Bad jobs*

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Abstract

We propose a definition of bad jobs and a competitive search model that addresses why workers seek such jobs, why employers create them and why market forces allow bad jobs to persist. The model features competitive search equilibria in which unemployed workers search for jobs that are unambiguously bad in a well defined sense. Concretely, these are jobs with suboptimal career prospects and jobs characterized by employers' underinvestment in labor. Our theory builds on the insight that when current employers can counter outside offers, potential employers who do not observe workers' productivity in their current jobs use wages as a signal of workers' willingness to switch jobs. In turn, this implies that the wage contracts that employers post in the market for unemployed workers not only direct job search but also signal career prospects. Bad jobs are a symptom of coordination failure stemming from a conflict between the signaling and allocative roles of wage contracts. Our analysis brings out potential difficulties inherent to the economics of bad jobs.

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1 Introduction

Most people seem to believe that there are bad jobs. Yet, job quality is an elusive concept. Lay people typically consider a job bad if it involves one or more of the following: low wages, temporary contracts, involuntary part time work, unpredictable or inflexible work schedules, poor working conditions, or little opportunity for career advancement. To an economist, however, the fact that a job has some undesirable characteristic does not necessarily make it bad. For example, a job may compensate for poor working conditions, costly effort, or the high cost of acquiring required skills by paying high wages (Rosen, 1986). To complicate matters further, a worker's assessment of the quality of a job cannot be understood without reference to both the worker's set of feasible jobs and the job's opportunities for advancement, which depend on the allocation of workers to jobs in the labor market more generally.

Surprisingly, the problem of bad jobs remains poorly understood. Empirical studies in economics and sociology tend to equate bad jobs with low wages, or some other undesirable job characteristic (e.g., Farber, 1997, Kalleberg et al., 2000), even though these are clearly not the same things. From a theoretical standpoint, typical random matching models and efficiency wage models of dual labor markets require that if homogeneous workers are employed in heterogeneous jobs, their preferred jobs must have been rationed according to luck (Acemoglu, 2001, Bulow and Summers, 1986). These are models of bad matches, rather than bad jobs, in the sense that the undesirable *ex post* outcomes are imposed on some workers at random, ignoring the workers' incentives to direct job search *ex ante*. Standard efficient sorting arguments, by contrast, allow workers to direct search across heterogeneous jobs, but assume that the market allocation of workers and jobs is socially optimal, subject to the relevant frictions. More generally, textbook economic arguments require that if a worker prefers another job to the one she has, it must be that she could not choose her preferred job for one reason or another. This may well be the case, but then should we characterize the best job in a worker's feasible set as a bad job for that worker even when there is no viable scenario in which that worker can possibly do better?

In our view, a theory of bad jobs must simultaneously explain why workers seek bad jobs, why employers create them instead of creating jobs that suit the preferences of the workforce and why market forces allow bad jobs to persist. It must also spell out why bad jobs are bad. In this paper, we provide a theory of bad jobs that addresses these questions.

Our analysis builds on previous work on competitive search equilibria with private information (Guerrieri et al., 2010) to address the interaction between job creation and job mobility under incomplete information about workers' outside options.[1] Our specification of search on the job combines elements of directed search that are standard in competitive search models (Menzio and Shi, 2011) and elements of bargaining that are standard in random matching models (Postel-Vinay and Robin, 2002). We allow employers to counter outside offers, which is both the most natural

[1] Other interesting analyses of competitive search equilibria with private information include Inderst and Müller (2002), Faig and Jerez (2005), Guerrieri (2008), Moen and Rosen (2011), Delacroix and Shi (2013), Guerrieri and Shimer (2014), Stacey (2016) and Chang (2018).

assumption and it renders our framework remarkably tractable by limiting the scope for job quits, thereby eliminating the sorts of wage ladders found in Delacroix and Shi (2006). Our model is sufficiently tractable that we are able to work with the decentralized equilibrium directly, rather than by way of a planning problem. This enables us to characterize pooling equilibria, which are neither block-recursive nor constrained efficient.[2]

One methodological contribution of this paper is to provide a definition of unambiguously bad jobs, as distinct from goods jobs that have some undesirable characteristics. Put simply, we say that *a job is bad for a worker if there is an equilibrium allocation in which a comparable worker can do better*. For a job to be bad for a given worker, our definition requires that there be a viable incentive structure where a comparable worker, that is, a worker with the same labor market history, can actually do better. We define viable to mean compatible with an equilibrium incentive structure. This is not the only possible definition, but it is the most natural to an economist. To the best of our knowledge, our approach to job quality is new to the literature, not only in economics, but also sociology and psychology.[3] The novelty of our approach lies both in our focus on workers' welfare, as opposed to job characteristics like current wages, respect, or job satisfaction, and in the requirement that a worker's meaningful evaluation of job quality must be relative to the set of jobs that are realistically viable for that worker. We believe this is a promising avenue for the study of the economics of bad jobs.

Perhaps the most crucial question is how bad jobs survive in the face of market competition. The main contribution of our model is to provide an answer to this question. Our proposed theory formalizes the idea that a conflict between the signaling and allocative roles of wage contracts prevents the market from eliminating bad jobs. We characterize two overlapping categories of bad jobs: jobs in which workers' career prospects are suboptimal and jobs in which employers underinvest in labor.

Suboptimal career prospects for jobs that are part of an equilibrium allocation are inherently linked to *non-revealing equilibria* in our model. To understand the nature of non-revealing equilibria, suppose that both workers and employers are *ex ante* homogeneous, but that matches are subject to *ex post* match-specific risk. Further suppose that employment contracts specify starting wages conditional on match productivity, but employers can counter credible outside offers. Finally, suppose that match productivity is observed by both the worker and the employer involved in the match, but not by other employers. We say an equilibrium is non-revealing if it exhibits *pooling contracts*. A pooling contract is one that specifies the same wage independently of match productivity. Potential future employers do not observe workers' actual productivity in their current matches and, under pooling contracts, cannot infer match productivity from observing wages. Such information is relevant because it determines workers' outside options. Workers who are poorly matched *ex post* search on the job with the intention of switching jobs, while workers who are well matched seek outside offers solely in order to elicit a retention offer from their current employer. Here, the

[2]Shi (2009) analyzes block-recursive competitive search equilibria with search on the job.
[3]See Kalleberg (2011) for a review of the literature.

screening problem takes an extreme form. With pooling contracts, both well matched and poorly matched workers have identical incentives to direct their search and it is simply impossible for potential employers to separate them. Accordingly, markets with pooling contracts offer suboptimal career prospects because adverse selection depresses the returns to on-the-job search.

The central question, then, is why pooling contracts are posted in the first place. To address this question note that, whereas adverse selection in the market for employed workers takes the form of a screening problem, it also gives rise to *a related signaling problem in the market for unemployed workers.* Specifically, the main feature of our analysis is that screening problems in the market for employed workers imply that potential employers in those markets use workers' current wages as a signal of the workers' outside options and, hence, their willingness to switch jobs *ex post*. Anticipating the future screening problems, the wage contracts employers post in the market for unemployed workers not only direct job search, but they also signal career prospects. In turn, the signaling and allocative roles of wage contracts may be in conflict, because the informational content of the signal is determined in equilibrium.

Importantly, what matters is the career prospects of a job posted in equilibrium relative to the career prospects of alternative jobs that could have been posted but were not. The problem is that when agents believe that jobs that are not actually posted have sufficiently poor career prospects (e.g., they are "dead-end" jobs), then suboptimal jobs can be part of an equilibrium allocation and attract workers, as they correctly understand that those are in fact the best jobs that are available to them. Ironically, it is the threat of poorer career prospects for jobs that are not created in equilibrium that can support the creation of jobs with unambiguously poor career prospects. This is related to but different from the textbook market signaling problem, where individuals can signal their exogenously given type (Spence, 1973).[4] Here, employers' decisions to create jobs and workers' job search decisions take place *before* they actually posses any private information; match productivity is revealed to both parties *after* matching takes place and such information is contractible.

We formalize the above market signaling problem and show that it underlies a coordination problem involving job creation and job mobility that arises from the interaction of two externalities that competitive search fails to internalize. The informational externality works as follows: employers in the market for unemployed workers do not take into account the value of the informational content of their wages to future employers. This interacts with a second externality: future employers do not internalize their effect on the outside option of workers hired in previous periods. Guerrieri (2008) was the first to show that competitive search fails to internalize the latter externality. Our contribution here is to show that the combination of these two externalities gives rise to multiple stationary competitive search equilibria. Different beliefs can be self-enforcing because future potential employers, who take the distribution of wages as given, assign a zero probability to wage contracts that have not been posted in the past. This is problematic because unemployed workers

[4]Delacroix and Shi (2013) show that competitive search equilibria can be inefficient when sellers post prices that not only direct buyers' search, but also signal product quality.

will only search for jobs they believe have sufficiently good career prospects. If they search for jobs offering separating wage contracts, then their wages will indeed reveal productivity in their current matches, in which case career prospects are good. However, if unemployed workers believe that jobs offering separating contracts are "dead-end", they will seek pooling contracts instead, because those are indeed the best jobs there are, and employers will offer them, because the demand for separating contracts is in fact insufficient. This is self-enforcing because future employers will then assign zero probability to separating contracts having been posted in the past.[5]

While it is evident that career prospects must be a key dimension of job quality, contemporaneous job characteristics, such as current wages and working conditions, are also important aspects of job quality. To formalize a second important dimension, we suppose that employers can choose to invest in the organization of the workplace in ways that make the likelihood that a worker is well matched in the job higher. It will become clear that our main results continue to hold if one assumes that investments in labor include amenities that are valued by workers, even if they do not influence labor productivity. For each type of employer, wage contracts are valuable signals of the likelihood of a worker's future job mobility, just as before. This gives rise to market failure when jobs created by high quality employers are believed to have sufficiently poor career prospects, in which case unemployed workers search for jobs where the firm has underinvested because those are in fact the best jobs they can get, whereas employers choose not to invest in labor because there is insufficient demand for such jobs. Note that the key issue here is not pooling, but coordination failure. Pooling contracts that are part of a non-revealing equilibrium allocation are essential to generate jobs that have suboptimal career prospects. Bad jobs associated with underinvestment in labor, however, can arise even if all contracts are separating.

We argue that bad jobs are not a theoretical curiosity, but rather a fundamental socio-economic problem that has been under-researched in economics.[6] In Section 5 of the paper, we show how our framework provides a number of insights about the creation and allocation of bad jobs. We begin by providing a formal definition of bad jobs and discussing some of our theory's immediate implications: From our perspective, bad jobs are a symptom of coordination failure. Consequently, market forces alone cannot eliminate bad jobs, which involve both private and social costs. Furthermore, job creation cannot be understood independently of job mobility, because career prospects are a key dimension of job quality. Importantly, although our definition of bad jobs is conceptually straightforward, we argue that identifying job quality empirically is challenging. We also argue that appropriately targeted taxes and subsidies can raise social welfare, but the successful implementation of such policies relies on the ability to identify job quality empirically. Finally, we illustrate a link between labor market discrimination and the creation of bad jobs.

In Section 2 we present the model. In Section 3 we consider a simplified version of the model, in order to facilitate a more intuitive presentation of some of the key mechanisms of the model. In Section 4 we allow for endogenous investments in labor. In Section 5 we discuss the main

[5]Burdett et al. (2004) offer an interesting analysis of long-term bilateral relationships when both parties can search while they are matched, which can create a different coordination problem.

[6]See Green (2015) for a related discussion.

implications of our analysis. Section 6 concludes. Technical proofs are in the Appendix.

2 The model

2.1 Environment

Consider a labor market with *ex ante* homogeneous workers and *ex ante* homogeneous employers trading *ex ante* homogeneous labor services. Time is discrete. All agents are risk neutral and discount the future at a rate $r > 0$. There is a unit measure of workers who are either employed or unemployed. An unemployed worker searches for a job and receives a flow benefit from unemployment equal to $b \geq 0$. An employed worker produces. Subsequently, a separation shock makes an employed worker become unemployed with probability $\delta > 0$. Otherwise, the worker can search for a different job while employed.

The measure of jobs is determined endogenously by free entry. The quality of a worker-employer match is also endogenous. Specifically, we assume that employers can make an investment to improve the likely fit of an average worker, which improves the expected productivity of the match. We assume that this investment in labor is costless in order to highlight the employer's choice of workplace organization, which is commonly associated with job quality. For simplicity, we model this as the choice of a production technology and we consider the case of two possible technologies, indexed by $j = 1, 2$. The type-1 technology is unambiguously superior, in the sense that workers are more likely to be productive in jobs using the type-1 technology. Formally, a worker-employer match produces y_h units of output with probability α_j and y_l units of output with probability $1 - \alpha_j$, for $j = 1, 2$, where $b < y_l < y_h$ and $0 < \alpha_2 < \alpha_1 < 1$. The symmetry in the realizations of labor productivity across technologies simplifies the analysis by helping to limit potential job-to-job transitions. It will be convenient to identify employers with technologies, where type-2 employers are those who underinvest in labor. We assume that employer types are observable.

Employers incur an entry cost $k > 0$ in order to post a vacancy. Each period there is a continuum of markets. Each market is associated with a single type of job x, which is specified below as a wage contract together with a production technology. Each employer can post any feasible job and each worker can direct her search to any market. Let $Q : X \rightarrow \mathbb{R}_+$, where X is the set of feasible jobs and $Q(x)$ denotes the queue length associated with a job x, which is defined as the ratio of workers searching for x to employers posting x. Matching is bilateral, so each employer meets at most one worker and vice versa. Workers who search in a market where $Q(x) = q$ meet an employer with probability $f(q)$ and employers in the same market meet a worker with probability $qf(q)$. We assume that $f(q)$ is twice differentiable, strictly decreasing and convex, with $f(0) = 1$ and $f(\infty) = 0$. We also assume that $qf(q)$ is strictly increasing and concave, approaching 1 as q converges to ∞. These assumptions ensure that the elasticity of job creation, given by $\eta(q) = -qf'(q)/f(q)$, is such that $0 = \eta(0) < \eta(1) \leq 1$, with $\eta'(q) > 0$. For simplicity, we also assume that $\eta(q)$ is concave, with $\eta(\infty) = 1$.[7]

[7] An example of a matching technology that satisfies these assumptions is $M(u, v) = uv/(u + v)$.

When a worker and an employer meet, both the worker's labor market status and her wage, if currently employed, are observed by the potential employer. Then, the productivity of the potential match is drawn randomly and observed by both parties. However, if the worker is already employed, the actual productivity of her current match is not observed by the potential employer. Subsequently, employers decide whether or not to make formal offers. We assume that employers make take-it-or-leave-it offers, they can counter outside offers and wages can only be renegotiated by mutual agreement. Workers then decide whether to accept any offers. New matches start producing in the next period.

Since workers are unable to commit not to search on the job, and employers are unable to commit not to counter outside offers, it will facilitate presentation to specify contracts in terms of fixed entry wages, taking into account that wages can be renegotiated by mutual agreement, rather than including hiring and retention policies as part of the contract. A worker that gets a credible outside offer can choose to terminate the current fixed wage contract and agree to a "new" contract with a different wage, which lasts until a new outside offer arrives. If the outside offer is credible and if a better counteroffer is feasible, the employer then commits to a new fixed wage contract. Retention policies will depend on history only through the worker's current wage, which is a sufficient statistic for the payoff-relevant history of the current contract.

A job $x = \{j, w_l, w_h\}$ specifies a type of production technology $j \in \{1, 2\}$ and a pair of wages, where $w_l \in [0, y_l]$ and $w_h \in [0, y_h]$ denote the entry wages to be offered when the realizations of match productivity in the new match are $y' = y_l$ and $y' = y_h$, respectively.

Our assumptions imply that employed and unemployed workers in effect do not compete for the same jobs. Moreover, neither workers nor employers can be forced to participate in a match before observing match productivity. That is, employers cannot commit to make a formal job offer and workers cannot commit to accept such an offer before observing the realized match productivity. In this sense, matches are pure inspection goods, rather than experience goods. These assumptions are made to highlight the role of incomplete information about workers' outside options. In Section 5, we argue that our main results continue to apply more generally, as long as match productivity in jobs that use the inferior technology is sufficiently easier to observe upon inspection.

We focus on the adverse selection problem that arises from the combination of limited commitment and asymmetric information. Since match productivity is unobserved by third parties, a worker's current labor productivity is private information to the worker *vis-a-vis* potential new employers. Consequently, poaching offers cannot discriminate between workers with different outside options, unless (equilibrium) wages reveal match productivity. Since workers are unable to commit not to search on the job and employers are unable to commit not to counter outside offers, workers in high-productivity matches have an incentive to seek outside offers solely to elicit retention offers from their current employers.

We assume that $(r + \delta) k < \alpha_2 (y_h - y_l)$ to allow for positive job-to-job transitions from and to either type of job. Furthermore, we assume that employers face a small cost of making a credible offer, so they will never make offers that they know will be rejected with certainty. This assumption rules out potential equilibria where workers in poor matches are able to elicit retention offers. For simplicity, we assume these costs are negligible and so we are not explicit about them.

2.2 Competitive search equilibrium

Let $s \equiv \{i, w, y\} \in S$ denote a worker's payoff-relevant state, where a worker can be unemployed ($i = 0$), working for a type-1 employer ($i = 1$) or working for a type-2 employer ($i = 2$); $w \in [0, y_h]$ denotes her current wage and $y \in \{y_l, y_h\}$ denotes current match productivity. Unemployed workers are associated with the state $s_u = \{0, b, b\}$ by convention, and the feasible state space is given by $S = \{s_u\} \cup S_e$, where $S_e = \{1, 2\} \times [0, y_h] \times \{y_l, y_h\}$.

Focus on stationary equilibria. A competitive search equilibrium (Moen, 1997) specifies a mapping Q from feasible jobs to market queues. Workers direct their search across all feasible jobs, taking as given the market queue length $Q(x)$ for all $x \in X$. Workers' decisions must be optimal at any information set, which includes their own state $s \in S$ and the distribution of workers across states, that is, the aggregate state of the economy $\psi : S \to [0, 1]$, where $\psi(s)$ is the proportion of state-s workers in the economy. It will become clear that competitive search equilibria need not be *block recursive*. That is, the agents' value functions and therefore equilibrium strategies may be a function of the aggregate state. However, in order to minimize clutter, we are not explicit about the potential dependence of the agents' value functions on the aggregate state ψ.

