cleanup cost for government of \$85 million. In addition, the total value of the mine under the endogenous bankruptcy scenario is lower than the total value under the solvent firm scenario, indicating a net efficiency loss to society. The potential to declare bankruptcy, in the absence of a bond, changes the firm's optimal strategy in terms of waste creation and ore production, which is non-optimal from society's point of view.

Table 4 also indicates that the exogenous bankruptcy cases are of significantly lower value, as already noted. The expected cost to government in the absence of a bond is \$17 million. However, total value is quite close between the two cases indicating a similar optimal strategy between the bond and liability policies in the exogenous bankruptcy scenario. When bankruptcy is not under the control of the firm as in the exogenous bankruptcy case, the firm does not get much extra benefit from not being required to post a bond. The bond protects the government from the cleanup cost, but there is little net gain in efficiency to society. This conclusion hinges on our assumption that the government's cleanup cost will be the same as for the firm. However as discussed in Section 3.1.1, if government cleanup costs exceed those of the firm then there may be a significant efficiency gain from imposing a bond, even for the exogenous bankruptcy scenario.

8.2 Optimal choice of project stages

The numerical solution of Equations (20) - (24) allows us to determine the optimal operating stage at each point in time and for all values of state variables. Our results show significant differences in the choice of optimal operating stage across the different cases. Critical prices at which it is optimal to switch stages at time $t = t_0$ are depicted in Figure 5. It should

Case	Firm Value	Government Value	Total Value
Solvent (liability or bond) (or Endog bnk, bond)	574	0	574
Endogenous bankruptcy, liability	650	-85	565
Exogenous bankruptcy, liability	447	-17	430
Exogenous bankruptcy, bond	432	0	432

Table 4: *Expected values at time zero, millions of U.S. dollars.*

be noted that the choice of operating stage is determined by Equation (23). Critical prices are not part of the solution approach and are inferred after the fact by comparing values at different operating stages. Critical prices may be difficult to determine accurately when values between different stages are close over a range of prices, given the approximation error in the numerical solution.[35] Our numerical results do indicate unique critical prices at time zero, which we present here for useful intuition about the optimal controls. Figure 5(a) shows critical prices to launch the project versus the waste stock created when the mine opens. If the copper price is equal to or greater that the critical price, then it is optimal to start the project immediately. We observe that critical prices rise with the initial waste stock. Intuitively, the larger the initial footprint in terms of waste created, the higher the initial copper price that is required for it to be economic to start the project. Critical prices for the endogenous bankruptcy case are the lowest, followed by the solvent firm, and then the exogenous bankruptcy (liability), and the exogenous bankruptcy (bond). The option to declare bankruptcy embedded in the endogenous bankruptcy case increases project value and results in the project starting sooner. Under exogenous bankruptcy the firm is more cautious about the project launch decision.

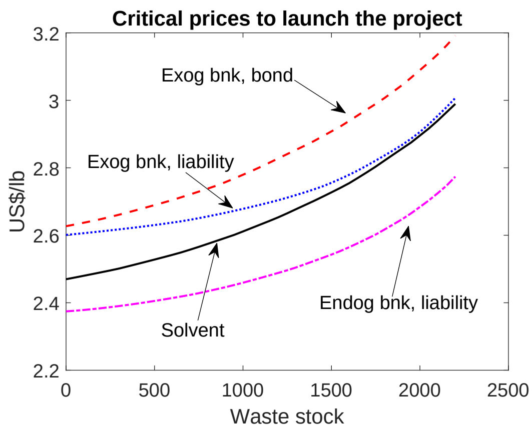
Figure 5(b) shows the critical prices to abandon the mothballed mine at time $t = t_0$. Our

[35] Further, there is no guarantee that critical prices are unique, implying that at low prices it is optimal to switch from stage i to stage j but at high prices it is optimal to switch back. Numerical tests show this is not a common occurrence but was observed for some t in the exogenous bankruptcy case.