Let $V(s)$ denote the value function of a worker evaluated in state s. Let $U(s, x, Q(x))$ denote the expected surplus to a worker with current state s from searching for x, with associated queue length $Q(x)$. The worker meets an employer with probability $f(Q(x))$, in which case a draw y' of match productivity is realized and a wage offer w_o is made. Workers reject any offer w_o from a type-j employer such that $V(s) > V(s_o)$, where $s_o = \{j, w_o, y'\}$. Instead, if $V(s) < V(s_o)$, the decision of a worker with current state $s = \{i, w, y\}$ amounts to choosing whether to accept the offer, in which case her state becomes $s_o = \{j, w_o, y'\}$, or reject the offer, in which case her state remains unchanged, if the worker was unemployed (if $s = s_u$), or it becomes $s_c = \{i, w_c, y\}$ if the worker was employed (if $s \neq s_u$) and she got a wage counteroffer w_c. Employed workers only renegotiate contracts if they have a credible outside option; so $V(s_c) > V(s)$ if and only if $V(s_o) > V(s)$.

Thus, we have

$$V(s) = w + \frac{\delta V(s_u)}{1+r} + (1-\delta)\left(\frac{V(s)}{1+r} + \max_{x \in X \cup \varnothing} U(s, x, Q(x))\right), \tag{1}$$

for all s, where $x = \varnothing$ denotes the choice of not searching, and where we have restricted attention to pure search policies, for simplicity, with

$$U(s, x, Q(x)) = \begin{cases} f(Q(x))\,\mathbb{E}_{y'}\left\{g_h(i, w, s_o)\max\left\{0, \frac{V(s_o)}{1+r} - \frac{V(s)}{1+r}\right\}\right\} & \text{if} \quad s = s_u \\ f(Q(x))\,\mathbb{E}_{y'}\left\{g_h(i, w, s_o)\max\left\{0, \frac{V(s_o)}{1+r} - \frac{V(s)}{1+r}, \frac{V(s_c)}{1+r} - \frac{V(s)}{1+r}\right\}\right\} & \text{if} \quad s \neq s_u, \end{cases}$$

where $\mathbb{E}_{y'}$ denotes the expectation taken with respect to the exogenous variable y'; $s = \{i, w, y\}$,

$s_o = \{j, w_o, y'\}$ and $s_c = \{i, w_c, y\}$, where the worker anticipates both the hiring policy $g_h(i, w, s_o)$ and the wage offer w_o of the potential new employer, taking as given that $w_c = g_r(s, j, w_o)$, where g_r is her current employer's retention policy, and where g_h and g_r are specified below.

A worker's optimal search policy is given by

$$g_x(s) \in \arg\max_{x \in X \cup \varnothing} U(s, x, Q(x)). \tag{2}$$

Furthermore, restricting attention to pure acceptance policies, for simplicity, we let

$$g_a(s, s_o, w_c) \in \begin{cases} \arg\max_{a \in \{0,1\}} \{aV(s_o) + (1-a)V(s)\} & \text{if} \quad s = s_u \\ \\ \arg\max_{a \in \{0,1\}} \{aV(s_o) + (1-a)\max\{V(s), V(s_c)\}\} & \text{if} \quad s \neq s_u \end{cases} \tag{3}$$

for all $s \in S$, $s_o \in S_e$ and $w_c \in [0, y_h]$, where $g_a(s, s_o, w_c) = 1$ if a worker in state s accepts an offer to work for a type-j employer at the wage w' when w_c is her current employer's counteroffer, with $s \equiv \{i, w, y\}$, $s_o \equiv \{j, w_o, y'\}$ and $s_c = \{i, w_c, y\}$ We let $w_c = b$ if $s = s_u$, by convention.

We next define the present value of an ongoing match to the employer. Note that employers in an ongoing match need to anticipate the worker's search policy (g_x) and her acceptance policy (g_a), as well as the hiring policy of the worker's potential new employer (g_h), which are common knowledge in equilibrium. Also note that the retention policy g_r is contingent on the worker's state $s = \{i, w, y\}$, which includes her type y, since this becomes contractible once the match is formed. Retention offers cannot be made contingent on the realized match productivity associated with an outside offer (i.e., the worker's potential future type), since this is unobserved by the incumbent employer. Of course, the observation that an offer was made and the observed wage offer may reveal match productivity in equilibrium.

Thus, the present value of an ongoing match to the employer, denoted by $J_f(s)$, solves

$$\begin{aligned} \frac{J_f(s)}{1+r} &= \frac{y - w}{r + \delta + (1-\delta) f(Q(g_x(s))) \mathbb{E}_{y'}\{g_h(i, w, s_o)\}} \\ &+ \frac{(1-\delta) f(Q(g_x(s)))}{r + \delta + (1-\delta) f(Q(g_x(s))) \mathbb{E}_{y'}\{g_h(i, w, s_o)\}} \\ &\times \ \mathbb{E}_{y'}\left\{g_h(i, w, s_o) \mathbb{E}_{s_o}\left\{\max_{w_c}\left\{(1 - g_a(s, s_o, w_c)) \frac{J_f(s_c)}{1+r}\right\} | j, w_o\right\}\right\} \end{aligned} \tag{4}$$

subject to $w_c \geq w$,

for all $s = \{i, w, y\} \neq s_u$, where $s_o = \{j, w_o, y'\}$ and $s_c = \{i, w_c, y\}$. The denominator on the right hand side reflects the three sources of discounting: the discount rate (r), the exogenous probability of job destruction (δ), and the probability that the worker receives an outside offer from a poaching firm. Let $g_r(s, j, w_o)$ denote a solution to problem (4).

Given $Q(x)$, workers searching for x do not need to account for the composition of workers in that market. By contrast, employers posting x need to anticipate not only the likelihood of meeting a worker, given by $Q(x) f(Q(x))$, but also the composition of the pool of workers searching for that contract. We let $\mu(\cdot|x)$ denote a probability distribution on S, for each $x \in X$. An employer posting x incurs a flow cost k and meets a worker with probability $Q(x) f(Q(x))$, in which case the expected surplus to the employer is given by $\mathbb{E}_s \{J(s,x)|x\}$, where $J(s,x)$ is the expected value of the employer's surplus conditional on meeting a state-s applicant and $\mathbb{E}_s\{\cdot|x\}$ is taken with respect to $\mu(\cdot|x)$. Thus, the value of posting x to an employer is given by

$$-k + Q(x) f(Q(x)) \mathbb{E}_s \{J(s,x)|x\},$$

where $\mathbb{E}_s \{J(s,x)|x\} = \mathbb{E}_s \{\mathbb{E}_s \{J(s,x)|i,w,x\}|x\}$.

In order to form the latter expectation, note that employers cannot commit to participate in the match before observing the realization of match productivity y'. Hence the decision to make an offer to a worker is made conditional on the realized match productivity. Note further that potential employers observe the workers' current labor market status and their wages, if currently employed, but not their labor productivity in their current match. This implies that potential employers must form expectations concerning the worker's current state, as given by the inner conditional expectation. This implies that:

$$\mathbb{E}_s \{J(s,x)|i,w,x\} = \mathbb{E}_{y'} \left\{ \max_{h\in\{0,1\}} \left\{ h\mathbb{E}_s \left\{ g_a(s, s_o, g_r(s,j,w_o)) \frac{J_f(s_o)}{1+r} |i,w,y',x \right\}\right\}\right\}, \tag{5}$$

with $s = \{i,w,y\}$, $s_o = \{j,w_o,y'\}$, where $J_f(s_o)$ satisfies equation (4) and where by convention, we set $g_r(s_u,j,w_o) = b$ for all (j,w_o), with $s_u = \{0,b,b\}$. Let $g_h(i,w,s_o)$ denote a solution to the problem in (5). Equation (5) reflects the fact that poachers anticipate the current acceptance policies of the workers they attract (g_a) and the retention policies of their current employers (g_r). In order to minimize clutter, we do not include these explicitly as arguments in the value function J. Similarly, recall that we have assumed that the cost of making an offer is positive, but negligible.

Definition 1 *A stationary equilibrium $\mathcal{E} = \{X^*, S^*, V, J, g_x, g_a, g_h, g_r, Q, \mu, \psi\}$ consists of a set of posted jobs $X^* \subseteq X$, a set of workers' states $S^* \subseteq S$, value functions $V : S \to R_+$ and $J : S \times X \to R_+$, policy functions $g_x : S \to X \cup \varnothing$, $g_a : S \times S_e \times [0, y_h] \to \{0,1\}$, $g_h : \{0,1,2\} \times [0,y_h] \times S \to \{0,1\}$ and $g_r : S_e \times \{1,2\} \times [0,y_h] \to [0,y_h]$, a function $Q : X \to R_+$, a distribution $\mu : S \times X \to [0,1]$ and a distribution $\psi : S \to [0,1]$, such that:*
(A) Atomistic agents: For all $x \in X$, (A1) all agents take $Q(x)$, $\mu(\cdot|x)$ and ψ as given, and (A2) $\mu(\cdot|x)$ has support on S^*.
(B) Workers' optimal search and acceptance: V satisfies (1); g_x satisfies (2); g_a satisfies (3).
(C) Optimal job posting and retention with free entry: g_h, g_r and J solve (4) and (5). Moreover, for any $x \in X$, $Q(x) f(Q(x)) \int_S J(s,x)\, d\mu(s|x) \le k$, with equality if $x \in X^*$.

(D) Consistent beliefs: For any $x \in X^*$,

$$\mu(s|x) = \frac{\psi(s)\,\mathbb{I}_x(g_x(s))}{\int_S \mathbb{I}_x(g_x(s))\,d\psi(s)}, \text{ with } \int_S \mathbb{I}_x(g_x(s))\,d\psi(s) > 0,$$

for all $s \in S$, *where* $\mathbb{I}_x(g_x(s)) = 1$ *if* $g_x(s) = x$ *and* $\mathbb{I}_x(g_x(s)) = 0$ *if* $g_x(s) \neq x$.
(E) Consistent allocations: For all $s \in S$,

$$\int_{S^*} \Pr(s_{t+1} = \tilde{s}\,|s_t = s)\,d\psi(\tilde{s}) = \int_{S^*} \Pr(s_{t+1} = s\,|s_t = \tilde{s})\,d\psi(\tilde{s}),$$

where $\Pr(s_{t+1}|s_t)$ *is the unique distribution associated with* g_x, g_h, g_a *and* g_r, *with* $g_x(s) \in X^* \cup \varnothing$, *for all* $s \in S^*$ *and* $S^* = \{s \in S : \psi(s) > 0\}$.

We refer to the pair (X^*, ψ), where S^* is the support of ψ, as an *equilibrium allocation*.

Definition 1 requires that markets are complete in the sense that employers can post any job in the set of feasible jobs and workers direct their search across all feasible jobs. Moreover, all agents are atomistic in the sense that they cannot influence aggregate variables. As is standard in the literature, we use the language of job posting (e.g., Guerrieri et al., 2010). However, as is well known, the competitive search equilibrium describes the equilibrium of a market, rather than a game. As this distinction turns out to be crucial in the present context, Part (A) of our equilibrium definition makes it explicit.

There are two important aspects to the assumption of atomistic agents in the present context. First, when workers search for a job $x \in X$, they take as given the probability that they will be rationed, which is $f(Q(x))$. Similarly, when firms post a job $x \in X$, they take as given the probability that they will be rationed. For the firms, this probability has two distinct dimensions, namely, the meeting probability $Q(x) f(Q(x))$ and the distribution of workers $\mu(\cdot|x)$ who are expected to search for the job. Rather than including the rationing probabilities in the description of a contingent commodity, as is normally done in the context of Walrasian markets, we treat them as a description of beliefs that all agents share about the trading process. Furthermore, note that, upon meeting a worker, the employers' assessment of the worker's unobservable type is, with a slight abuse of notation, given by $\mu(y|i, w, x)$, for $y \in \{y_l, y_h\}$, which can be constructed from the equilibrium mapping $\mu(\cdot|x)$.

The second important aspect of atomistic agents in our context arises because the presence of on-the-job search implies not only that workers are heterogeneous in observable as well as unobservable dimensions, but also that the distribution of workers (i.e., the aggregate state) itself is an equilibrium object. Thus, agents take as given not only $Q(x)$ and $\mu(\cdot|x)$, for all $x \in X$, but also ψ, where $\psi(s)$ is the proportion of state-s workers in the economy. This imposes some natural restrictions on beliefs. For $x \in X^*$, beliefs must be consistent in the sense that they satisfy Bayes' rule, as usual. Furthermore, $\mu(\cdot|x)$ must have full support on S^*, for all $x \in X$. In particular, when a firm considers posting a job that is not part the equilibrium allocation, it understands how both the market queue and the pool of searchers associated with $x \notin X^*$ will vary with the job posted, but it takes as given that its posting of a different job does not influence the distribution of jobs in the economy.

Part (B) of Definition 1 ensures that workers' search and acceptance policies are optimal for all states, taking as given the market queue length for all jobs. Part (C) ensures that employers' posting behavior and their subsequent retention policies are optimal, and employers posting equilibrium contracts make zero profits. Part (D) ensures that employers' beliefs are consistent with the workers' equilibrium search policies through Bayes' rule. It ensures that any contract that is posted in equilibrium attracts a positive mass of workers and that the distribution of workers searching for any equilibrium contract is exactly what the employers posting those contracts expect. Free entry of employers then ensures the correct market clearing queue. Part (E) ensures that employers' and workers' equilibrium strategies generate a stationary distribution of workers and jobs and characterizes the sets of equilibrium jobs X^* and worker states S^*. The latter consists of the unemployed plus the support of the equilibrium wage distribution for workers working for each type of employer. This condition requires that the aggregate flows in and out of any state in S^* must be equal to each other at all times. A formal statement of the transition probability $\Pr(s_{t+1} = s' \,|s_t = s\,)$ is straightforward, but cumbersome. In the Appendix, we provide one in the specific context of the equilibria we characterize.

2.3 Equilibrium refinement

Clearly, the above equilibrium definition allows for more or less arbitrary off-equilibrium beliefs (other than the restriction that $\mu(\cdot\,|x)$ and ψ must have common support, for all $x \in X$) and so it allows for many equilibria, each of which is supported by particular beliefs in the markets where no trade takes place. The issue is that some contracts may not be traded because employers fear they would attract only undesirable types of workers. If workers expect the labor market queue associated with those contracts to be sufficiently high then those contracts would in fact not attract any workers and so the employers' pessimistic beliefs are never contradicted. We propose the following refinement of equilibrium, restricting agents' beliefs about contracts that are not traded in equilibrium.

Definition 2 *A refined equilibrium is a stationary equilibrium $\mathcal{E}$ such that, for any $x \notin X^*$ there does not exist any queue $q \in R_+$ and any beliefs $\mu'(\cdot\,|x)$ on S with support on S^* such that $qf(q)\int J(s,x)\,d\mu'(s|x) \geq k$, where for any $s \in S^*$, $\mu'(s|x) > 0$ if and only if $U(s,x,q) > U(s, g_x(s), Q(g_x(s)))$.*

Our equilibrium refinement is in the spirit of the Intuitive Criterion proposed in Cho and Kreps (1987). It eliminates equilibria if there is some off-equilibrium job x and some pair of labor market queues and beliefs $(q, \mu'(\cdot|x))$ that yield some firm offering the deviating job non-negative profits and some worker seeking the deviating job a payoff above her equilibrium payoff as long as the firm does not assign a positive probability to the deviation having been made by any type for whom this action is (weakly) equilibrium dominated.

The refined equilibrium proposed in Definition 2 builds on the concepts proposed in Gale (1992, 1996) and Guerrieri et al. (2010), by requiring that beliefs must be such that, for any $s \in S$ and any $x \notin X^*$, $\mu(s|x) = 0$ if $U(s, x, Q(x)) < U(s, g_x(s), Q(g_x(s)))$, where U is given by (1). This condition amounts to requiring that employers posting an off–equilibrium job must believe that the only workers the job would ever attract must be indifferent between the off-equilibrium job and their preferred equilibrium job. For if they strictly preferred the off-equilibrium job, then condition (B) in the equilibrium definition would be violated. In turn, this requires that firms believe that off-equilibrium jobs will attract only those workers who are willing to endure the highest labor market queue.

Our equilibrium refinement also rules out a continuum of equilibria where employers do not post some jobs because they believe they will not attract workers while workers do not search for those jobs because they believe too many other workers would be searching for them as well. An alternative restriction that eliminates all these equilibria is the following: for any $x \notin X^*$, $Q(x) = 0$ if $\mu(s|x) = 0$ for all $s \in S$.

Moreover, our refinement rules out equilibria where off-equilibrium poaching jobs are not posted because potential employers believe they would only attract unemployed workers, while employed workers do not search on the job because they believe that there are no off-equilibrium poaching jobs. This is important in our setting because beliefs need to be specified over observable worker characteristics as well as unobservable worker types and, because of the possibility of search on the job, workers in different equilibrium states $s \in S^*$ are not indifferent between all equilibrium jobs $x \in X^*$.

The above requirement applies only to workers who participate in the labor market in equilibrium, that is, only if $s \in S^*$. Intuitively, if a state s is not in the support of the distribution ψ, then employers should assign probability zero to the event that such a worker would ever search for any job. Specifically, this implies that potential future employers, who take the distribution of wages as given, assign probability zero to all wage contracts that have not been posted in the past. It will become clear that this feature is crucial to support the pooling equilibrium that we characterize below.

3 Equilibrium jobs with suboptimal career prospects

In this section we examine an economy with only one type of employer. Addressing this simplified version of the model allows us to introduce both the relevant informational externality that affects firm wage setting and the adverse selection problem that underlies the possibility of market failure without the complexity of the full model. Furthermore, by first presenting a one-type-of-employer version of the model, we can highlight an important technical problem that has impeded research in this area to date as well as present our resolution of this problem. Throughout this section we suppose that employers are of type 1.

Our assumptions about the nature of counteroffers impose a lot of structure on the problem. First, we assume that employers cannot commit to not counter outside offers. In the absence of commitment, incumbent firms will match any offer up to the worker's current productivity. Second, we assume that poaching firms never make offers that will be rejected with certainty. Collectively, these assumptions imply that workers who are known to be in high productivity matches cannot profit from on-the-job search. The reason is that a worker in a high productivity match who receives an outside offer will elicit a retention offer from her current employer, who will be willing to pay up to y_h to retain the worker. Consequently, there are no gains from trade between workers in high productivity matches and potential poaching firms. This implies, for example, that workers in high productivity matches cannot search in separate markets, as no poaching firm would enter a market populated solely by such job seekers.

In principle, workers known to be in high productivity matches could pool with workers in poor matches, which would crowd out those workers. However, well-matched workers seek outside options only to elicit a retention offer. Since poaching firms do not make offers that will be rejected with certainty, such workers will not get outside offers. This possibility is ruled out by our equilibrium refinement, since there would be alternative contracts that could be posted where employers would make non-negative profits and poorly matched workers searching for them would be strictly better off, without making well-matched workers worse off. Note that this means that workers in high productivity matches can only profit from on-the-job search if their match productivity is not revealed in equilibrium.

We further assume that poaching firms cannot commit to offers before observing match productivity, which implies that the maximum wage a poaching firm can offer a worker with whom it has a low productivity match is y_l. Note that, since incumbent firms are willing to make retention offers up to the total amount of worker productivity, poaching firms never make offers to workers with whom they would form a low productivity match.

This shapes the set of possible job and wage transitions as follows: all job switches occur when a worker in a low productivity match meets a firm with which she has a high productivity match. A worker in a high productivity match (who can search on the job by pooling) who meets a firm with which she would also form a high productivity match will elicit both a job offer from the poaching firm and a retention offer from the incumbent firm. Therefore, all job switches and wage changes reveal that a worker is now employed in a high productivity match. An implication of this is that the job ladder has at most one rung. A worker who moves reveals that she has moved into a high productivity match and will no longer be the target of poaching firms while a worker who accepts a retention offer reveals that she is currently employed in a high-productivity match and will also no longer be the target of poaching firms.

Our model shares the well known property that the allocation supported by a competitive search equilibrium can be characterized as the solution of a corresponding dynamic programming problem. However, in our model there exist two equilibria: in one wages reveal match productivity, while in the other wages do not reveal match productivity. While each equilibrium outcome corresponds to the solution of a distinct dynamic programming problem, we will present these two problems using one set of Bellman equations.

To that end, let $\rho \in \{1-\alpha_i, 1\}$ denote the fraction of poorly matched workers among all on-the-job searchers, where we can restrict attention to two types of situations: one where wages are revealing and, consequently, only poorly matched workers search on the job ($\rho = 1$), and another where wages are non-revealing and, consequently, both well-matched and poorly matched workers search on the job ($\rho = 1-\alpha_i$).

We begin with the search problem of an employed worker. Even though in this section we are assuming that $i = j = 1$, it will be convenient to index employer types by i and j, and to assume that unemployed workers search for job offers from type-i employers while employed workers search for job offers from type-j employers, so that our analysis in this section will extend readily to the case with multiple types of employers.

For a given value of $\rho \in \{1-\alpha_i, 1\}$, the value of employment to a worker in state $s = \{i, w, y\} \neq s_u$, for $w \in [0, y_h]$ and $y \in \{y_l, y_h\}$, who seeks a job offer from a type-j employer is given by:

$$\overline{V}(s,\rho) = w + \frac{\delta \overline{V}(s_u,\rho)}{1+r} + (1-\delta)\left\{\frac{\overline{V}(s,\rho)}{1+r} + U_j(s,\rho)\right\}, \tag{P1}$$

where

$$U_j(s,\rho) = \max_{w',q'}\left\{f(q')\,\alpha_j\left[\max\left\{0, \frac{w'}{r+\delta} + \left(\frac{\delta}{r+\delta}\right)\frac{\overline{V}(s_u,\rho)}{1+r} - \frac{\overline{V}(s,\rho)}{1+r}\right\}\right]\right\}$$

subject to

$$k \leq q' f(q')\,\alpha_j\left(\frac{y_h - w'}{r+\delta}\right)\rho,$$

$$w' \geq y_l,$$

$$U_j(\{i, w, y_h\}, 1) = 0.$$

Denote a solution to Problem (P1) by $\{w_e(s,\rho), q_e(s,\rho)\}$, with $q_e(\{i,w,y_h\},1) = \infty$ and $w_e(\{i,w,y_h\},1) = 0$. This normalization simply captures the fact that workers searching on the job from high productivity matches do not crowd out workers searching from low productivity matches in an equilibrium where entry wages reveal match productivities. To avoid clutter, we are not explicit about the dependence of w_e and q_e on j.

$U_j(s,\rho)$ represents the option value, to an employed worker, of on-the-job search for a job offer from a type-j employer. As discussed above, the structure of our counteroffer game implies that an employed worker whose wage reveals her to be in a high productivity match cannot profit from on-the-job search. Workers in low productivity matches and workers who are indistinguishable from them can search on the job, and the option value of this search is given by the constrained optimization problem above.

Workers who can profit from on-the-job search face a relatively straightforward competitive search problem. The first constraint imposes that poaching firms must make non-negative expected profits. In this constraint, $\rho \in \{1-\alpha_i, 1\}$ is used to index the two problems. When $\rho = 1$, the non-negative profit constraint is written as if all poaching offers are accepted by workers. This version of the problem corresponds to the equilibrium where wages reveal match productivity, in which case workers in high productivity matches cannot profit from on-the-job search. When $\rho = 1-\alpha_i$, the non-negative profit constraint is written as if poaching offers are accepted by workers with probability $1-\alpha_i$. This version of the problem corresponds to the equilibrium in which wages do not reveal productivity, high productivity workers search on the job, and a fraction $1-\alpha_i$ of applicants to poaching firms reject job offers in favor of retention offers. The other constraint, $w' \geq y_l$, reflects the assumption that poachers recognize the fact that employed workers can only be recruited if the poaching offer exceeds the worker's current productivity.

One can verify that a solution to Problem (P1) is such that

$$q_e\left(\{i, w, y_h\}, 1-\alpha_i\right) = q_e\left(\{i, w, y_l\}, 1-\alpha_i\right)$$

and

$$w_e\left(\{i, w, y_h\}, 1-\alpha_i\right) = w_e\left(\{i, w, y_l\}, 1-\alpha_i\right),$$

which reflects the fact that workers searching on the job from high and low productivity matches have identical incentives in an equilibrium where entry wages do not reveal match productivities. Both types of workers compete for the same outside offers, where subsequent retention offers elicited by well-matched workers will just match the outside offers that will be accepted by poorly matched workers.

The value of unemployment to a worker who seeks a job offer from type-i employers is given by:

$$\overline{V}\left(s_u, \rho\right) = b + V_i\left(\rho\right), \tag{P2}$$

where

$$V_i\left(\rho\right) = \frac{\overline{V}\left(s_u, \rho\right)}{1+r} + \max_{w_l, w_h, q}\left\{f\left(q\right)\left[\alpha_i \frac{\overline{V}\left(\left\{i, w_h, y_h\right\}, \rho\right)}{1+r} + \left(1-\alpha_i\right)\frac{\overline{V}\left(\left\{i, w_l, y_l\right\}, \rho\right)}{1+r} - \frac{\overline{V}\left(s_u, \rho\right)}{1+r}\right]\right\}$$

subject to

$$k \leq qf(q)\left[\alpha_i\left(\frac{y_h}{r+\delta} - \frac{w_e(\left\{i, w_h, y_h\right\}, \rho)}{r+\delta} + \frac{w_e(\{i, w_h, y_h\}, \rho) - w_h}{r+\delta+(1-\delta)\alpha_j f(q_e(\{j, w_h, y_h\}, \rho))}\right)\right.$$
$$\left. + \left(1-\alpha_i\right)\left(\frac{y_l - w_l}{r+\delta+\left(1-\delta\right)\alpha_j f\left(q_e\left(\left\{j, w_l, y_l\right\}, \rho\right)\right)}\right)\right],$$

$$w_l \leq y_l \ , \ w_h \leq y_h \text{ and } w_h \begin{cases} = w_l & \text{if } \rho = 1 - \alpha_i \\ \neq w_l & \text{if } \rho = 1. \end{cases}$$

Denote a solution to Problem (P2) by $\left\{w_u^l(i,\rho), w_u^h(i,\rho), q_u(i,\rho)\right\}$. Once again, to avoid clutter, we are not explicit about the dependence of w_u^l, w_u^h and q_u on j.

The last constraints in Problem (P2) reflect the facts that employers cannot commit to pay wages that exceed the worker's marginal product and that wages reveal a worker's current match productivity if and only if entry wages vary across realizations of match productivity.

The first constraint is the non-negative profits constraint, incorporating all possibilities for on-the-job search allowed under our assumptions about counteroffers. The two terms within the parentheses in the first line reflect the profits an employer enjoys when it forms a high productivity match with an unemployed job seeker. The first term is the expected discounted value of the profits received if the employer were to pay the future retention offer $w_e(\{i, w_h, y_h\}, \rho)$. The second term reflects the temporary extra profits due to the fact that the entry wage w_h of a high productivity worker is lower than the retention offer the worker will elicit as soon as she receives an outside offer. The denominator reflects the three sources of discounting: the discount rate (r), the exogenous probability of job destruction (δ), and the probability that such a worker receives an outside offer from a poaching firm $((1-\delta)\alpha_j f(q_e(\{j, w_h, y_h\}, \rho)))$, in which case the incumbent firm will match and the worker's wage will change. The term within the parentheses in the second line represents the profits a firm enjoys when if forms a low productivity match with an unemployed job seeker. The structure of our counteroffer game implies that such workers are always able to search on the job, never elicit retention offers, and quit whenever they meet a poaching firm with which they form a high productivity match.

The following proposition implies that any allocation supported by a refined equilibrium with positive quits must solve a version of the above problems.

Proposition 1 *Consider a refined equilibrium with positive quits. If the equilibrium is revealing, the equilibrium allocation solves problems (P1) and (P2) with $\rho = 1$. If the equilibrium is non-revealing, the equilibrium allocation solves problems (P1) and (P2) with $\rho = 1 - \alpha_i$.*

The two possible types of equilibrium correspond to the cases where entry wages either reveal or do not reveal the relevant match productivity realization. Whether wages do or do not reveal this information is critical because it determines whether workers with a high productivity realization can profit from on-the-job search. We employ the terminology of the traditional rational expectations equilibrium literature to refer to these equilibria as *revealing* and *non-revealing*. Revealing equilibria correspond to typical competitive search (separating) equilibria, in which all wage contracts are separating contracts. With respect to this, our contribution is to provide a characterization of a non-revealing (pooling) equilibrium, which exhibits pooling contracts.

Using problems (P1) and (P2) to characterize equilibrium allocations is non-trivial due to the fact that the objective function in problem (P2) is not generally concave in $\{w_l, w_h, q\}$. The main complication arises because poachers do not take workers' future quit rates as given, but rather they understand that workers' future quit rates are a function of their current wages. To see why, consider how a worker' current wage affects her trade-off between quit rates and future wages. For a given current wage, a worker is willing to quit at a relatively slower rate only in exchange for relatively higher future wages. The higher her current wage, the lower the *ex post* surplus she can obtain from a given wage and thus, the lower the worker's quit rate. Since a given (future) wage represents a smaller proportional share of the wage gain in the worker expected surplus for workers with higher current wages, a worker's quit rate declines with her current wage at a decreasing rate. While this property is as one would expect, it implies that the worker's value function $\bar{V}(\{i,w,y\},\rho)$ may not be a concave function of w, which is problematic. In general, it is unclear whether or not the properties of $q_e(\{i,w,y\},\rho)$ ensure that both the worker's surplus and the employer's surplus are well-behaved with respect to w.

The above problem complicates significantly the analysis of competitive search on the job (e.g., Delacroix and Shi, 2006). In the appendix, we show that this problem can be addressed by viewing the solution to (P1) as a mapping from the workers' quit rates to their current wages, rather than the reverse. This approach is crucial as it allows us to solve directly for the equilibrium, as opposed to characterizing a constrained efficient outcome that corresponds to the equilibrium allocation. This approach allows us to examine both efficient and inefficient equilibria.

3.1 Revealing equilibrium

It is instructive to begin with the separating equilibrium, in which entry wages reveal match productivity. We refer to this kind of equilibria as (fully) revealing.

Proposition 2 *Assume that* $(y_h - b)/(y_h - y_l) \geq (r+\delta+\alpha_1)/(r+\delta+(1-\delta)\alpha_1)$*. There is a number* $k_0 > 0$ *such that for all* $k \leq k_0$ *there is a refined equilibrium that is revealing. The corresponding equilibrium allocation is uniquely characterized by equations (6)-(9) and (11) below, and it maximizes the present value of aggregate production net of search costs.*

In a revealing equilibrium the wage distribution has three mass points: one wage for each productivity realization for workers who find jobs out of unemployment, and one wage for workers who find jobs via on-the-job search. Equilibrium transitions are as follows: All job offers made to unemployed workers are accepted. Unemployed workers who meet a firm with which they form a low productivity match conduct on-the-job search. These workers change jobs upon meeting another firm with which they form a high productivity match. Our equilibrium refinement implies that workers in high productivity matches will not crowd out poorly matched workers in a revealing equilibrium. Accordingly, such workers do not profit from search on the job and never change jobs. Jobs are destroyed both exogenously (at rate δ) and, for the case of low productivity matches, endogenously by quits.

In the revealing equilibrium low productivity workers are paid their marginal product:

$$w_u^l(i,1) = y_l. \tag{6}$$

This is an important feature of separating equilibria and is at the core of the constrained efficiency of the revealing equilibrium. Intuitively, the *ex ante* match surplus is maximized when the employer assigns all of the match surplus to poorly matched workers *ex post*, in which case they quit exactly when it is efficient to do so. Such a surplus division is optimal from the viewpoint of employers, because they are able to maximize surplus extraction when workers are well-matched *ex post*.

Otherwise, the revealing equilibrium satisfies the usual zero profit and matching efficiency conditions of competitive search models. In particular, $\{w_e(s,1), q_e(s,1)\}$ is the unique pair $\{w', q'\}$ that solves

$$q' f\left(q'\right) \alpha_j \left(\frac{y_h - w'}{r+\delta}\right) = k \tag{7}$$

and

$$\frac{w' - y_l}{r + \delta + (1-\delta)\,\alpha_j f\left(q'\right)} = \left(\frac{1-\eta\left(q'\right)}{\eta\left(q'\right)}\right)\left(\frac{y_h - w'}{r+\delta}\right). \tag{8}$$

Equation (7) requires that the expected value of a vacancy to potential poachers equals the cost of posting the vacancy. It implies that employers are willing to offer higher wages and suffer reductions in the net present value of their profits only if they expect to fill their vacancies at a faster rate.

Equation (8) is the familiar condition of matching efficiency from standard competitive search equilibrium models. The left side of the equation is the present value of forgone wages while a poorly matched worker searches on the job. It is easy to verify that the Bellman equation in problem (P1) implies that this value is equal to the worker surplus in the new match. Recalling that $\eta\left(q\right)$ is the elasticity of job creation and $1-\eta\left(q\right)$ is the elasticity of job finding, this matching-efficiency condition implies that the ratio of the worker's surplus to the firm's surplus in new matches equals the ratio of their matching elasticities.

Similarly, $\left\{w_u^h(i,\rho), q_u(i,\rho)\right\}$ is the unique pair $\{w, q\}$ that satisfies

$$q f(q) \alpha_i \left(\frac{y_h - w}{r+\delta}\right) = k \tag{9}$$

and

$$\alpha_i \frac{\overline{V}\left(\left\{i, w, y_h\right\}, 1\right)}{1+r} + \left(1-\alpha_i\right)\frac{\overline{V}\left(\left\{i, y_l, y_l\right\}, 1\right)}{1+r} - \frac{\overline{V}\left(s_u, 1\right)}{1+r} = \left(\frac{1-\eta\left(q\right)}{\eta\left(q\right)}\right)\alpha_i\left(\frac{y_h - w}{r+\delta}\right) \tag{10}$$

together with the Bellman equation in Problem (P2).

Note that the condition for matching efficiency in the market for unemployed workers (i.e., equation (10)) is completely standard. This is because of the result that, in equilibrium, firms earn no profit from low productivity workers. Consequently, the firm's match surplus is entirely a function of the profits it makes when employing high productivity workers. Since these workers cannot profit from on-the-job search in a revealing equilibrium, employers have no incentive to set wages in order manipulate their quit rates.

One can verify that equations (9) and (10), together with the Bellman equation in Problem (P2) imply that $q_u(i, \rho)$ is the unique value of q that solves

$$\frac{y_h - b}{r + \delta} - \frac{(1 - \alpha_i)\, k}{\eta\left(q_e(s,1)\right) q_e(s,1) f\left(q_e(s,1)\right) \alpha_j} = \frac{k}{\eta\left(q\right) q f\left(q\right)} + \left(\frac{1 - \eta\left(q\right)}{\eta\left(q\right)}\right) \frac{k}{\left(r + \delta\right) q}. \tag{11}$$

This completes the characterization of the unique equilibrium allocation associated with a revealing equilibrium.

The assumption that $(y_h - b) / (y_h - y_l) \geq (r + \delta + \alpha_1) / (r + \delta + (1 - \delta)\, \alpha_1)$ made in Proposition 2 is sufficient to ensure that unemployed workers are willing to accept job offers when match productivity is low. Otherwise, a revealing equilibrium with positive job creation may not exist if k is sufficiently low. The assumption requires that the difference between y_l and b be sufficiently large. The smaller the job destruction rate the less restrictive the assumption is.

In equilibrium, beliefs about the composition of the pool of applicants must be correct, both in the market for unemployed workers as well as the separate market for workers searching on the job. Moreover, in the latter employers have the most optimistic beliefs, as they believe that their job offers will be accepted with certainty. Therefore, it is straightforward to support the above equilibrium allocation. Clearly, neither the equilibrium mapping Q nor off-equilibrium beliefs that support the equilibrium allocation are unique.

3.2 Non-revealing equilibrium

We now turn to the pooling equilibrium, in which entry wages do not reveal match productivity. We refer to this kind of equilibria as non-revealing.

Proposition 3 *Assume that $(1 - \alpha_1)(1 - \delta) > (r + \delta)$. There is a number $k_1 > 0$ such that for all $k \leq k_1$ there is a refined equilibrium that is non-revealing. The allocation supported by such a non-revealing equilibrium is uniquely characterized in the Appendix.*

In a non-revealing equilibrium the wage distribution has two mass points. Since wages do not differ across productivity realizations, all entry jobs pay an identical wage. In principle, there could be two wages in the on-the-job search market, as poorly matched workers accept poaching offers whereas well-matched workers transition to retention wages. However, since both types of workers have the same current wage, their incentives to search on the job are identical, so the equilibrium poaching and retention wages are also identical.