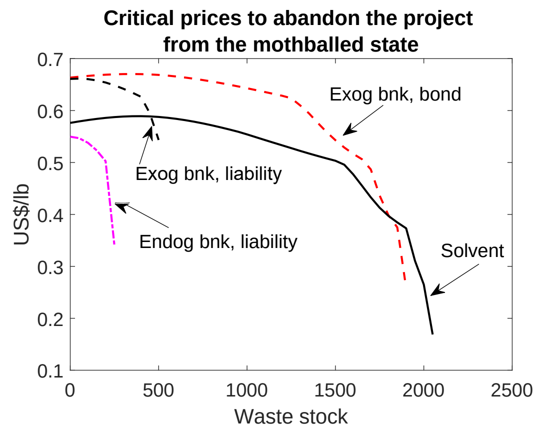
normal assumption is that the firm starts in stage 1 at time zero. But our PDE results also provide the solution for the firm starting in the mothballed state at time zero. If the price of copper drops to a value less than or equal to the critical price, then the firm will choose to shut down permanently. A higher critical abandonment price therefore implies a firm is more likely to abandon the project. For all cases, critical prices fall as w increases, implying that abandonment is less likely for a larger waste stock. This is due to the increased cleanup costs that are incurred at higher waste stock levels. The exogenous bankruptcy cases have the highest critical prices, indicating abandonment is more likely. In addition we observe that for the exogenous bankruptcy liability case, there is a wide range of waste stock where there are no critical prices. Recall the range of waste stock considered is from zero to 2200 million pounds. Where no critical prices are shown, the value of staying mothballed always exceeds the value of abandoning the mine. If exogenous bankruptcy occurs, the firm will no longer be responsible for cleanup costs under the liability policy; hence at high levels of waste it makes sense not to abandon the project at any price. The endogenous bankruptcy (liability) scenario is least likely to abandon the mine, compared to other cases, since it has the option to avoid cleanup costs if it is optimal to do so. At levels of waste above 300 million pounds the firm will never abandon, no matter what the price. At waste levels below 300 million pounds, critical abandonment prices do exist since there is a cost to staying in the mothballed state.

To reduce clutter we do not show graphs of critical prices to temporarily mothball the project. The probability of being in the mothballed state is addressed through the Monte Carlo analysis.

We now consider results of the Monte Carlo analysis showing the evolution of project stage over time. Figure 6(a) shows the probability of remaining in stage 1 - i.e. not launching the mine. This probability is calculated as the percent of Monte Carlo realizations at each time period for which the project remains in stage 1. Recall that the decision to change



(a) Project launch.

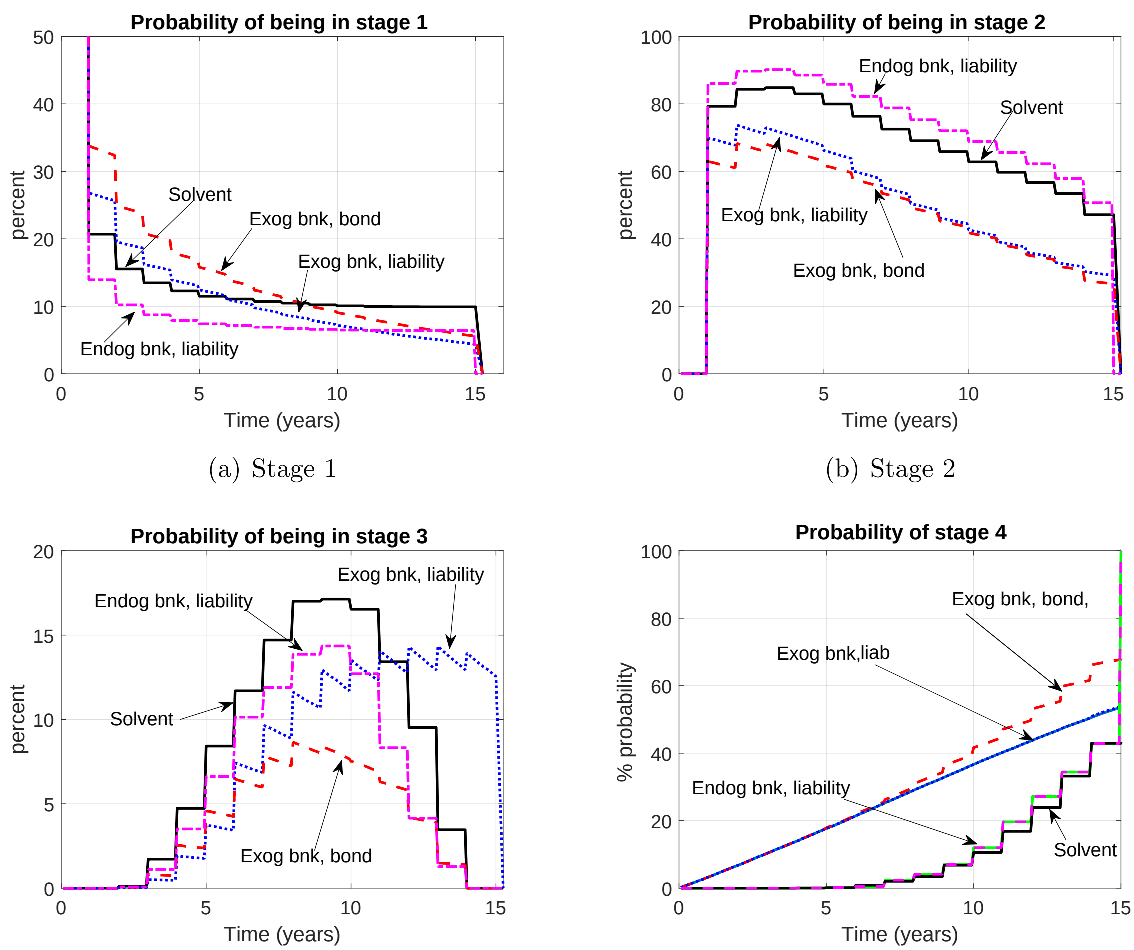


(b) Abandon from the mothballed state

Figure 5: Critical prices to switch stages at time $t = t_0$ versus waste stock (million pounds). Left panel: Waste stock refers to the level of waste created upon mine opening at $t_s > t_0$. Right panel: Waste stock refers to waste level in mothballed state.

stages happens only once a year, meaning that the first time the project can be launched is after one year. At the one year mark, we observe that the probability of remaining in stage one drops significantly. Consistent with Figure 5(a), the endogenous bankruptcy case is the least likely to still be in stage 1 after one year. The probability of remaining in stage 1 declines over time for all cases, but by year 15, there is still a 5 to 10 percent probability that the project would not be launched at all. This reflects the possibility of depressed copper prices over these 15 years.

Figure 6(b) shows the probability of being in stage 2, the phase of active extraction. The exogenous bankruptcy cases are least likely to be this phase, while the endogenous bankruptcy case is most likely. The probability of being temporarily mothballed, shown in Figure 6(c), has an inverted U-shape for all cases. One point of interest is that the exogenous bankruptcy liability case is less likely to be in the mothballed state early in the life of the mine, but it is more likely to be temporarily mothballed in the last few years of the project compared to the exogenous bankruptcy bond case. The logic here is that as the project nears completion, it is advantageous for the firm to go bankrupt as in the absence of a bond, this would avoid cleanup costs. Hence as time T gets close, the firm will refrain from voluntarily closing.