Equilibrium transitions are as follows: All job offers made to unemployed workers are accepted, and all workers employed in jobs found out of unemployment search on the job, with workers who are well-matched *ex post* mimicking the on-the-job search behavior of workers who are poorly matched. As a result of pooling, all workers searching on the job face the same matching probabilities. Workers with low productivity realizations in their first jobs change jobs upon meeting another employer with which they form a high productivity match. Workers with high productivity realizations in their first jobs receive retention offers upon meeting another employer with which they form a high productivity match. Jobs are destroyed both exogenously (at rate δ) and, in the case of workers in low productivity matches, endogenously by quits.

Consider the search problem of a worker who currently works for a type-i employer earning a wage w and searching for a job offer from a type-j employer. Once again, it will be convenient to distinguish between different types of employers even though we are assuming that $i = j = 1$ throughout this section. It is easy to verify that an interior solution of Problem (P1) with $\rho = 1$, $\{w', q'\}$, satisfies the familiar matching efficiency condition

$$\frac{w' - w}{r + \delta + (1 - \delta)\,\alpha_j f(q)} = \left(\frac{1 - \eta(q')}{\eta(q')}\right)\left(\frac{y_h - w'}{r + \delta}\right),$$

according to which the ratio of the worker's surplus to the employer's surplus equals the ratio of their matching elasticities. Notice that this condition is identical to (8), which is the corresponding matching efficiency condition in the revealing equilibrium, though the entry wage (w) is generally different. A solution of Problem (P1) also satisfies the usual zero-profit condition

$$q' f(q')\,(1 - \alpha_i)\,\alpha_j\left(\frac{y_h - w'}{r + \delta}\right) = k. \tag{12}$$

Note that, in a non-revealing equilibrium, potential poachers need to anticipate that a fraction $(1 - \alpha_i)$ of their pool of applicants are poorly matched in their current jobs, and so a fraction α_i will turn down their job offers because they are only searching to elicit a retention offer from their current employer.

Solving Problem (P2) is non-trivial because the objective function is not generally concave in $\{w, q\}$. Fortunately, one can address this problem by viewing the solution to Problem (P1) as a mapping from the workers' quit rates to their entry wages, rather than the reverse, and then treat current and future quit rates as the relevant choice variables in Problem (P2). We follow this approach in the proof of Proposition 2 to characterize the equilibrium allocation in the constrained efficient equilibrium. In the Appendix, we show that this approach can be followed more generally to characterize the allocation in the non-revealing equilibrium and prove Proposition 3.

To understand the properties of the non-revealing equilibrium allocation, it is useful to consider the transformed problem in some detail. To that end, use the above first-order conditions to express the worker's entry wage as a function of (the future) q':

$$\widetilde{W}(q') \equiv y_h - \left(\frac{k}{q' f(q')\,(1 - \alpha_i)\,\alpha_j}\right)\left(r + \delta + \left(\frac{1 - \eta(q')}{\eta(q')}\right)\left(r + \delta + (1 - \delta)\,\alpha_j f(q')\right)\right). \tag{13}$$

Observe that employers understand i) that all workers search on the job, and ii) that job finding probabilities in the on-the-job search market depend on the wages earned by workers in their current jobs. Employers take this effect into account and set current wages, in part, in order to influence future quit rates. Let $\widetilde{V}_0(i,q')$ denote the value of a match with a type-i employer to an employed worker expressed as a function of q':

$$\begin{aligned}\widetilde{V}_0(i,q') &\equiv \alpha_i \overline{V}\left(\left\{i,\widetilde{W}(q'),y_h\right\},1-\alpha_i\right)+(1-\alpha_i)\overline{V}\left(\left\{i,\widetilde{W}(q'),y_l\right\},1-\alpha_i\right)\\ &=\overline{V}\left(\left\{i,\widetilde{W}(q'),y_h\right\},1-\alpha_i\right)=\overline{V}\left(\left\{i,\widetilde{W}(q'),y_l\right\},1-\alpha_i\right)\end{aligned}$$

and let $\widetilde{M}_0(i,q')$ denote the *ex ante* surplus associated with the match:

$$\widetilde{M}_0(i,q') \equiv \alpha_i \widetilde{M}\left(\left\{i,\widetilde{W}(q'),y_h\right\}\right)+(1-\alpha_i)\widetilde{M}\left(\left\{i,\widetilde{W}(q'),y_l\right\}\right),$$

where $\widetilde{M}\left(\left\{i,\widetilde{W}(q'),y_h\right\}\right)$ is the *ex post* surplus associated with a high-productivity match and $\widetilde{M}\left(\left\{i,\widetilde{W}(q'),y_l\right\}\right)$ is the *ex post* surplus associated with a low-productivity match.

It is useful to understand the connection between the total surplus of a match and its allocation between a worker and her employer. To that end, note first that the surplus in low-productivity matches is given by

$$\begin{aligned}\frac{\widetilde{M}\left(\left\{i,\widetilde{W}(q'),y_l\right\}\right)}{1+r} &= \frac{\overline{V}\left(\left\{i,\widetilde{W}(q'),y_l\right\},1-\alpha_i\right)}{1+r}-\frac{\overline{V}(s_u,1-\alpha_i)}{1+r}\\ &\quad+\left(\frac{r+\delta}{r+\delta+(1-\delta)\alpha_j f(q')}\right)\frac{y_l-\widetilde{W}(q')}{r+\delta}\end{aligned}$$

where the first line on the right side is the part of the match surplus that goes to the worker and the second line is the part that goes to the employer, which consists of a flow of profits equal to $y_l-\widetilde{W}(q')$ for as long as the worker stays with the employer (where the term in parentheses is the probability that the worker will find an outside offer), which the employer anticipates she will accept with probability one.

The surplus in high-productivity matches is more interesting. In particular,

$$\begin{aligned}\frac{\widetilde{M}\left(\left\{i,\widetilde{W}(q'),y_h\right\}\right)}{1+r} &= \frac{\overline{V}\left(\left\{i,\widetilde{W}(q'),y_h\right\},1-\alpha_i\right)}{1+r}-\frac{\overline{V}(s_u,1-\alpha_i)}{1+r}\\ &\quad+\left(1-\frac{r+\delta}{r+\delta+(1-\delta)\alpha_j f(q')}\right)\frac{y_h-w_e\left(\left\{i,\widetilde{W}(q'),y_h\right\},1-\alpha_i\right)}{r+\delta}\\ &\quad+\left(\frac{r+\delta}{r+\delta+(1-\delta)\alpha_j f(q')}\right)\frac{y_h-\widetilde{W}(q')}{r+\delta}.\end{aligned}$$

The first and the third lines in the right side are the obvious counterparts of those in low-productivity matches. The second line reflects the fact that *ex post* well-matched workers will search for outside offers solely to elicit a retention offer from their current employer.

It is easy to verify that an interior solution for the current and future labor market queues $\{q, q'\}$ must satisfy the following conditions:

$$\frac{\widetilde{V}_0(i, q')}{1+r} - \frac{\overline{V}(s_u, 1-\alpha_i)}{1+r} = \lambda_i q \left(\frac{1-\eta(q)}{\eta(q)}\right) \frac{k}{qf(q)}, \tag{14}$$

$$\lambda_i = \frac{f(q)\, \partial\widetilde{V}_0/\partial q'}{qf(q)\left(\partial\widetilde{V}_0/\partial q' - \partial\widetilde{M}_0/\partial q'\right)} \tag{15}$$

and

$$qf(q)\left(\frac{\widetilde{M}_0(i, q')}{1+r} - \frac{\widetilde{V}_0(i, q')}{1+r} + \frac{\overline{V}(s_u, 1-\alpha_i)}{1+r}\right) = k, \tag{16}$$

where λ_i is the multiplier associated with the employer's zero-profit constraint, given by equation (16). Equation (14) coincides with the standard matching efficiency condition if and only if the multiplier equals $1/q$. Consider equation (15). The multiplier is the expected value of surplus to the worker associated with a higher labor market queue at the margin ($f(q)\, \partial\widetilde{V}_0/\partial q'$) evaluated in terms of the employer's surplus ($qf(q)\left(\partial\widetilde{V}_0/\partial q' - \partial\widetilde{M}_0/\partial q'\right)$). The expected surplus of a match is maximized at $\partial\widetilde{M}_0/\partial q'$, which implies that $\lambda_i = 1/q$. In the Appendix, we show that this happens exactly at the corner when $\widetilde{W}(q') = y_l$.

In an interior non-revealing equilibrium $\lambda_i \geq 1/q$ and the match surplus is not maximized, except in the special case where the first-order conditions hold at the corner and $\lambda_i = 1/q$. The problem is that while employers can lower the workers' future quit rates by raising the entry wages they offer in the first place, they also have an impact on the outside offers the workers will get, because workers with higher wages have an incentive to elicit higher outside offers. Since they cannot prevent well-matched workers from seeking outside offers, the allocation of surplus at the margin is allocated disproportionately to the worker and so employers do not typically have an incentive to raise entry wages all the way to y_l.

The corner allocation is still inefficient because it induces too little entry of employers in the market for unemployed workers. This is because, relative to a revealing equilibrium, employers are forced to share too much surplus with the worker, as high productivity workers who receive outside offers stay with their current employer, but are able to extract some of the surplus. In the Appendix, we show that the allocation in a non-revealing equilibrium is uniquely characterized, although in general we cannot guarantee that the allocation is interior.

To understand why non-revealing wages can be supported in equilibrium note the existence of an informational externality, whereby firms in the market for unemployed workers do not take into account the informational value of wages to poachers. The consequence of this externality is that employers have no direct incentive to post revealing wages. This means that non-revealing wages can be equilibrium wages as long as unemployed workers choose to search for non-revealing contracts when revealing contracts are feasible. This occurs because the option to search on the job constitutes an important component of the value of a job, but the value of this option depends on the beliefs of both workers and potential poaching firms.[8]

To see this, consider a candidate non-revealing equilibrium. Suppose an unemployed worker considers searching for a contract with revealing wages. The value of on-the-job search for such a job, however, depends on the off equilibrium beliefs of poaching firms about the current match quality of their applicant pool. Our equilibrium refinement has no bite for beliefs about non-equilibrium states, therefore these beliefs are unrestricted. Consequently, if potential poachers are sufficiently pessimistic about the composition of the applicant pool in the on-the-job search market associated with a revealing contract (i.e. they believe it will contain a high percentage of well-matched workers looking for retention offers) then the returns to on-the-job search associated with deviations to a revealing contract are sufficiently low that such deviations are unprofitable to workers.

Formally, if $s \notin S^*$, beliefs are arbitrary and we may assume that employers believe that $\mu(s|x) = 0$ for all $s \notin S^*$. In the Appendix, we show that the assumption that $(1-\alpha_1)(1-\delta) > (r+\delta)$ made in Proposition 3 then ensures that no type-1 employer can profit from offering a deviating contract where wage offers to unemployed workers are made conditional on match productivity. The assumption requires that the probability of *ex post* mismatch of workers and type-1 employers is sufficiently high, that the jobs are sufficiently durable and that workers value future payoffs sufficiently. Of course, neither the equilibrium mapping Q nor off-equilibrium beliefs that support the equilibrium allocation are unique.

Observe that wages that do not reveal match productivity create adverse selection in the on-the-job search market since, under non-revealing wages, workers in matches with high productivity cannot be identified and, therefore, have an incentive to search on the job in order to elicit retention offers from their current employers. Non-revealing wages, therefore, increase the value of on-the-job search to workers with a high productivity realization relative to the case where wages reveal match quality, thereby preventing well-matched workers from searching on the job. The overall effect of this adverse selection problem, however, is to depress the returns to poaching firms, which reduces the entry of poachers and therefore, depresses the returns to on-the-job search overall. This reduction is concentrated on poorly matched workers. Note that this adverse selection problem is worse the higher is α_1, because a large value of α_1 implies that many workers are well matched to begin with, and therefore only searching on the job to elicit retention offers.

4 Equilibrium underinvestment in labor

In this section we turn to the analysis of the economy where employers make an investment in labor, which affects the likelihood that a match with an average worker will be productive. Recall that we

[8] Examples of pooling equilibria are found in Shi (2002) and Shimer (2005) in the context of labor markets and Chang (2018) and Guerrieri and Shimer (2014) in the context of asset markets.

assume that this investment is costless, for simplicity, and we model it as if employers unilaterally choose the type of production technology, which is observable. Formally, a worker-employer match produces y_h units of output with probability α_j and y_l units of output with probability $1-\alpha_j$, for $j=1,2$, where $b<y_l<y_h$ and $0<\alpha_2<\alpha_1<1$. Otherwise, our model of the labor market remains unchanged. We show that there are refined equilibria exhibiting underinvestment in labor.

It is easy to verify that an equilibrium allocation must solve the obvious analogues of Problems (P1) and (P2). First, note that our assumptions about counteroffers continue to restrict the possible job and wage transitions as explained in Section 3. It is straightforward to verify that any allocation supported by an equilibrium with positive quits must be such that employed workers only ever search for type-1 jobs. Intuitively, workers are expected to be more productive in type-1 jobs and, consequently, type-1 employers always drive type-2 employers out of any market where employed workers search.

Taking this into account, Problem (P1), with $j=1$, can be used to characterize equilibrium allocations, except that now it ought to be recognized that the value functions and the corresponding policy functions are functions of (ρ_1,ρ_2), rather than simply ρ, where $\rho_i\in\{1-\alpha_i,1\}$ denotes the fraction of poorly matched workers currently employed in type-i jobs among all those searching for type-1 jobs, for $i=1,2$. With a slight abuse of notation we will continue to denote those functions as before. One can verify that equilibrium allocations must satisfy the analogue of Problem (P2), with

$$\overline{V}\left(s_u,\rho_1,\rho_2\right)=b+\max\left\{V_1,V_2\right\},$$

where V_i is given by Problem (P2) with $\rho=\rho_i$, for $i=1,2$.

It is straightforward to extend our analysis of the problem with an exogenous employer's type to show the following.

Proposition 4 *Maintain the assumptions of Proposition 2 and 3. There are numbers* $\widehat{\alpha}\in(0,\alpha_1)$ *and* $\widehat{k}>0$ *such that for all* $\alpha_2\in(\widehat{\alpha},\alpha_1)$ *and all* $k\in\left(0,\widehat{k}\right)$ *there are four refined equilibria, each one supporting a unique equilibrium allocation such that all employed workers search for jobs with revealing wages posted by type-*1 *employers. There is one equilibrium allocation in which unemployed workers seek jobs with revealing wages posted by type-*1 *employers, one in which they seek jobs with revealing wages posted by type-*2 *employers, one in which they seek jobs with non-revealing wages posted by type-*1 *employers, and one in which they seek jobs with non-revealing wages posted by type-*2 *employers.*

Compare the two revealing equilibria. It is easy to see that the equilibrium such that both employed and unemployed workers seek jobs with revealing wages posted by type-1 employers exists under the assumption of Proposition 2, and it is constrained efficient in the sense that it maximizes the present value of aggregate production net of search costs. However, this is not the only revealing equilibrium in the class of refined equilibria. There is another one in which unemployed workers seek jobs with revealing wages posted by type-2 employers. The reason is that the incentive to post a type-1 job depends on the workers' off-equilibrium beliefs about the career prospects associated with the job. If unemployed workers believe that type-1 employers only post dead-end jobs, then they may prefer to work for employers that underinvest in labor, because those jobs are in fact associated with sufficiently valuable future job mobility. Clearly this will be the case if the likelihood of a more productive match is not too different across production technologies.

It is worth stressing that equilibria in which type-2 employers create jobs are such that not only some employers create technologically inferior jobs, but some workers direct their search towards these jobs. This suggests a link between the creation of bad jobs and the possibility of *ex ante* mismatch, as opposed to *ex post* mismatch due to random match quality, as is the case in random matching models of the labor market. Here, instead, workers search for technologically inferior jobs even though search for superior jobs is feasible. This happens because, in equilibrium, the latter come, endogenously, with poor career prospects and so they are in fact less valuable to workers. The problem is that the career prospects of a given job are determined in equilibrium and jobs that are technologically superior and offer better career prospects are only viable under a different equilibrium incentive structure.

A similar argument implies that there are conditions under which there are two non-revealing equilibria. Both of them are supported by similar off-equilibrium beliefs. That is, equilibrium jobs with underinvestment in labor, poor career prospects, or both, can arise if unemployed workers believe that jobs offering revealing wages and jobs posted by type-1 employers are attainable only via on-the-job-search, or else they are dead-end jobs.

Both technologically inferior jobs and jobs characterized by non-revealing contracts are the result of market failure and cannot be properly understood without reference to the underlying equilibrium incentive structure. The key to understanding the three inefficient equilibria characterized in Proposition 4 is to note that in each case the adverse selection problem associated with jobs that are posted in equilibrium is sufficiently less severe than the adverse selection problem affecting jobs offering revealing contracts and jobs posted by type-1 employers. Potential poaching firms understand this and, consequently, are less willing to enter markets where workers in those jobs search, which lowers the value of on-the-job search. When this effect is sufficiently strong, jobs offering non-revealing contracts and jobs posted by type-2 employers have an equilibrium advantage over all other jobs that are viable in a given equilibrium, despite their obvious drawbacks, which are common knowledge.

Arguably, the same mechanism underlying our results could also explain why employers' underinvestment in labor may result in workers' suboptimal human capital accumulation, providing yet another channel for suboptimal career prospects. We believe that human capital accumulation is an important driver of job mobility.[9] However, it is unlikely that job mobility can be well understood without reference to the functioning of the labor market, which is our focus here.

Similarly, it should be clear that our main arguments continue to hold if one assumes that

[9]Jovanovic and Nyarko (1997) is an insightful analysis of the role of human capital in job mobility, contrasting learning from experimentation versus human capital accumulation.

investments in labor include amenities that are valued by workers, even if they do not influence labor productivity. Then, employers may not offer amenities in equilibrium because of insufficient demand, whereas workers do not seek jobs that offer those amenities because they are dead-end jobs. We return to this issue below when we consider the link between discrimination and the creation of bad jobs.