(a) Stage 1

(b) Stage 2

(c) Stage 3

(d) Stage 3

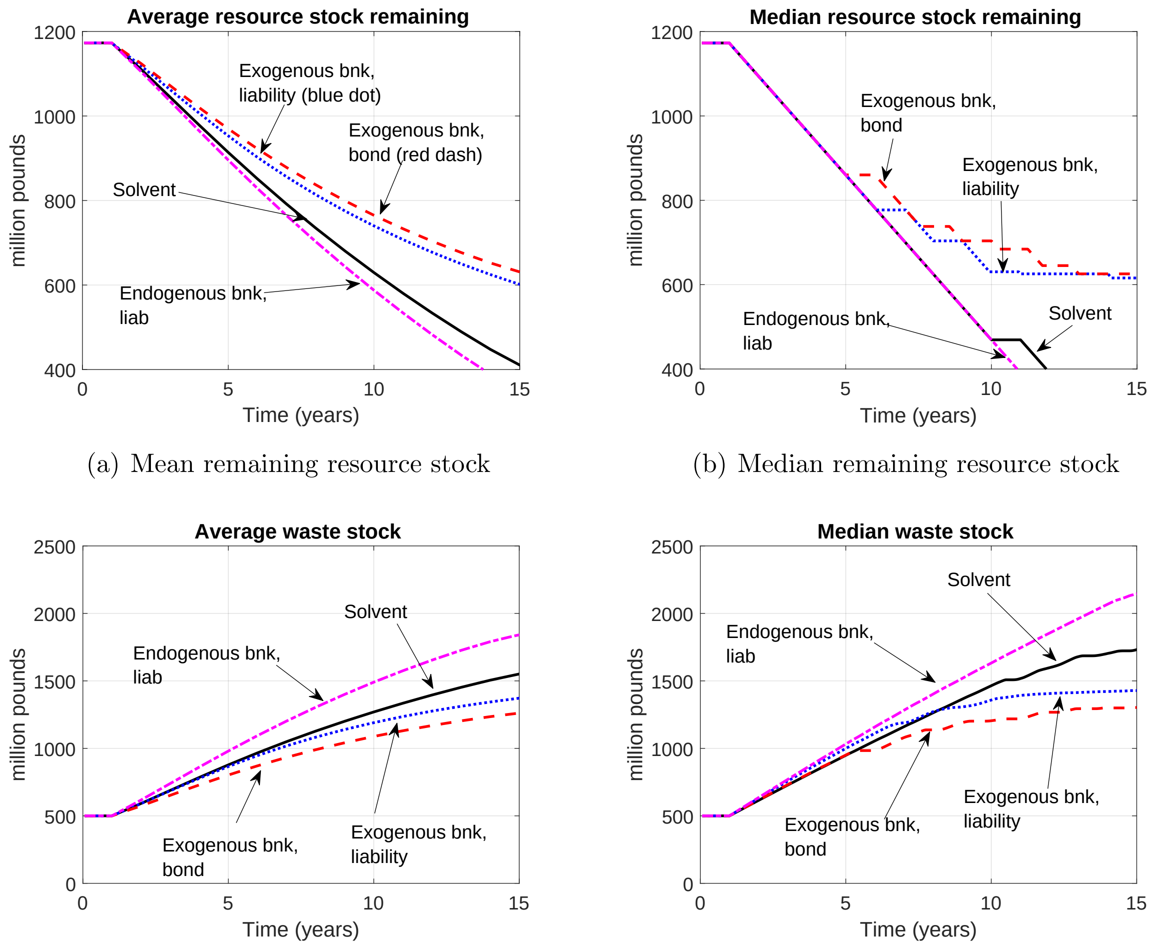
Figure 6: Monte Carlo results: Probabilities of being in the different stages.

Figure 6(d) shows the probability of being in stage 4 for the four cases. We observe that the exogenous bankruptcy cases are the most likely to be in stage 4. It turns out that the probability of being in stage 4 for the exogenous bankruptcy liability scenario coincides with the cumulative probability of exogenous bankruptcy. In contrast, under the bond policy towards the end of the mine life, the firm may choose to close prior to going bankrupt (the red dashed line is above the blue line). With a bond, the firm does not have an incentive to delay cleanup in anticipation of possible bankruptcy since cleanup costs have been prepaid. We also observe in Figure 6(d) that the probability of being in Stage 4 is higher for the endogenous bankruptcy liability scenario (pink/green line) than for the solvent scenario (black line). Under endogenous bankruptcy the firm will always choose bankruptcy rather than abandonment. The probability of being in stage 4 in this case is the same as the probability of endogenous bankruptcy.

8.3 Resource extraction, abatement and waste buildup

The decisions about the optimal operating stage determine the path of resource extraction. Ore extraction occurs only in stage 2, and as noted in Section 4.3, resource extraction is a bang-bang decision, implying that if the firm is in stage 2 it is producing at the maximum rate. Figure 7(a) shows the expected path of resource stock for the four cases. The average resource stock declines most slowly for the exogenous bankruptcy cases reflecting the fact that in these cases, the project is less likely to be in the operating stage (Figure 6(b)). In addition, the firm with the bond depletes its stock more slowly than under the liability scenario. The fastest depletion is shown for the endogenous liability scenario. Again this is consistent with Figure 6(b), showing that the endogenous bankruptcy case is more likely to be in stage 2 over the life of the project. Figure 7(b) depicts the path of the median resource stock, and we observe median paths are generally below the average paths.