5 Discussion

5.1 The creation of bad jobs

Job quality is necessarily a relative concept. To us, the question is how to define a bad job for a worker relative to whatever is actually viable. The main issue concerns the meaning of *viable*. Any definition of job quality that fails to be explicit about this issue is suspect. We propose the following definition.

Definition 3 *Consider a refined equilibrium $\mathcal{E}$. A job $x^* = g_x(s) \in X^*$ is bad for a worker in state $s \in S^*$ if (i) there exists a job x and a queue length q such that $U(s, x, q) > U(s, x^*, Q(x^*))$, and (ii) there exists a refined equilibrium $\widehat{\mathcal{E}}$ such that $x = \widehat{g}_x(s) \in \widehat{X}^*$, with $q = \widehat{Q}(x)$. A job $x^* = g_x(s) \in X^*$ is good for a worker in state $s \in S^*$ if it is not bad.*

First note that we define the quality of a job relative to the perspective of those workers who actually search for that job. This is because the evaluation of a job by workers who choose not to search for that job is trivial, in the sense that the search behavior of such workers reveals that they find alternative, available job to be superior. What matters is the quality of the jobs they do choose to search for. Furthermore, our definition restricts attention to states that are part of some equilibrium allocation, and therefore does not attempt to classify jobs that are never observed to exist in any equilibrium.

Condition (i) requires that a state-s worker prefers to search for a different job from the one she is searching for in equilibrium. Condition (ii) requires that the worker's preferred job is a viable alternative in the sense that a state-s worker would credibly search for it in some equilibrium. If Condition (ii) fails to be satisfied, then it must be that searching for the preferred job is incompatible with the structure of incentives arising in any equilibrium. Note that, a bad job can exist only if Condition (ii) is satisfied for some $\widetilde{\mathcal{E}} \neq \mathcal{E}$. For if $\widetilde{\mathcal{E}} = \mathcal{E}$, then Condition (i) and Condition (ii) are incompatible and so x must be a good job for a worker in state s. An immediate corollary is that there can be no bad jobs if the equilibrium allocation is unique.

Relative to the meaning of the phrase "bad jobs" in common parlance, our definition stresses the fact that a useful definition of job quality ought to consider not just the workers' welfare, but also whether it is actually viable to improve upon the job. Put simply, *a job is bad for a worker if there is an equilibrium where a comparable worker can do better*. Formally, two workers are comparable if they have the same payoff-relevant history. This is not the only possible definition of bad jobs, but it is the most natural to an economist. One could define the constrained efficient allocation as a viable structure, but this is problematic in cases where the socially optimal allocation cannot be decentralized. Observe that an inefficient labor market outcome is not a sufficient condition for bad jobs to exist. For a job to be bad, it must be possible to organize the same labor market in a way that a better job is compatible with the system of incentives arising in equilibrium. If a job is not dominated by another job that is compatible with equilibrium incentives, we define it as a good job, albeit a good job in a second best economy in cases where equilibria are not efficient. This implies that the evaluation of job quality is model-specific and so any attempt to identify good and bad jobs in practice requires an explicit account of the actual institutional environment.

It is straightforward to adapt our definition of good and bad jobs to encompass endogenous labor market regulation, and more generally endogenous government policy. Such an extension is important to understand cases where labor market regulation affects employment contracts directly. For example, the growth of temporary employment contracts in Spain and France has been linked to excessive firing costs associated with indefinite contracts. Are these new jobs bad jobs? The logic of our definition would say they are if and only if there is another politically viable equilibrium in which better jobs can be created.

Our notion of viability can help understand why labor market participants may agree that bad jobs are common without appealing to privately suboptimal behavior of some market participants or to purely random allocations of jobs to workers. In our model workers and employers are rational agents, search is directed and markets are complete. Consequently, the jobs employers create and the jobs workers seek in a competitive search equilibrium are necessarily optimal choices among all feasible choices in that equilibrium. Nevertheless, there is room for the possibility of bad jobs relative to the best viable alternative equilibrium allocation.

The possibility of multiple equilibria, while central to our notion of bad jobs, is obviously not specific to our model. For example, increasing returns in the production function, or demand complementarities, can lead to multiple labor market equilibria in the Neoclassical frictionless model and the standard random matching model of the labor market, in which case an equilibrium in those models can exhibit bad jobs with the flavor of underinvestment in labor. In these cases, however, market failures external to the labor market are the source of bad jobs, whereas in our model bad jobs arise due to fundamental failures in the labor market itself, arising from the conflict between the signaling and allocative role of wages. Naturally, these different sources of bad jobs have radically different policy implications. We return to this issue below.

Proposition 5 *Consider an equilibrium allocation that is part of a refined equilibrium. Jobs offered by type-*2 *employers and jobs involving non-revealing wages are bad jobs. Jobs involving revealing wages offered by type-*1 *employers are good jobs.*

Bad jobs in our model can take two forms. Jobs offered under pooling contracts are bad jobs because they generate suboptimal career prospects, as they fail to signal willingness to move. Jobs can also be bad when employers underinvest in labor. In this case, the jobs that exist in equilibrium have lower productivity, or lower human capital acquisition, or inferior workplace amenities, than other jobs that are technologically feasible. Good jobs, in contrast, are those in which employers invest in labor, and which are offered under separating contracts that ensure optimal career mobility for workers. Note that the existence of bad jobs, of both types, is driven by the fact that alternative jobs come with sufficiently poor career prospects that they are not created in equilibrium.

Our focus on job mobility is motivated, in part, by the observation that job-to-job flows are a sizable component of both new hires and job separations, which suggests that the effect of job mobility on job creation is likely to be a central feature of labor markets.[10] In spite of this, standard equilibrium analyses of job creation and destruction typically abstract from job mobility. What our analysis makes clear is that neither job creation nor job quality can be understood independently of job mobility. Since the option to search on the job is an important component of the value of a job, a firm's decision to create a job, the quality of the job, and an unemployed worker's decision to search for the job all depend crucially on the value of on-the-job search from that job. Our paper demonstrates the interaction between these factors.

Our theory also provides a novel lens to understand the role played by on-the-job search in labor markets. One possibility is that job mobility is an efficient response to labor market frictions. This perspective, which emphasizes the ability of on-the-job search to correct *ex-post* misallocation, corresponds to the conventional view.[11] Our analysis, however, shows that on-the-job search can also be an important *cause* of labor market dysfunction, as it gives workers an incentive to search for jobs for which they are otherwise not well suited.

Bad jobs, under our theory, tend to be found on the bottom rungs of job ladders. Empirical evidence suggests that most workers do indeed work their way up a job ladder. Consequently, job loss is costly because unemployed workers have to climb the job ladder anew (Carrington and Fallick, 2017). The evidence also indicates that job losers are significantly more likely than other workers to be subsequently employed in temporary jobs and involuntary part-time jobs, and this type of employment tends to be a stepping stone to regular full-time employment (Farber, 1999, Booth et al., 2002). Our theory explains these observations as the result of market discrimination against the unemployed. That is, in some markets there exist other equilibria in which the unemployed have direct access to better jobs. Consequently, the costs of job loss associated with the fact that unemployed workers have to climb the job ladder anew are excessively high.

Our model also helps clarify the link between job quality, job satisfaction, and compensating wage differentials. To see this, consider a version of the model with homogeneous employers, but rather than match specific productivities, suppose that jobs provide match-specific amenities that are valued by workers. In this version of the model, an equilibrium exists where jobs are offered under separating contracts, and therefore exhibit compensating differentials in the sense that workers who find matches with high amenities are penalized in terms of wages, whereas "unhappy" workers receive wages that allow them to extract all the surplus from a match. Note that in this equilibrium all jobs are good. Workers in jobs characterized by poor amenities may well report low job satisfaction,

[10] See Fallick and Fleischman (2004), Bjelland et al. (2011).

[11] See, for example, Menzio and Shi (2011) and Shi (2018).

but are compensated for this both by relatively high wages and attractive mobility prospects. In contrast, a bad job in this economy would be one offered under a pooling contract. Such a job would involve no wage differential and would hide the worker's utility value of work from the market. Despite that, those workers in jobs with high match-specific amenities might well report high levels of job satisfaction.

Finally, our theory has implications concerning the role played by posted wages in labor markets. Unlike conventional wage posting models, in which the allocative role for prices tend to be efficiency enhancing, wage posting in our model can be an indicator of a bad job. To the extent that ex-post productivity differences across workers are concealed by the existence of a single ex-ante posted wage, the posting of a single wage resembles our pooling contracts, which are associated with poor equilibrium job mobility.

5.2 The identification of bad jobs

What, in practice, is a bad job? Non-revealing equilibria of our model, where bad jobs are characterized by suboptimal career prospects, capture a situation that may be observed in some retail jobs. For example, retailers such as Wal-Mart have the reputation of being bad employers in the U.S., and retail jobs are often referred to as dead end jobs. Are these bad jobs? In a similar spirit to our model, Carre and Tilly (2017) argue that differences in job quality among retailers are largely the result of social norms and institutions, with Wal-Mart having a significantly better reputation as an employer in Mexico than in the U.S., for example. Similarly, the supermarket chain Mercadona has a reputation as a good employer in Spain, offering full-time jobs with significant training aimed at expanding the workers' career prospects (Ton, 2012). This suggests that retail jobs are consistent with better career prospects in some equilibria, in which case dead-end retail jobs are in fact bad jobs.

While examples are instructive, it is worth examining whether bad jobs can be identified more systematically. The central problem in identifying bad jobs empirically involves the difficulty of distinguishing between worker heterogeneity and job quality. In the remainder of this section we examine this issue, and identify an approach that could be implemented in practice.

The simplest way to illustrate the problem is to preserve the assumption that workers are homogeneous in terms of productivity and preferences for amenities, but heterogeneous along other observable dimensions, which allows for the existence of multiple distinct markets. This could be the case, for example, if markets are separated geographically or across occupations with equal productivity. In this case, our model can be applied market by market. Bad jobs may exist in some markets as a result of localized coordination failure, and the economy as a whole will have a mix of good and bad jobs, depending on which equilibrium obtains in each individual market.

By contrast, consider an economy in which the efficient equilibrium obtains in each market, but that workers in each market are heterogeneous, in terms of either productivity or preferences for amenities, and that this heterogeneity is unobserved by outsiders (such as econometricians). Observe that it will not be possible to distinguish between the bad jobs and the heterogeneous workers economy on the basis of observable job characteristics, like wages. This is because heterogeneity in any observed job characteristic, whether it be a wage or amenity, could be equally the result of worker heterogeneity across efficient markets, or localized coordination failure in some but not all markets, with homogeneous workers.

Corollary 1 *For any given equilibrium distribution of job quality in the economy with multiple markets, there exists a distribution of unobserved productivities and preferences for amenities that replicates the exact distribution of jobs and wages as the outcome of a constrained efficient competitive search equilibrium, in which all jobs are good jobs.*

Note that the problem becomes worse when both worker heterogeneity and bad jobs coexist. This is because coordination failure does not depend on worker characteristics, which means the model does not suggest any correlation between the productivity level and the production of good jobs. For example, suppose that high and low ability workers search on different markets. It could be the case that sectors of the economy with low productivity good jobs, while the jobs produced in higher productivity sectors are bad. This illustrates why it is not possible to identify good and bad jobs by comparing the search behavior of observationally equivalent workers. Within an equilibrium, all workers search for the best possible job. Consequently, observed differences in search behavior is driven by unobserved differences in workers' productivities or preferences. The question of whether the jobs created are good or bad is not related to the issue of high and low productivity occupations.

This highlights the problems associated with trying to identify good and bad jobs by comparing across jobs that are observed within a particular equilibrium. The key to identifying bad jobs lies in the fact that they are a market phenomenon and, consequently, they can only be identified by the use of cross market variation in jobs. In particular, a job can be identified as bad only in comparison to another job that could have been created in its place, which requires finding a comparable market in which a better job was in fact created. This could be done by identifying a comparable market in the cross section, such as a similar market in a separate geographical location, or the market for a different but comparable occupation of worker. It could also be done using intertemporal variation within a market, in cases where the same market is reorganized at some point in time.

5.3 Pitfalls of bad jobs

Even though the creation of each bad job is the choice of a single employer, job quality is intrinsically an equilibrium outcome. Given that employers create bad jobs for unemployed workers, no single employer has an incentive to create a good job for unemployed workers instead, because unemployed workers in fact demand bad jobs rather than good jobs. If technologically superior jobs come with poor career prospects, there is nothing a single employer can do about it. Similarly, if jobs with revealing contracts are considered dead-end jobs in a particular labor market, non-revealing contracts are in fact the best feasible contracts, given the equilibrium incentive structure. This is not to say that nothing can be done about bad jobs. Rather, the point is that this is not something that can be addressed without regard for the equilibrium incentive structure. Successful labor market policy needs to coordinate the creation of good jobs across employers in a given market.

One possibility is for large firms to develop internal labor markets to address the adverse selection problem that we have highlighted. For instance, extend our model to allow for an employer to offer a career path internally. Specifically, suppose that an employer can engineer a career change after observing the productivity of the worker-job match, in the sense that the employer can create a match that has productivity y_h with probability one the period after the match is formed. Note that there will not be search on the job in a refined equilibrium, since it is common knowledge that employers will resolve the problem of mismatched workers the period immediately following the formation of the match. Now consider the market for unemployed workers and suppose that employers offer non-revealing contracts. Note that workers who are well matched ex post will not receive retention offers and workers who are poorly matched will not receive wage increases even as their productivity rises in the period after the match is formed. Consequently, all that matters, both for unemployed workers and employers, is the expected wage. The main implication is that pooling contracts are in fact efficient in this case, since contracts do not affect the revelation of information. Hence, even if the equilibrium continues to involve pooling, rather than separating wages, the adverse selection problem is absent.

Note that, in this case, the resulting wage posting is not a symptom of a bad job, but rather a reflection of the fact that the form of wage contracts does not influence the revelation of information. From an empirical standpoint, the problem is that wage posting may reflect the case where large firms are solving the problem, or simply the case where productivity does not vary across workers or matches. Addressing the extent to which wage posting is a symptom of bad jobs seems a fruitful area of future research.

In general, however, firms may not be able to internalize the relevant externalities, either because they are not sufficiently large or because they suffer from other problems.[12] For instance, Milgrom and Oster (1987) argue why informational frictions may give employers and incentive to discriminate against certain groups of workers by hiding their productivity from other potential employers.

In principle, the problem we have highlighted will disappear if employers can commit not to counter outside offers. However, depending on how this is achieved, it may give rise to other problems. For instance, if a group of firms enter a no-poaching agreement, as it happens in some markets, then the adverse selection problem disappears, but this is so at the expense of job mobility, which hurts workers. Note that, in practice, this can happen across widely different occupations. For example, note that eight fast food companies, including Burger King, Dunkin' Donuts and Five Guys, are the subject of investigations of 11 state attorney generals concerning the imposition of non-poaching agreements about low wage workers (Washington Post, July 12, 2018, "7 fast food chains agree to drop 'no-poaching' clauses"). Similarly, Silicon Valley firms, including Apple and Google, recently paid $ 415 million to settle a suit concerning non-poaching agreements regarding

[12] See, for example, Nalbantian and Guzzo (2009) for a comparison of successful and unsuccessful cases of internal career mobility programs.

engineers (Fortune, July 12, 2015, "Tech workers will get average of $ 5,770 under final anti-poaching settlement").

Note that taxing retention offers, or job mobility more generally, is not a solution either, since the possibility of some retention offers, like job mobility in some cases, is efficiency enhancing. In particular, the threat of retention offers is important in preventing employers from inefficiently expending resources on jobs designed to poach workers who are already well matched.

When bad jobs are associated with underinvestment of labor, taxing employers who underinvestment in labor can be a useful tool to combat the problem of bad jobs. Essentially, by making the creation of these jobs more costly it is possible to destroy the bad jobs equilibrium, thereby enabling the market to coordinate on an equilibrium where firms invest in labor. In a world with heterogeneity, however, the implementation of such policies is complicated by the fact that they backfire to the extent that they impact good, but low-productivity, jobs instead of bad jobs. Furthermore, the nature of underinvestment may vary across markets, which may call for differing policy responses across markets. By contrast, note that subsidizing certain investments in labor can increase the opportunity cost of creating bad jobs without interfering with the creation of good jobs.

Addressing the problem of bad jobs when the problem is the contract requires different tools. In this case, it seems that a minimum wage can be a particularly useful tool to combat the problem of bad jobs. Since the wage in non-revealing contracts is low, the minimum wage increases the cost of offering these contracts, thereby enabling the market to coordinate on revealing contracts. Again, however, the difficulty lies in implementation. In particular, a minimum wage is not the most appropriate tool to address the problem of underinvestment in labor in our setting. *Ex post*, workers employed with type-1 and type-2 employers will have the same range of productivity realizations. The difference is just that the latter are more likely to be employed in an unproductive match. In order to increase the cost of creating type-2 jobs, however, the minimum wage must exceed y_l, as this is the wage received by low productivity workers in type-2 jobs. However, since this is also the wage earned by low productivity workers in type-1 jobs, raising the minimum wage above y_l will distort both good and bad matches.

Our model suggests there is room for closer integration of vocational training and work, and for some cooperation among employers designed to improve career prospects associated with some jobs. For example, government coordinated programs under which employers pay some or all of the cost of training apprentices, even if those apprentices end up working elsewhere, help solve the adverse selection problem. This type of approach appears to have been successful in eliciting the creation of good jobs in Germany. Furthermore, the fact that employers appear to believe these programs to be beneficial to them suggests that they alter the equilibrium so as to render the creation of good jobs incentive compatible to employers (The Atlantic, Oct 16, 2014, "Why Germany Is So Much Better at Training Its Workers.").

Our theory of bad jobs may help to understand why traditional government policy, including job-training programs and unemployment insurance, sometimes struggle to produce expected results. For example, from the viewpoint of human capital theory, the impact of job training programs has been somewhat disappointing, in the sense that they have typically failed to have significant wage impacts in the short run (Card et al., 2017). Similarly, random-matching models suggest that unemployment insurance may improve match quality (Marimon and Zilibotti, 1999, Acemoglu and Shimer, 2000 and Acemoglu, 2001). The prevalence of low-quality, temporary jobs in countries with relatively generous unemployment insurance such as Spain and France is problematic for these theories. From our perspective, these policies have not been successful because they do not address the underlying adverse selection problem, and therefore may fail to mitigate the creation of bad jobs. Finally, note that simple redistributive schemes, while potentially desirable for other reasons, will not eliminate the existence of bad jobs in our setting.

5.4 Bad jobs and discrimination

That the issue of bad jobs is more than an academic issue can also be seen by considering how other important issues, such as equality of opportunity and discrimination, interact with the problem of bad jobs. To examine these issues, we extend our model to incorporate unobserved heterogeneity. The heterogeneity we are interested in here is unrelated to worker productivity.

Precisely, suppose that unemployed workers face a cost of searching for jobs offered by type-2 employers. Formally, suppose that an unemployed worker incurs a cost c if she searches for type-2 jobs in a given period, where c is the realization of a random variable that is independent across workers and over time. For simplicity, we assume that c is a drawn from an exponential distribution: $F(c) = 1 - \exp\{-\theta c\}$, for $c \geq 0$, with $\theta > 0$.

It is easy to verify that an equilibrium allocation must solve the obvious analogues of Problems (P1) and (P2). First, note that our assumptions about counteroffers continue to restrict the possible job and wage transitions as explained in Section 3. It is straightforward to verify that any allocation supported by an equilibrium with positive quits must be such that employed workers only ever search for type-1 jobs. Intuitively, workers are expected to be more productive in type-1 jobs and, consequently, type-1 employers always drive type-2 employers out of any market where employed workers search.

Taking this into account, Problem (P1), with $j = 1$, can be used to characterize equilibrium allocations, except that now it ought to be recognized that the value functions and the corresponding policy functions are functions of (ρ_1, ρ_2), rather than simply ρ, where $\rho_i \in \{1 - \alpha_i, 1\}$ denotes the fraction of poorly matched workers currently employed in type-i jobs among all those searching for type-1 jobs, for $i = 1, 2$. With a slight abuse of notation we will continue to denote those functions as before.

One can verify that equilibrium allocations must satisfy the analogue of Problem (P2), with

$$\overline{V}(s_u, \rho_1, \rho_2) = b + \max\{V_1, V_2\}, \tag{17}$$

where V_i is given by Problem (P2) with $\rho = \rho_i$, for $i = 1, 2$.

In order to characterize an equilibrium allocation, first note that, due to the technological inferiority of type-2 jobs, if wages in type-1 matches reveal productivity, the equilibrium is constrained efficient. In this case, no type-2 jobs are created, the equilibrium allocation solves (P1) and (P2), with $\rho = 1$, and is as given by Proposition 2, with $j = 1$. Note that, if wages in both types of jobs are revealing, then neither type of job suffers from the adverse selection problem. In this case, the higher productivity of type-1 jobs makes them more attractive to all searchers. If wages in type-1 jobs are revealing, but wages in type-2 jobs are non-revealing, then type-2 jobs, in addition to being less productive, also suffer from the adverse selection problem. Clearly, the allocation in a revealing equilibrium is unique within the class of revealing equilibria, but it can be supported in a continuum of different ways.

Thus, the existence of an equilibrium where type-1 and type-2 employers coexist in the market for unemployed workers requires that wages in type-1 jobs are non-revealing, in which case on-the-job search from these jobs suffers from the adverse selection problem created by well-matched workers searching for retention offers. The key to understanding these equilibria is to note that the adverse selection problem is more severe in markets for type-1 jobs, because workers in these jobs are relatively less likely to be poorly matched, and therefore more likely to be searching on the job in order to elicit retention offers than are workers in type-2 jobs. Potential poaching firms understand this and, consequently, are less willing to enter markets where workers in type-1 jobs search on the job. This lowers the value of on-the-job search for workers in type-1 jobs. When this effect is sufficiently strong, type-2 jobs have an equilibrium advantage over type-1 jobs despite being inferior along traditional technical dimensions.

One can prove the following.

Proposition 6 *Maintain the assumptions of Proposition 2 and 3. There are numbers* $\widehat{\alpha} \in (0, \alpha_1)$ *and* $\widehat{k} > 0$ *such that for all* $\alpha_2 \in (\widehat{\alpha}, \alpha_1)$ *and all* $k \in \left(0, \widehat{k}\right)$ *there is a refined equilibrium in which some unemployed workers search for jobs with revealing contracts offered by type-*2 *employers, whereas other unemployed workers search for jobs with non-revealing contracts offered by type-*1 *employers.*

The proposition characterizes an equilibrium in which bad type-1 and type-2 employers coexist. In equilibrium, all unemployed workers would prefer to search for jobs with revealing contracts offered by type-2 employers, if they were not facing search costs. However, the i.i.d. realization of costs of searching for jobs with type-2 employers implies that some unemployed workers search for type-1 jobs. These jobs are superior in a technological sense but, as a consequence of being offered under non-revealing contracts, come with sufficiently poor career prospects that they are actually worse than the available type-2 jobs. Both jobs created are bad jobs, in the sense that there exists an equilibrium in which type-1 employers offer jobs with revealing contracts to unemployed workers, and all workers would prefer those jobs to the ones they search for in equilibrium.

It is easy to see that the unemployed workers' optimal search policy is characterized by a cutoff c_0 such that they search for type-2 jobs if and only if their current realization of the search cost c is smaller than the cutoff. Noting that unemployed workers will search for type-2 jobs if and only if their idiosyncratic search cost c is smaller than the utility gain $V_2 - V_1$, we have that

$$\overline{V}(s_u, 1-\alpha_1, 1) - b = V_1 + F(c_0)(V_2 - \mathbb{E}(c|c \le c_0) - V_1). \tag{18}$$

That is, the net value of unemployment ($\overline{V}(s_u, 1-\alpha_1, 1) - b$) to a worker is equal to the value of searching for jobs where she is more productive (V_1) plus the option value of searching for jobs where she is less productive, which consists of the expected utility gain $F(c_0)(V_2 - V_1)$ minus the expected search costs $F(c_0)\mathbb{E}(c|c \le c_0)$.

Since searching for type-1 jobs is costless, an equilibrium allocation must have $V_2 - V_1 \le c_0$, with equality if and only if $V_2 \ge V_1$. Thus, either the option value of searching for type-2 jobs is non-negative, or else only type-1 jobs are created in equilibrium. In this sense constructing the equilibrium is non-trivial: it requires that the value of search for jobs where the unemployed is relatively less productive to be relatively higher in equilibrium. That is, it requires that $c_0 > 0$. The assumption that $\alpha_2 \in (\widehat{\alpha}, \alpha_1)$ in the proposition ensures that the two types of jobs are sufficiently similar that unemployed workers would strictly prefer to search for type-2 jobs if it were costless to do so. It is clear that there is a number $\widehat{\alpha} \in (0, \alpha_1)$ such that this is the case.

In the equilibrium with two types of entry jobs, workers in type-1 and type-2 jobs conduct on-the-job search on separate markets, because job type is an observable component of a worker's labor market state. It is then easy to see that the equilibrium allocation is such that V_i is given by Problem (P2) subject to (18), for $i = 1, 2$, with $\rho = 1 - \alpha_1$ for $i = 1$, and $\rho = 1$ for $i = 2$.

Suppose the search costs are viewed as credit constraints. In this case the model can be viewed as capturing differences in equality of opportunity, in the sense that some workers face external demands that require them to prioritize current over lifetime earnings. This interacts with the issue of job quality because, even though both jobs in this equilibrium are bad jobs (type-2 jobs because they are low productivity and type-1 jobs because come with poor career prospects), they can be ranked ex-anted. In particular, unconstrained workers prefer the type-2 jobs and only choose to apply to type-1 jobs to the extent that they face constraints that make searching for type-2 jobs more costly. That the jobs can be ranked implies that inequality of opportunity matters, in that workers with unfavorable starting conditions may be forced to take worse jobs than workers in more favorable starting conditions. That both jobs are bad distinguishes our equilibrium from a standard efficient sorting perspective on this issue: in our equilibrium, inequality of opportunity would be eliminated if the market were able to coordinate on the production of good jobs, in which both current productivity and career prospects are high.

Under this interpretation, the equilibrium characterized in Proposition 6 can be viewed as capturing a situation akin to that facing workers with a law degree or an accounting degree. Such workers can choose to take internships or even entry jobs in law firms and accounting firms, for example, where jobs at the bottom of the ladder are characterized by low productivity and long hours, but career prospects are good (The Atlantic, May 10, 2012, "Unpaid Internships: Bad for Students, Bad for Workers, Bad for Society"). These would correspond to type-2 employers in our equilibrium. Such workers could search for jobs in smaller firms in which they are more productive today, but which offer less attractive career prospects. These would correspond to type-1 employers in our equilibrium. Note that while jobs in large law and accountancy firms, which we are calling type-2 jobs, are commonly viewed as good jobs, most likely because they are only available to highly educated workers and offer good career prospects, they are bad jobs in this equilibrium.

This extension can also be used to examine issues related to discrimination and gender. For example, suppose that type 1 employers are those which invest in labor in the sense of organizing the workplace to allow greater flexibility in terms of hour. Type 2 employers, on the other hand, do not invest in labor, and consequently workers must work inflexible hours. Now, modify our previous model to allow for two different sources of heterogeneity. First, suppose that workers are either men or women. Gender is observable, but it is productivity irrelevant in that all workers have the same ability. Second, suppose that unemployed women face a cost of accepting jobs offered by type-2 employers, because these jobs involve long and inflexible hours of work, and make family-work balance costly. For simplicity, suppose this cost is just paid once. Formally, suppose that an unemployed woman incurs a fixed cost c if she accepts a type-2 job in a given period. As above, we assume that c is a drawn from an exponential distribution: $F(c) = 1 - \exp\{-\theta c\}$, for $c \geq 0$, with $\theta > 0$, where is independent across workers and over time.

The observability of gender means that the economy generates separate markets by gender. There exists and equilibrium in which all men search for type-2 jobs with revealing contracts and, as the cost of posting a vacancy goes to zero, the labor market for women exhibits the equilibrium described by Proposition 6. In words, all men and some women search for jobs with poor working conditions, but good career prospects, while other women search for jobs with more flexible working conditions but poor career prospects. As before, both jobs are bad jobs, but type-2 jobs would be preferred by all workers in absence of the idiosyncratic desire for workplace flexibility.

What then should one make of the observation that some occupations heavily reward the investment of long working hours at the start of a worker's career (Goldin, 2014)? One possibility, is that inflexible working hours are a technological requirement of these jobs, in which case it may be that there is no way to create jobs with more flexible working hours in an equilibrium. If this is the case, these jobs are indeed good jobs. Our theory, however, suggests that the long and inflexible working hours observed in these jobs could be equilibrium objects, rather than technological constraints, which opens up the possibility that these jobs could be improved on within the context of an equilibrium. This view is supported by developments in other markets, in which firms have managed to move toward the creation of jobs with more flexible working hours without requiring a tradeoff in either pay or job mobility. For example, pharmacists can now choose to work shorter or longer hours at no penalty, and larger medical practice organizations tend to provide physicians with greater schedule and career flexibility (Briscoe, 2006).

This is relevant from a policy standpoint if the hours worked in those jobs are excluding women or exacerbating the gender wage gap, as argued by Goldin (2014). If the technological constraints interpretation is correct then both jobs are good, and the gender gap is fundamentally a reflection of different preferences on the part of men and women over those jobs. Our interpretation, however, suggests that the problem of bad jobs interacts with differences in preferences in a way that disproportionately affects women. Importantly, it also implies that better jobs, for both women and men, can be implemented as an equilibrium outcome.

However, to the extent that the problem is a bad jobs problem, simple anti-discrimination or female-friendly regulations are not likely to solve the problem. Anti-discrimination policy, per se, does nothing to guarantee that the market coordinates on good jobs. In our equilibrium, a requirement that firms offer the same jobs to men and women may well result in both men and women seeking jobs created by type-2 employers offering separating contracts, which are bad jobs for everyone: this policy reinforces a situation that penalizes women. Similarly, narrowly focusing on improving job flexibility in the example above may result in both men and women seeking jobs created by type-1 employers offering pooling contracts, which are also bad jobs for everyone: this solution neglects the distinction between job quality and specific job characteristics. What are needed are policies that better allow firms to coordinate on equilibria in which good jobs are created for both men and women.

6 Conclusion

In this paper we have developed a theory of bad jobs driven by the interaction of job creation and job mobility in the presence of asymmetric information about workers' outside options. Specifically, we argue that bad jobs are a symptom of coordination failure stemming from a conflict between the signaling and allocative roles of wage contracts.

In the process of developing our theory, we have proposed a definition of bad jobs: a job that is part of an equilibrium allocation is bad for a worker if there is an equilibrium allocation in which a comparable worker can do better. We believe our definition of bad jobs is of general interest, independently of our concrete theory of bad jobs. Furthermore, we identify two broad categories of bad jobs: jobs with suboptimal career prospects and jobs characterized by employers' underinvestment in labor.

Our analysis calls for further research into the economics of bad jobs. Perhaps the biggest challenge ahead is to develop methods to identify job quality in practice. As our definition makes clear, the difficulty lies in determining what is actually viable for a given worker. On the empirical side, our model suggests that trying to identify changes in equilibrium incentives within a market over time, or across markets seems fruitful. On the theoretical side, we have characterized one source of bad jobs, but there may be others. One implication of our definition of bad jobs is that future research ought to consider the possibility of multiple labor market equilibria seriously. Of course, understanding the source of the multiplicity of equilibria matters both for understanding the source of bad jobs and the appropriate policy response. In any case, it should be noted that non-targeted labor market policies will not only fail to address the problem of bad jobs we have highlighted here, but they may well backfire.

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Appendix

Proof of Proposition 1

Consider a competitive search equilibrium with positive quits. If the equilibrium allocation does not solve problems (P1) and (P2), for a given value of $\rho \in (0,1]$, it is easy to see that the proposed equilibrium must violate the condition of the equilibrium refinement given in Definition 2. Hence, it cannot be a refined equilibrium. The discussion leading to Proposition 1 implies that a revealing equilibrium must have $\rho = 1$ and a non-revealing equilibrium must have $\rho = 1 - \alpha_i$, as required. **QED**

Proof of Proposition 2

Throughout this proof we maintain the assumption that $\rho = 1$ and we drop the argument ρ from all functions. We keep track of employer types under the assumption that unemployed workers search for jobs posted by type-i employers and employed workers search for jobs posted by type-j employers, where it is understood that $i = j = 1$ throughout this proof. We first prove existence and uniqueness of the candidate equilibrium allocation. It will become clear that it can be supported by a revealing equilibrium. Then, we show that the revealing equilibrium is constrained efficient.

We begin by characterizing the solution to Problem (P1) as a function of a worker's wage.

Lemma 1 *Let $s = \{i, w, y_l\}$. For any $w \in [0, y_l]$, $\{w_e(s), q_e(s)\}$ is given by the unique pair (w', q') with $y_l \leq w' < y_h$ and $0 < q_a \leq q \leq q_b < \infty$ that solves the following conditions:*

$$q' f(q') \alpha_j \left(\frac{y_h - w'}{r + \delta} \right) = k,$$

$$\frac{w' - w}{r + \delta + (1 - \delta) \alpha_j f(q')} \geq \left(\frac{1 - \eta(q')}{\eta(q')} \right) \left(\frac{y_h - w'}{r + \delta} \right)$$

and $q' \geq q_a$ with complementary slackness, where q_a is given by

$$q_a f(q_a) \alpha_j \left(\frac{y_h - y_l}{r + \delta} \right) = k$$

and $q_b > q_a$ is given by

$$\frac{y_h - y_l}{r + \delta} = \left(\frac{k}{q_b f(q_b) \alpha_j} \right) \left(1 + \left(\frac{1 - \eta(q_b)}{\eta(q_b)} \right) \left(\frac{r + \delta + (1 - \delta) \alpha_j f(q_b)}{r + \delta} \right) \right). \tag{19}$$

Proof: The first-order conditions for an interior solution of problem (P1) with $\rho = 1$ are given by:

$$\lambda q' = 1,$$

where λ is the relevant Lagrange multiplier, and

$$\frac{w'}{r + \delta} + \left(\frac{\delta}{r + \delta} \right) \frac{\overline{V}(s_u)}{1 + r} - \frac{\overline{V}(\{i, w, y_l\})}{1 + r} = \lambda q' \left(\frac{1 - \eta(q')}{\eta(q')} \right) \left(\frac{y_h - w'}{r + \delta} \right),$$

together with the zero-profit constraint

$$q'f\left(q'\right)\alpha_j\left(\frac{y_h-w'}{r+\delta}\right)=k.$$

This is the first condition stated in the lemma. The second condition follows from combining the first two first-order conditions above and the fact that the Bellman equation implies that a solution to the problem must be such that

$$\frac{w'}{r+\delta}+\left(\frac{\delta}{r+\delta}\right)\frac{\overline{V}\left(s_u\right)}{1+r}-\frac{\overline{V}\left(\{i,w,y_l\}\right)}{1+r}=\frac{w'-w}{r+\delta+(1-\delta)\,\alpha_j f\left(q'\right)}.$$

Clearly, $w_e\left(\{i,w,y_l\}\right)\geq y_l$ if and only if $q_e\left(\{i,w,y_l\}\right)\geq q_a$. Our assumption that $(r+\delta)\,k<\alpha_2\left(y_h-y_l\right)$ ensures that $0<q_a<\infty$.

Combining the two conditions stated in the proposition implies that an interior solution $q_e\left(\{i,w,y_l\}\right)$ is the unique value of q' that solves

$$\frac{y_h-w}{r+\delta}=\left(\frac{k}{q'f\left(q'\right)\alpha_j}\right)\left(1+\left(\frac{1-\eta\left(q'\right)}{\eta\left(q'\right)}\right)\left(\frac{r+\delta+(1-\delta)\,\alpha_j f\left(q'\right)}{r+\delta}\right)\right). \tag{20}$$

It follows that $w\leq y_l$ implies that $q_e\left(\{i,w,y_l\}\right)\leq q_b$. Clearly, $\infty>q_b>q_a>0$. **QED**

Invert (20) to express the worker's current wage as a function of q':

$$W\left(q'\right)\equiv y_h-\left(\frac{k}{q'f\left(q'\right)\alpha_j}\right)\left(r+\delta+\left(\frac{1-\eta\left(q'\right)}{\eta\left(q'\right)}\right)\left(r+\delta+(1-\delta)\,\alpha_j f\left(q'\right)\right)\right), \tag{21}$$

for all $q'\in\left[q_a,q_b\right]$, and note the following.