The accumulation of waste depends on both the production and abatement decisions. Figures 7(c) and 7(d) show the mean and median paths of the waste stock, respectively. Average and median waste stock are highest for the endogenous bankruptcy case and lowest for the exogenous bankruptcy cases. Under exogenous bankruptcy, the bond gives a lower path for the waste stock than under liability alone.



(a) Mean remaining resource stock

(b) Median remaining resource stock

(c) Mean waste stock

(d) Median waste stock

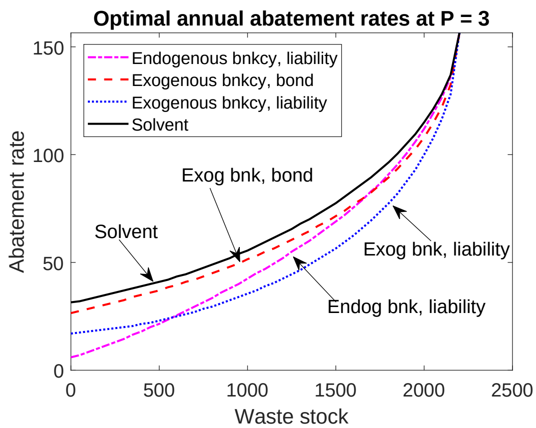
Figure 7: Monte Carlo results for resource and waste stocks. Starting price = \$3/lb, Initial waste stock = 500 million pounds, Upper waste stock limit = 2200 million pounds, Starting resource stock of 1173 million lb

Figure 8(a) compares the optimal abatement rate, as revealed by the solution of the HJB equation, in the production phase (Stage 2) versus the initial waste stock at time $t = t_0$ for all cases. Recall the optimal abatement condition expressed in Equation (29), whereby the marginal cost of increasing abatement by one unit is set equal to the marginal cost of waste buildup. The marginal cost of waste buildup to the firm includes the marginal effect of a larger waste stock, $\frac{\partial V}{\partial w}$, as well as the payment to a bond for an additional unit of waste, $\theta(W)$, when relevant. The marginal cost of waste buildup, $dV/dw - \theta(w)$, is depicted in Figure 8(b). The marginal cost of waste buildup is largest for the solvent firm, and hence the optimal abatement rate shown in 8(a) is highest over all waste stock levels. In the other cases, the possibility of bankruptcy reduces the cost of waste buildup. For the exogenous bankruptcy cases, the value of having spare capacity in the waste facility is reduced as the firm may not be around to take advantage of it. For the endogenous bankruptcy liability case, waste buildup is less costly for the firm compared to the solvent firm scenario, since it has the option to declare bankruptcy and avoid cleanup costs.

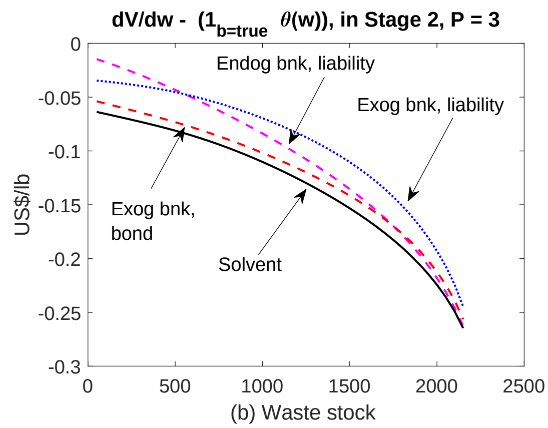
For a view of the abatement over time, we compared mean and median abatement paths for the four cases in Figures 8(c) and 8(d), respectively. Consistent the above discussion, the solvent firm undertakes significantly more abatement than in the other scenarios. It is interesting to note that mean and median abatement in the endogenous liability scenario rise over time. For this scenario, the only reason to abate is to maintain capacity in the waste facility. Consistent with this observation, in Figure 7(d), we see that the median waste stock approaches its upper limit at T for the endogenous bankruptcy liability case.