Lemma 2 *$W(q)$ and $\overline{V}\left(\{i,W(q),y_l\}\right)$ are strictly increasing and concave functions of q on $\left[q_a,q_b\right]$.*

Proof: It is easy to verify that the Bellman equation for $\overline{V}\left(\{i,w,y_l\}\right)$ implies that

$$\begin{aligned}\frac{\overline{V}\left(\{i,W(q),y_l\}\right)}{1+r}&=\left(\frac{\delta}{r+\delta}\right)\frac{\overline{V}\left(s_u\right)}{1+r}+\left(\frac{r+\delta}{r+\delta+(1-\delta)\,\alpha_j f(q)}\right)\frac{W(q)}{r+\delta}\\&\quad+\left(1-\frac{r+\delta}{r+\delta+(1-\delta)\,\alpha_j f(q)}\right)\frac{w_e\left(\{i,W(q),y_l\}\right)}{r+\delta}\end{aligned} \tag{22}$$

and, using the first-order conditions stated in Lemma 1, one can write

$$\frac{\overline{V}\left(\{i,W(q),y_l\}\right)}{1+r}=\frac{y_h}{r+\delta}+\left(\frac{\delta}{r+\delta}\right)\frac{\overline{V}\left(s_u\right)}{1+r}-\frac{k}{\eta(q)\,qf(q)\,\alpha_j}. \tag{23}$$

One can verify that

$$\frac{\partial}{\partial q}\left(\frac{\overline{V}\left(\{i,W(q),y_l\}\right)}{1+r}\right)=\frac{k}{qf(q)\,\alpha_j}\left(\frac{\eta'(q)}{\left(\eta(q)\right)^2}+\frac{1}{q}\left(\frac{1-\eta(q)}{\eta(q)}\right)\right),$$

which is positive on $[q_a, q_b]$. A sufficient condition for it to be strictly decreasing on $[q_a, q_b]$ is that $\eta'(q) / (\eta(q))^2$ is a decreasing function, which follows from the concavity of η. Hence $\overline{V}(\{i, W(q), y_l\})$ is strictly concave on $[q_a, q_b]$, as required.

Next, differentiating equation (20) with respect to w and q one can verify that

$$\frac{\partial W(q)}{\partial q} = (r + \delta + (1-\delta)\alpha_j f(q)) \frac{\partial}{\partial q}\left(\frac{\overline{V}(\{i, W(q), y_l\})}{1+r}\right),$$

which is positive and strictly decreasing on $[q_a, q_b]$, because both f and $\partial\overline{V}/\partial q$ are positive and strictly decreasing on $[q_a, q_b]$. Hence, $W(q)$ is strictly increasing and concave *on* $[q_a, q_b]$, as required. **QED**

Let $M(s)$ denote the match surplus as a function of the worker's state and note that

$$\frac{M(\{i, w, y_h\})}{1+r} = \frac{\overline{V}(\{i, w, y_h\})}{1+r} - \frac{\overline{V}(s_u)}{1+r} + \frac{y_h - w}{r+\delta} \tag{24}$$

and

$$\frac{M(\{i, W(q), y_l\})}{1+r} = \frac{\overline{V}(\{i, W(q), y_l\})}{1+r} - \frac{\overline{V}(s_u)}{1+r} + \frac{y_l - W(q)}{r+\delta+(1-\delta)\alpha_j f(q)}. \tag{25}$$

Lemma 3 *$M(\{i, w, y_h\})$ is independent of w; $M(\{i, W(q), y_l\})$ is a strictly concave function of q on $[q_a, q_b]$ and it is maximized at $q = q_b$; $M(\{i, W(q), y_l\}) - \overline{V}(\{i, W(q), y_l\})$ is a strictly decreasing and convex function of q on $[q_a, q_b]$.*

Proof: Fix $\overline{V}(s_u)$. Noting that

$$\frac{\overline{V}(\{i, w, y_h\})}{1+r} = \frac{w}{r+\delta} + \frac{\delta}{r+\delta}\frac{\overline{V}(s_u)}{1+r}$$

one can write

$$\frac{M(\{i, w, y_h\})}{1+r} = \frac{y_h}{r+\delta} - \frac{r\overline{V}(s_u)}{1+r},$$

which is independent of q. Using (23), together with (21) and (25), one can write

$$\begin{aligned}\frac{M(\{i, W(q), y_l\})}{1+r} = {} & \frac{y_h}{r+\delta} - \frac{r\overline{V}(s_u)}{1+r} - \frac{r+\delta}{r+\delta+(1-\delta)\alpha_j f(q)}\left(\frac{y_h - y_l}{r+\delta}\right) \\ & - \left(1 - \frac{r+\delta}{r+\delta+(1-\delta)\alpha_j f(q)}\right)\left(\frac{k}{q f(q)\alpha_j}\right).\end{aligned} \tag{26}$$

where $M(\{i, w, y_h\}) > M(\{i, W(q), y_l\})$ whenever $y_h > y_l$. Differentiating equation (26) one can verify that

$$\frac{\partial}{\partial q}\left(\frac{M(s)}{1+r}\right)=\left(\frac{1-\delta}{q^2\left[r+\delta+(1-\delta)\alpha_j f(q)\right]}\right)$$
$$\times\left((1-\eta(q))k-\left(\frac{(r+\delta)\eta(q)}{r+\delta+(1-\delta)\alpha_j f(q)}\right)\left(qf(q)\alpha_j\left(\frac{y_h-y_l}{r+\delta}\right)-k\right)\right).$$

for $s=\{i,W(q),y_l\}$. The term in the first line is decreasing in q since both $qf(q)$ are strictly increasing on $[q_a,q_b]$. The terms in the second line are also decreasing in q since $f(q)$ is decreasing and $\eta(q)$ and $qf(q)$ are increasing on $[q_a,q_b]$, and $qf(q)\alpha_j(y_h-y_l)\geq(r+\delta)k$ for $q\geq q_a$. Hence $M(\{i,W(q),y_l\})$ is strictly concave on $[q_a,q_b]$. It is now easy to verify that equation (19) is a necessary and sufficient condition for $\partial M(\{i,W(q),y_l\})/\partial q=0$. Hence $M(\{i,W(q),y_l\})$ is maximized at $q=q_b$.

Using equations (23) and (26) one can write

$$\frac{M(s)-\left(\overline{V}(s)-\overline{V}(s_u)\right)}{1+r}=\left(\frac{k}{qf(q)\alpha_j}\right)\left(\frac{r+\delta}{r+\delta+(1-\delta)\alpha_j f(q)}+\frac{1-\eta(q)}{\eta(q)}\right)$$
$$-\left(\frac{y_h-y_l}{r+\delta+(1-\delta)\alpha_j f(q)}\right),$$

for $s=\{i,W(q),y_l\}$, and differentiating this equation one can verify that

$$\frac{\partial}{\partial q}\left(\frac{M(s)-\left(\overline{V}(s)-\overline{V}(s_u)\right)}{1+r}\right)=\left(\frac{(1-\delta)\alpha_1 f'(q)}{\left[r+\delta+(1-\delta)\alpha_j f(q)\right]^2}\right)\left(y_h-y_l-\frac{(r+\delta)k}{qf(q)\alpha_j}\right)$$
$$-\left(\frac{k}{qf(q)\alpha_j}\right)\left(\left(\frac{1-\eta(q)}{q}\right)\left(\frac{r+\delta}{r+\delta+(1-\delta)\alpha_j f(q)}+\frac{1-\eta(q)}{\eta(q)}\right)+\frac{\eta'(q)}{(\eta(q))^2}\right),$$

for $s=\{i,W(q),y_l\}$. The term in the first line of the right side is negative since $f'(q)<0$ and $qf(q)\alpha_j(y_h-y_l)\geq(r+\delta)k$ for $q\geq q_a$. The term subtracted in the second line is positive since $\eta(q)<1$ and $\eta'(q)>0$. Hence $M(s)-\left(\overline{V}(s)-\overline{V}(s_u)\right)$, for $s=\{i,W(q),y_l\}$, is a strictly decreasing function of q on $[q_a,q_b]$. Moreover, the term in the first line of the right side is an increasing function of q, because $f'(q)$ and $qf(q)$ are increasing and $f(q)$ is decreasing. The term subtracted in the second line is a decreasing function of q, since $qf(q)$ and $\eta(q)$ are increasing and $f(q)$ and $\eta'(q)/(\eta(q))^2$ are decreasing. Hence, $M(s)-\left(\overline{V}(s)-\overline{V}(s_u)\right)$, for $s=\{i,W(q),y_l\}$, is a strictly convex function of q. **QED**

Next, note that Problem (P2) can be formulated as

$$\overline{V}(s_u)=b+V_i, \tag{P3}$$

where

$$V_i=\frac{\overline{V}(s_u)}{1+r}+\max_{w,q,q'}\left\{f(q)\left(\frac{V_0(i,w,q')}{1+r}-\frac{\overline{V}(s_u)}{1+r}\right)\right\}$$

subject to

$$k \leq qf(q)\left(\frac{M_0(i,w,q')}{1+r} - \frac{V_0(i,w,q')}{1+r} + \frac{\overline{V}(s_u)}{1+r}\right),$$
$$q' \in [q_a, q_b]\ ,\ w \leq y_h,\ w \neq W(q')$$

where

$$V_0(i,w,q') = \alpha_i \overline{V}(\{i,w,y_h\}) + (1-\alpha_i)\overline{V}(\{i,W(q'),y_l\}),$$

and

$$M_0(i,w,q') = \alpha_i M(\{i,w,y_h\}) + (1-\alpha_i) M(\{i,W(q'),y_l\}).$$

With a slight abuse of notation, we let $\{w_u^h(i), q_u(i), q_e^l(i)\}$ denote a solution to Problem (P3) while disregarding the constraint $w \neq W(q')$. Even though the objective is not concave in $\{w,q,q'\}$, we prove below that the solution is unique (and it is such that $w_u^h(i) \neq W(q_e^l(i))$). It is then easy to see that $\{w_u^h(i), W(q_e^l(i)), q_u(i)\}$ solves problem (P2), since $q_e^l(i) = q_e(\{i, W(q_e^l(i)), y_l\})$.

One can readily verify that an *interior* solution to Problem (P3) is such that the total surplus of the match is maximized. Specifically, it must be that $\partial M_0(i,w,q')/\partial q' = 0$, which requires that $\partial M(\{i,W(q'),y_l\})/\partial q' = 0$. Hence, Lemma 3 implies that $q_e^l(i) = q_b$, where q_b is given by equation (19). Comparing (19) and (20), it follows that $W(q_e^l(i)) = y_l$. Hence, $w_u^l(i,1) = y_l$ as indicated in (6).

Next, note that

$$V_i = (1 - f(q_u(i)))\frac{\overline{V}(s_u)}{1+r} + f(q_u(i))\frac{V_0(i, w_u^h(i), q_e^l(i))}{1+r}$$
$$= (1 - f(q_u(i)))\frac{\overline{V}(s_u)}{1+r} + f(q_u(i))\left(\frac{\overline{V}(s_u)}{1+r} + \left(\frac{1-\eta(q_u(i))}{\eta(q_u(i))}\right)\frac{k}{q_u(i) f(q_u(i))}\right),$$

where the first equality comes from the Bellman equation in Problem (P3) and the second equality follows from the matching-efficiency condition (10) and the zero-profit condition (9). It follows that

$$V_i = \frac{\overline{V}(s_u)}{1+r} + \left(\frac{1-\eta(q_u(i))}{\eta(q_u(i))}\right)\frac{k}{q_u(i)},$$

which, together with the fact that $\overline{V}(s_u) - b = V_i$, implies that

$$\frac{r\overline{V}(s_u)}{1+r} = b + \left(\frac{1-\eta(q_u(i))}{\eta(q_u(i))}\right)\frac{k}{q_u(i)}.$$

Using this equation, together with equations (9) and (10) and the fact that

$$\frac{V_0\left(i, w_u^h(i), q_e^l(i)\right)}{1+r} - \frac{\overline{V}(s_u)}{1+r} = \alpha_i \frac{w_u^h(i)}{r+\delta} + (1-\alpha_i)\left(\frac{y_h}{r+\delta} - \frac{k}{\eta\left(q_e^l(i)\right) q_e^l(i) f\left(q_e^l(i)\right)\alpha_j}\right) - \left(\frac{r}{r+\delta}\right)\frac{\overline{V}(s_u)}{1+r},$$

it follows that $q_u(i)$ satisfies equation (11) in the text.

The right side of (11) is strictly decreasing in $q_u(i)$, it converges to ∞ as $q_u(i)$ approaches 0 and it converges to k as $q_u(i)$ approaches ∞. Hence, there is a unique solution $q_u(i) \in (0,\infty)$ that solves the equation if and only if

$$\frac{y_h - b}{r+\delta} - \frac{(1-\alpha_i)k}{\eta(q_b) q_b f(q_b)\alpha_j} > k.$$

There is a number $k_a > 0$ such that this inequality holds for all $k \in (0, k_a)$. To prove this, differentiate (19) to verify that

$$\frac{\partial q_b}{\partial k} > 0 \text{ and } \frac{\partial}{\partial k}\left(\frac{k}{\eta(q_b) q_b f(q_b)}\right) > 0,$$

with

$$\lim_{k\to 0} q_b = 0 \text{ and } \lim_{k\to 0}\left\{\frac{k}{\eta(q_b) q_b f(q_b)\alpha_j}\right\} = \frac{y_h - y_l}{r+\delta+(1-\delta)\alpha_j} < \frac{y_h - y_l}{r+\delta} < \frac{y_h - b}{r+\delta}.$$

Next, we verify that $\overline{V}(s_u) \leq \min\left\{\overline{V}\left(\left\{i, w_u^h(i), y_h\right\}\right), \overline{V}\left(\{i, y_l, y_l\}\right)\right\}$. To that end, note that

$$\overline{V}(\{i, y_l, y_l\}) - \overline{V}(s_u) = V_0\left(i, w_u^h(i), q_e^l(i)\right) - \overline{V}(s_u) - \alpha_i\left[\overline{V}\left(\left\{i, w_u^h(i), y_h\right\}\right) - \overline{V}(\{i, y_l, y_l\})\right]$$

and

$$\overline{V}\left(\left\{i, w_u^h(i), y_h\right\}\right) - \overline{V}(s_u) = V_0\left(j, w_u^h(i), q_e^l(i)\right) - \overline{V}(s_u) + (1-\alpha_i)\left[\overline{V}\left(\left\{i, w_u^h(i), y_h\right\}\right) - \overline{V}(\{i, y_l, y_l\})\right],$$

where

$$V_0\left(i, w_u^h(i), q_e^l(i)\right) = \alpha_i \overline{V}\left(\left\{i, w_u^h(i), y_h\right\}\right) + (1-\alpha_j)\overline{V}\left(\left\{i, w_l\left(j, q_e^l(i)\right), y_l\right\}\right),$$

and use the fact that

$$\frac{V_0\left(i, w_u^h(i), q_e^l(i)\right)}{1+r} - \frac{\overline{V}(s_u)}{1+r} = \left(\frac{1-\eta(q_u(i))}{\eta(q_u(i))}\right)\frac{k}{q_u(i) f(q_u(i))}$$

and the fact that

$$\frac{\overline{V}\left(\left\{i,w_u^h\left(i\right),y_h\right\}\right)}{1+r}-\frac{\overline{V}\left(\left\{i,y_l,y_l\right\}\right)}{1+r}=\frac{k}{\eta\left(q_b\right)q_bf\left(q_b\right)\alpha_j}-\left(\frac{k}{\alpha_iq_u\left(i\right)f\left(q_u\left(i\right)\right)}\right)$$

to write

$$\frac{\overline{V}\left(\left\{i,y_l,y_l\right\}\right)-\overline{V}\left(s_u\right)}{1+r}=\frac{k}{\eta\left(q_u\left(i\right)\right)q_u\left(i\right)f\left(q_u\left(i\right)\right)}-\alpha_i\left(\frac{k}{\eta\left(q_b\right)q_bf\left(q_b\right)\alpha_j}\right), \tag{27}$$

where

$$\frac{\overline{V}\left(\left\{i,w_u^h\left(i\right),y_h\right\}\right)-\overline{V}\left(s_u\right)}{1+r}=\left(\frac{1}{\eta\left(q_u\left(i\right)\right)}-\frac{1}{\alpha_i}\right)\frac{k}{q_u\left(i\right)f\left(q_u\left(i\right)\right)}$$
$$+\left(1-\alpha_i\right)\frac{k}{\eta\left(q_b\right)q_bf\left(q_b\right)\alpha_j}.$$

Differentiating equation (11), one can verify that $\partial q_u\left(i\right)/\partial k>0$, with

$$\lim_{k\to 0}q_u\left(i\right)=\lim_{k\to 0}\left\{\frac{k}{q_u\left(i\right)f\left(q_u\left(i\right)\right)}\right\}=0$$

and

$$\lim_{k\to 0}\left\{\frac{k}{\eta\left(q_u\left(i\right)\right)q_u\left(i\right)f\left(q_u\left(i\right)\right)}\right\}=\left(\frac{r+\delta}{1+r+\delta}\right)\left(\frac{y_h-b}{r+\delta}-\left(1-\alpha_j\right)\frac{y_h-y_l}{r+\delta+\left(1-\delta\right)\alpha_j}\right).$$

It follows that there is a number $k_b>0$ such that $\overline{V}\left(\left\{i,w_u^h\left(i\right),y_h\right\}\right)>\overline{V}\left(s_u\right)$ for all $k\in\left(0,k_b\right)$. Moreover,

$$\lim_{k\to 0}\left\{\frac{\overline{V}\left(\left\{i,y_l,y_l\right\}\right)-\overline{V}\left(s_u\right)}{1+r}\right\}=\frac{y_h-b}{1+r+\delta}-\left(\frac{\alpha_j+r+\delta}{1+r+\delta}\right)\left(\frac{y_h-y_l}{r+\delta+\left(1-\delta\right)\alpha_j}\right).$$

This limit is positive if and only if $\left(y_h-b\right)/\left(y_h-y_l\right)\geq\left(r+\delta+\alpha_j\right)/\left(r+\delta+\left(1-\delta\right)\alpha_j\right)$, which is ensured by the assumption in Proposition 2. It follows that there is a number $k_c>0$ such that $\overline{V}\left(\left\{i,y_l,y_l\right\}\right)>\overline{V}\left(s_u\right)$ for all $k\in\left(0,k_c\right)$.