8.4 Numerical results summary

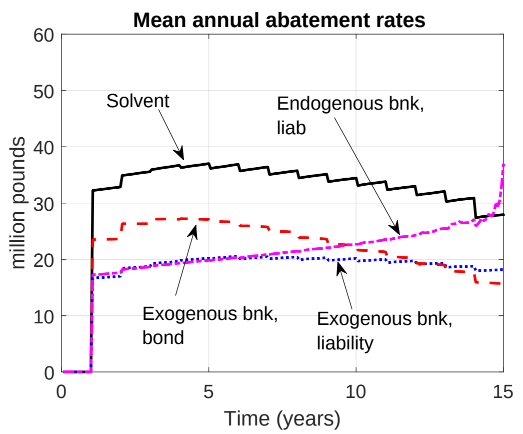
Endogenous bankruptcy (liability) scenario. The option to declare bankruptcy increases the value of the mine to the firm relative to the benchmark. This option changes the firm's optimal operating strategy. Relative to the benchmark, the firm is more likely to launch the mine and will abandon it sooner. It will deplete the resource stock more quickly, undertake less abatement, resulting in a larger buildup of waste. The option to declare bankruptcy imposes a significant expected cleanup cost on government. There is also an efficiency loss to society overall, which is caused by the incentive provided to the firm under



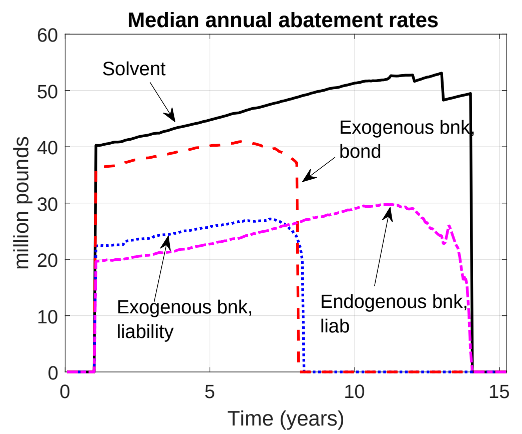
(a) Optimal control for abatement



(b) Marginal cost of waste buildup



(c) Mean abatement in Monte Carlo runs



(d) Median abatement in Monte Carlo runs

Figure 8: Optimal control and Monte Carlo results for abatement. Upper panels at time $t = t_0$. Marginal cost of waste buildup is defined as $[dV/dw - \mathbf{1}_{b=true}\theta(w)]$ (see Equation (28)) Starting price = \$3/lb, Initial waste stock = 500 million pounds, Upper waste stock limit = 2200 million pounds, Starting resource stock of 1173 million lb. Solvent refers to either bond or liability case.

the bankruptcy option to generate more waste.

Exogenous bankruptcy (liability and bond) scenarios. The possibility of exogenous bankruptcy reduces the firm's expected mine value relative to the benchmark, with the reduction greater for the firm subject to a bond. Under exogenous bankruptcy risk, the firm is less likely to open the mine, while resource depletion and waste buildup proceed at a slower pace. Under exogenous bankruptcy with no bond, the firm depletes the resource more rapidly, undertakes less abatement, and hence builds up a larger waste stock relative to the firm with a bond. With no bond in place, the government faces a cleanup liability, but the overall efficiency loss is small relative to the endogenous bankruptcy scenario.

9 Conclusions

This paper is motivated by the observation that many resource extraction projects leave behind a toxic legacy and taxpayers are left to fund the cleanup. If designed appropriately, an environmental bond is one mechanism to ensure that adequate funds are set aside by private firms to undertake site cleanup. In practice, funds set aside as environmental bonds are often less than needed to cover actual cleanup costs, resulting in a significant buildup over time of environmental liabilities. While minimizing bond requirements may be popular among mining and other resource extraction firms, taxpayers may be left "holding the bag".

This paper undertakes an in depth study of the effect of an environmental bond on mine value, the optimal operating strategy of a firm, and the expected cleanup liability for government under different assumptions about a firm's bankruptcy risk. Since our focus is on cleanup costs, we assume the waste stock generates no environmental damage prior to mine closure, and by regulation must be cleaned up once it is permanently abandoned. The study formulates a stochastic optimal control problem for the firm with copper prices modelled as an Ito process. Parameter estimates are under the $\mathcal{Q}$-measure, and it is assumed that government pays interest on the bond at the risk free rate. There are no costs for setting up the bond.

In reality, reasons for declaring bankruptcy are complex, and dependent on firm structure and relevant legislation. To capture a flavour of the different causes of bankruptcy, we defined two different bankruptcy types: exogenous bankruptcy, modeled as a Poisson process, and endogenous bankruptcy whereby the firm can declare bankruptcy if it is optimal to do so. With two policies, bond and no-bond (i.e. liability only), and three scenarios regarding bankruptcy (no bankruptcy, endogenous bankruptcy, exogenous bankruptcy), there are potentially six cases for consideration. The paper presents a theoretical results proving that the value of the mine at time zero is the same for the solvent firm (liability), solvent firm (bond), and endogenous bankruptcy (bond) cases. At times greater than zero, the mine value under the solvent firm (liability) and the other two cases differs only by the value of the bond. Further, the optimal controls in these cases are all identical.

Given these theoretical results we are left with four cases to consider: solvent firm liability (no bankruptcy allowed, no bond), endogenous bankruptcy liability (no bond), exogenous bankruptcy bond, exogenous bankruptcy liability (no bond). Results for our numerical example are summarized in Section 8.4. Most notable of the results is the efficiency loss imposed on society when no bond is imposed, and the firm faces bankruptcy risk. The efficiency loss was most significant in endogenous bankruptcy scenario due to the impact of the firm's behaviour. In the no-bond case our results showed that the mine will open sooner, less abatement will be undertaken and more waste will be created relative to the benchmark solvent firm case. The incentive for the firm to generate more waste creates a significant expected cleanup cost for government and an efficiency loss for society as a whole. In the exogenous bankruptcy case, if no bond is imposed, the government faces an expected cleanup liability. However the efficiency loss is much less than in the endogenous bankruptcy cases since in the former case the advantage to the firm of allowing the waste stock to build up is much lower. The efficiency losses described above were based on the assumption that the government can clean up the mine site at the same cost as the firm. However, as noted in the Introduction, there is evidence that cleanup costs by government tend to exceed those of the firm. This would add an extra efficiency loss in the event of bankruptcy.

Our results are predicated on the assumption that the government pays an appropriate interest rate on the bond[36] and there are no extra charges for setting up the bond. In practice the governments may not pay interest on the bond, or there may be extra expenses to set up the bond including a risk premium if the firm finances the bond through borrowing. Additional costs to maintain and finance the bond will increase the cost of the bonding policy to the firm. With an extra premium on the bond, the bond and no-bond policies will no longer be equivalent for the solvent firm. Extra costs for the bond will provide an incentive for the firm to abate more waste and cleanup the site sooner in order to reduce the cost of maintaining the bond. Aghakazemjourabbaf (2019) explores the impact these additional costs on firm behaviour.

The bonding policy we have analyzed is stringent in that the full cost of cleanup must be deposited, and hence the government is fully protected from bankruptcy risk. This is more demanding than many bond policies in practice, which are not regularly updated as the waste stock changes. Imposing a stringent bonding policy such as the one in our paper will reduce the number of resource extraction projects that are developed. This is a common dilemma for governments - the trade off between promoting local economic activity versus efficient environmental policies. However, given the number of orphan waste sites in North American and elsewhere, a more stringent bonding policy seems long overdue.

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[36]Since our analysis is undertaken in the $\mathcal{Q}$-measure, the risk-free rate is the appropriate rate in our model.

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A Boundary Conditions

Boundary conditions at upper and lower bounds of p, r, w, and t are described in this section.

- Evaluation of Equations (20) and (21) as the commodity price $\boldsymbol{p} \rightarrow \boldsymbol{0}$ implies the following for the solvent or endogenous bankruptcy scenarios.

$$\frac{\partial V}{\partial t} + \big(r + \lambda(p)\big)V + \max_{q,a}\left\{\pi - q\,\frac{\partial V}{\partial s} + (\phi q - a)\,\frac{\partial V}{\partial w}\right\} = 0. \tag{49}$$

  No special boundary condition is needed as there is no term involving p. For the exogenous bankruptcy case, the firm will be bankrupt: $\lambda(p \rightarrow 0) \rightarrow \infty$, implying $V = 0$.

- As $\boldsymbol{p} \rightarrow \boldsymbol{p_{max}}$, we assume $\frac{\partial^2 V}{\partial p^2} \rightarrow 0$, which from Equation (20) implies:

$$0 = \frac{\partial V}{\partial t} + \kappa(\hat{\mu} - \ln p)p\,\frac{\partial V}{\partial p} + rV + \max_{q,a}\left\{\pi - q\,\frac{\partial V}{\partial s} + (\phi q - a)\,\frac{\partial V}{\partial w}\right\} \tag{50}$$

  The assumption that V is linear in p is common in the literature (In't Hout, 2017). The value chosen for p_{max} is intended to approximate an infinite upper limit. Note that for the exogenous bankruptcy scenario $\lambda(p \rightarrow p_{max}) \rightarrow 0$, so that there is no possibility of bankruptcy.

- As $\boldsymbol{s} \rightarrow \boldsymbol{0}$, the admissible set of q collapses to zero as shown in Equation (6). No boundary condition is needed.

- As $\boldsymbol{s \to s_{max}}$, no special boundary conditions is required as Equation (20) has outgoing characteristics in the s direction.

- For the boundary $\boldsymbol{w = 0}$, no boundary condition is required as Equation (20) has outgoing characteristics in the w direction.

- At the boundary $\boldsymbol{w = \bar{w}}$, Equation (7) implies that Equation (20) has outgoing or zero characteristics in the w direction. Hence no special boundary condition is needed.

- At $\boldsymbol{t = T}$, and assuming $T < T^*$ (the firm has not gone bankrupt), the obligation to clean up the site from Stages 2 and 3, under the liability rule and the bond, implies that

$$\begin{aligned} &V(p, s, w, \delta_i, T) = 0, && i = 1, 4,\ \ T < T^* \\ &V(p, s, w, \delta_i, T) = \mathbf{1}_{b=true} C^f(W) - C^f(W,) && i = 2, 3,\ \ T < T^*. \end{aligned} \tag{51}$$

B Numerical solution of the optimal control problem and Monte Carlo analysis

This section briefly describes the numerical approach to solving the stochastic optimal control problem given by Equations (20)–(24). The computational domain of Equation (20) is $(p, s, w, \bar{\delta}, t) \in \Gamma$ where $\Gamma \equiv [0, p_{max}] \times [0, s_0] \times [0, \bar{w}] \times Z_\delta \times [0, T]$. p_{max} is chosen large enough to approximate an infinite domain. $\mathcal{L}V$ in Equation (20) can be discretized using a standard finite difference approach while the other terms in the equation are discretized using a semi-Lagrangian scheme. These approaches are described in Chen & Forsyth (2007) and references therein. The numerical computations increase in accuracy as the solution grid is refined, but at the cost of a significant increase in computation time. We chose a level of grid refinement such that the maximum error from the approximation of the PDE is 2%.

Recall that the optimal control for q which we denote by q^* is bang-bang so that $q^* \in \{0, \bar{q}\}$. To determine the optimal control we search over the set $(q, a) \in \{0, \bar{q}\} \times Z_a$. We discretize the controls $a \in Z_a$ and determine the optimal control by exhaustive search at each point in the state space (p, s, w, t). In the event of a tie between one or more values of a or q, the lowest a or q is selected.

The numerical solution of Equations (20)–(24) specifies the firm's strategy in terms of abatement, production and operating stage depending on the state variables, including copper price, waste stock, copper reserves and time. We compute and store the optimal controls as a function of state variables. We then integrate the stochastic differential equation for copper prices (Equation (1)) using a Monte Carlo analysis for a given starting price, as well as chosen starting values for other state variables. At each time step we look up the stored optimal controls associated with the new price level and current state variables, and then update all other state variables. This proceeds until the lease end date at year 15. We do this for a large number of possible price paths (64000), and then calculate the expected value, median and other percentiles of key variables of interest as they evolve over time. The expected values calculated from the Monte Carlo analysis should agree with the values from the solution of the HJB equation, to an acceptable level of accuracy. In effect, the Monte Carlo analysis provides a check on the HJB equation solution. Our Monte Carlo results are within 1% of the PDE results, indicating a good level of accuracy.