Furthermore, note that

$$\lim_{k\to 0}\left\{\frac{\overline{V}\left(\left\{i,w_u^h\left(i\right),y_h\right\}\right)}{1+r}-\frac{\overline{V}\left(\left\{i,y_l,y_l\right\}\right)}{1+r}\right\}=\frac{y_h-y_l}{r+\delta+\left(1-\delta\right)\alpha_j}>0,$$

which implies that $\lim_{k\to 0}w_u^h\left(i\right)>y_l$.

The above arguments together imply that there is a number $k_0>0$ such that $k\in\left(0,k_0\right)$ is sufficient for $q_u\left(i\right)\in\left(0,\infty\right)$ and $\overline{V}\left(s_u\right)\leq\min\left\{\overline{V}\left(\left\{i,w_u^h\left(i\right),y_h\right\}\right),\overline{V}\left(\left\{i,y_l,y_l\right\}\right)\right\}$, and $w_u^h\left(i\right)>y_l$. Since $w_u^h\left(i\right)\neq y_l$, equilibrium wages reveal the current productivity of employed workers, as required.

It is straightforward to characterize ψ. The unemployment rate is given by

$$\psi\left(s_u\right)=\frac{\delta}{\delta+f\left(q_u^*\right)},$$

where $q_u^* \equiv q_u(i,1)$. The wage distribution has three mass points: two wages for workers who find jobs out of unemployment — a wage w_h^* for those who are well matched and a wage w_l^* for those who are poorly matched — and a wage w_e^* for those workers who find jobs via search on the job. The mass of workers earning the wage w_l^* is

$$\psi\left(\left\{i,w_l^*,y_l\right\}\right)=\left(\frac{\left(1-\alpha_i\right)f\left(q_u^*\right)}{\delta+\left(1-\delta\right)\alpha_j f\left(q_e^*\right)}\right)\psi\left(s_u\right),$$

where $w_l^* \equiv w_u^l(i,1)=y_l$ and $q_e^* \equiv q_e\left(\left\{i,w_l^*,y_l\right\},1\right)$. The mass of workers earning the wage $w_h^* \equiv w_u^h(i,1)$ is

$$\psi\left(\left\{i,w_h^*,y_h\right\}\right)=\left(\frac{\alpha_i f\left(q_u^*\right)}{\delta}\right)\psi\left(s_u\right)$$

and the mass of workers earning the wage $w_e^* \equiv w_e\left(\left\{i,w_l^*,y_l\right\},1\right)$ is

$$\psi\left(\left\{j,w_e^*,y_h\right\}\right)=\left(\frac{\left(1-\delta\right)\alpha_j f\left(q_e^*\right)}{\delta}\right)\psi\left(\left\{i,w_l^*,y_l\right\}\right).$$

One can verify that $f\left(q_u^*\right)$ is an increasing function of y_l, y_h and α_1, and $f\left(q_e^*\right)$ is an increasing function of $\left(y_h-y_l\right)$ and α_1 in the revealing equilibrium.

It is straightforward to verify that there are functions Q and μ that support the allocation characterized by equations (6)-(9) and (11). If $s \in S^*$, beliefs must be correct and the construction of Q is standard. If $s \notin S^*$, beliefs are arbitrary and we may assume that employers believe that $\mu\left(s\,|x\right)=0$ for all $s \notin S^*$.

Lemma 4 *The revealing equilibrium allocation maximizes the present value of aggregate production net of search costs.*

Proof: First, note that the state of the economy at the beginning of each period can be summarized by $\{u,m\}$, where $u \in [0,1]$ is the measure of unemployed workers, and $m:\{y_l,y_h\}\rightarrow[0,1]$, where $m(y)$ denotes the measure of employed workers with match productivity y. Let $p(y)$ denote the probability with which a match has productivity realization y. Let $x_u(y)$ denote the probability with which a meeting between an unemployed worker and a job is turned into a match given the productivity realization y, and $x_e(y'|y)$ denote the probability with which a meeting between a worker and a job with productivity realization y' is turned into a match given that the worker is currently employed in a job with match productivity y. Finally, let q_u denote the labor market queue where unemployed workers search for jobs, and $q_e(y)$ denote the labor market queue where employed workers search given that they are currently employed in jobs with productivity y.

Aggregate output can be written as:

$$Y(u,m)=bu+\sum_y ym(y)-k\frac{u}{q_u}-(1-\delta)k\sum_y\frac{m(y)}{q_e(y)}. \tag{28}$$

Denote by $\widehat{u}$ the measure of unemployed workers one period ahead, and by $\widehat{m}(y)$ the measure of employed workers with match productivity y one period ahead. Then,

$$\widehat{u} = \left(1 - \sum_y f(q_u)x_u(y)\right) u + \delta \sum_y m(y) \tag{29}$$

and

$$\begin{aligned}\widehat{m}(y) &= p(y)f(q_u)x_u(y)u + (1-\delta)m(y)\left[1 - p(y')f(q_e(y))x_e(y'|y)\right] \\ &\quad + (1-\delta)m(y')p(y)f(q_e(y'))x_e(y|y').\end{aligned} \tag{30}$$

The allocation that maximizes aggregate output net of search costs can be characterized as the solution to the planning problem:

$$J(u,m) = \max_{q_u,x_u,q_e,x_e} \left\{Y(u,m) + \frac{J(\widehat{u},\widehat{m})}{1+r}\right\}, \tag{31}$$

subject to equations (28)–(30). $J(u,m)$ is the unique solution to the planner's problem and can be written as:

$$J(u,m) = J_u u + \sum_y m(y)J_e(y),$$

where

$$J_u = \max_{q_u,x_u}\left\{b - \frac{k}{q_u} + \sum_y p(y)f(q_u)x_u(y)\frac{J_e(y)}{1+r} + \left(1 - \sum_y p(y)f(q_u)x_u(y)\right)\frac{J_u}{1+r}\right\} \tag{32}$$

and

$$\begin{aligned}J_e(y) = \max_{x_e,q_e}\Bigg\{ & y - (1-\delta)\frac{k}{q_e(y)} + \delta\frac{J_u}{1+r} \\ & + (1-\delta)\left(1 - \sum_{y'} p(y')f(q_e(y))x_e(y'|y)\right)\frac{J_e(y)}{1+r} \\ & +(1-\delta)\sum_{y'} p(y')f(q_e(y))x_e(y'|y)\frac{J_e(y')}{1+r}\Bigg\}.\end{aligned} \tag{33}$$

It is easy to verify that at the optimum $q_e(y_h) = \infty$. This implies:

$$J_e(y_h) = y_h + \delta\frac{J_u}{1+r} + (1-\delta)\frac{J_e(y_h)}{1+r} > J_e(y_l). \tag{34}$$

It is also easy to verify that $x_e(y_h|y_l) = 1$ and $x_e(y_l|y_l) \in [0,1]$ at the optimum. This means that the planner's problem has multiple solutions, all of which yield the same optimal value. The multiplicity concerns the probability with which the planner instructs workers to accept or reject lateral job moves. We characterize the solution when $x_e(y_l|y_l) = 0$.

The necessary condition of (33) with respect to $q_e(y_l)$ can be written:

$$\frac{J_e(y_h)}{1+r} - \frac{J_e(y_l)}{1+r} = \frac{k}{\alpha_1 q_e(y_l) f(q_e(y_l))\eta(q_e(y_l))} \tag{35}$$

and the Bellman equation for $J_e(y_l)$ gives:

$$\frac{J_e(y_l)}{1+r} = \frac{1}{r+\delta+(1-\delta)f(q_e(y_l))\alpha_1}\left(y_l - y_h - (1-\delta)\frac{k}{q_e(y_l)}\right) + \frac{J_e(y_h)}{1+r}.$$

The above equations, along with the expression for $J_e(y_h)$, yield equation (19), which defines the equilibrium value of $q_e(y_l)$.

Conjecture that $x_u(y) = 1$ for $y = \{y_l, y_h\}$. The necessary condition of (32) with respect to q_u can be written:

$$\frac{r}{r+\delta}\frac{J_u}{1+r} = \frac{y_h}{r+\delta} - \frac{k}{q_u f(q_u)\eta(q_u)} - \frac{(1-\alpha_1)k}{\eta(q_b) q_b f(q_b)\alpha_1}. \tag{36}$$

From the Bellman equation for J_u:

$$\frac{rJ_u}{1+r} = b + \frac{k}{q_u}\left(\frac{1-\eta(q_u)}{\eta(q_u)}\right). \tag{37}$$

Combining these two equations yields equation (11) from the text, where $i = j = 1$, and $q_e(s,1) = q_b$. This characterizes the equilibrium value of q_u.

To show that $x_u(y) = 1$ for $y = \{y_l, y_h\}$, combine equations (35) and (36) to obtain:

$$\frac{J_e(y_l)}{1+r} - \frac{J_u}{1+r} = \frac{k}{q_u f(q_u)\eta(q_u)} - \alpha_1 \frac{k}{q_b f(q_b)\eta(q_b)\alpha_1}.$$

The right side is identical to the right side of (27), so it is positive under the same conditions. Since $J_e(y_h) > J(y_l)$, if follows that when all low productivity matches are accepted, all high productivity matches are accepted. This concludes the proof of Lemma 4. **QED**

This concludes the proof of Proposition 2. **QED**

Proof of Proposition 3

This proof parallels that of the first part of Proposition 2. Throughout the proof we maintain the assumption that $\rho = 1-\alpha_i$ and we drop the argument ρ from all functions. As before, we keep track of employer types under the assumption that unemployed workers search for jobs posted by type-i employers and employed workers search for jobs posted by type-j employers, where it is understood that $i = j = 1$ throughout this proof. We first prove existence and uniqueness of the candidate equilibrium allocation. Then we show how it can be supported by a non-revealing equilibrium.

The first-order conditions for an interior solution of Problem (P1) are given in the main text. We now have that $q_e(s) = q_e(\{i, w, y_l\}) = q_e(\{i, w, y_h\})$ and it is easy to verify that $q_e(s) \in [\widehat{q}_a, \widehat{q}_b]$, where $w_e(s) \geq y_l$ if and only if $q_e(s) \geq \widehat{q}_a$ and $\widetilde{W}(q_e(s)) \leq y_l$ if and only if $q_e(s) \leq \widehat{q}_b$ and where

$\widehat{q}_a$ and $\widehat{q}_b$ are given by

$$\widehat{q}_a f\left(\widehat{q}_a\right)\left(1-\alpha_i\right)\alpha_j\left(\frac{y_h-y_l}{r+\delta}\right)=k \tag{38}$$

and

$$\frac{y_h-y_l}{r+\delta}=\left(\frac{k}{\widehat{q}_b f\left(\widehat{q}_b\right)\left(1-\alpha_i\right)\alpha_j}\right)\left(1+\left(\frac{1-\eta\left(\widehat{q}_b\right)}{\eta\left(\widehat{q}_b\right)}\right)\left(\frac{r+\delta+(1-\delta)\alpha_j f\left(\widehat{q}_b\right)}{r+\delta}\right)\right), \tag{39}$$

respectively. Clearly, $\infty>\widehat{q}_b>\widehat{q}_a>0$.

Proceeding as before, Problem (P2) can be formulated in the present case as

$$\overline{V}\left(s_u\right)=b+V_i, \tag{P4}$$

where

$$V_i=\frac{\overline{V}\left(s_u\right)}{1+r}+\max_{q,q'}\left\{f\left(q\right)\left(\frac{\widetilde{V}_0\left(i,q'\right)}{1+r}-\frac{\overline{V}\left(s_u\right)}{1+r}\right)\right\}$$

subject to

$$k\leq qf\left(q\right)\left(\frac{\widetilde{M}_0\left(i,q'\right)}{1+r}-\frac{\widetilde{V}_0\left(i,q'\right)}{1+r}+\frac{\overline{V}\left(s_u\right)}{1+r}\right),$$
$$q'\in\left[q_a,q_b\right],\ w\leq y_h,$$

where $\widetilde{V}_0\left(i,q'\right)$ and $\widetilde{M}_0\left(i,q'\right)$ are defined in the main text. Let $\left\{q_u\left(i\right),q_e^l\left(i\right)\right\}$ denote a solution to Problem (P4).

Noting that

$$\frac{\widetilde{V}_0\left(i,q'\right)}{1+r}-\frac{\overline{V}\left(s_u\right)}{1+r}=\frac{y_h}{r+\delta}-\left(\frac{r}{r+\delta}\right)\frac{\overline{V}\left(s_u\right)}{1+r}-\frac{k}{\eta\left(q'\right)q'f\left(q'\right)\left(1-\alpha_i\right)\alpha_j},$$

and using (12)–(13) and the definition of $\widetilde{M}\left(\left\{i,\widetilde{W}\left(q'\right),y_l\right\}\right)$ given in the main text, one can verify that

$$\begin{aligned}\frac{\widetilde{M}_0\left(i,q'\right)}{1+r}=&\frac{y_h}{r+\delta}-\left(\frac{r}{r+\delta}\right)\frac{\overline{V}\left(s_u\right)}{1+r}\\&-\left(1-\alpha_i\right)\left(\frac{r+\delta}{r+\delta+(1-\delta)\alpha_j f\left(q'\right)}\right)\left(\frac{y_h-y_l}{r+\delta}\right)\\&-\left(1-\alpha_i\right)\left(1-\frac{r+\delta}{r+\delta+(1-\delta)\alpha_j f\left(q'\right)}\right)\left(\frac{k}{q'f\left(q'\right)\left(1-\alpha_i\right)\alpha_j}\right).\end{aligned}$$

Lemma 5 *(i) $\widetilde{W}\left(q\right)$ and $\widetilde{V}_0\left(i,q\right)$ are strictly increasing and concave functions of q on $\left[\widehat{q}_a,\widehat{q}_b\right]$. (ii) $\widetilde{M}_0\left(i,q\right)$ is a strictly concave function of q on $\left[\widehat{q}_a,\widehat{q}_b\right]\subset\left(0,\infty\right)$ and it is maximized at $q=\widehat{q}_b$;*

$\widetilde{M}_0(i,q) - \widetilde{V}_0(i,q)$ is a strictly decreasing and convex function of q on $[\widehat{q}_a, \widehat{q}_b]$.

Proof: It replicates the arguments in Proposition 2 with minor changes. **QED**

The first-order conditions for an interior solution of Problem (P4) are given by equations (14)–(16) in the main text.

Following similar steps as in the proof of Proposition 2 one can verify that an interior solution to Problem (P4) satisfies

$$\frac{y_h - b}{r+\delta} - \frac{k}{\eta(q')\, q' f(q')(1-\alpha_i)\alpha_j} = \lambda_i q \left(\frac{1-\eta(q)}{\eta(q)}\right) \frac{k}{q} \left(\frac{1}{f(q)} + \frac{1}{r+\delta}\right), \tag{40}$$

where λ_i is given by (15), and

$$\begin{aligned}\frac{k}{qf(q)} &= -(1-\alpha_i)\left(\frac{r+\delta}{r+\delta+(1-\delta)\alpha_j f(q')}\right)\left(\frac{y_h - y_l}{r+\delta}\right) \\ &\quad + \left(\frac{k}{q' f(q')(1-\alpha_i)\alpha_j}\right)\left(\frac{(1-\alpha_i)(r+\delta)}{r+\delta+(1-\delta)\alpha_j f(q')} + \frac{1-\eta(q')}{\eta(q')} + \alpha_i\right).\end{aligned} \tag{41}$$

Lemma 6 *Assume that $(r+\delta)k < (1-\alpha_1)\alpha_1(y_h - y_l)$. Equations (15), (40) and (41) have a unique solution (λ_i, q, q'), with $q \in (0,\infty)$, $q' \in (q_c, q_d)$, and $\lambda_i q \geq 1$, where*

$$\frac{y_h - b}{r+\delta} - \frac{k}{\eta(q_c)\, q_c f(q_c)(1-\alpha_i)\alpha_j} = 0,$$

$$\frac{\widetilde{M}_0(i,q_d)}{1+r} - \frac{\widetilde{V}_0(i,q_d)}{1+r} + \frac{\overline{V}(s_u)}{1+r} = k$$

and where $q_c < \widehat{q}_b < q_d$.

Proof: Differentiating equation (15) one can verify that the following inequality is necessary and sufficient for $\partial \lambda_i q / \partial q' < 0$:

$$\frac{-\partial^2 \widetilde{M}_0 / \partial q'^2}{-\partial^2 \widetilde{V}_0 / \partial q'^2} > \frac{\partial \widetilde{M}_0 / \partial q'}{\partial \widetilde{V}_0 / \partial q'}.$$

The left side of the inequality is greater than one, since $\widetilde{M}_0 - \widetilde{V}_0$ *is a strictly convex function of* q'. The right side is smaller than one, since $\widetilde{M}_0 - \widetilde{V}_0$ *is a strictly decreasing function of* q'. Hence, $\partial \lambda_i q / \partial q' < 0$. Moreover, note that $\lambda_i q \geq 1$ if and only if $\partial \tilde{M}_0 / \partial q' \geq 0$. Accordingly, (40) characterizes q as a strictly decreasing function of q', where the right side converges to 0 as q approaches ∞ and it converges to ∞ as q approaches 0. Thus, $\infty > q > 0$ if and only if $q' > q_c$. Similarly, (41) characterizes q as a strictly increasing function of q', where the left side converges to ∞ as q approaches 0 and it converges to k as q approaches ∞. Thus, $\infty > q > 0$ if and only if $q' < q_d$. Together, (40)-(41) imply that $q' \in (q_c, q_d)$ and, hence, $\infty > q > 0$.

To verify that $\widehat{q}_b < q_d$, write (39) as

$$\frac{\alpha_i k}{\eta(\widehat{q}_b)\widehat{q}_b f(\widehat{q}_b)(1-\alpha_i)\alpha_j} = -(1-\alpha_j)\left(\frac{r+\delta}{r+\delta+(1-\delta)\alpha_j f(\widehat{q}_b)}\right)\left(\frac{y_h-y_l}{r+\delta}\right) + \left(\frac{k}{\widehat{q}_b f(\widehat{q}_b)(1-\alpha_i)\alpha_j}\right)\left(\frac{(1-\alpha_i)(r+\delta)}{r+\delta+(1-\delta)\alpha_j f(\widehat{q}_b)} + \frac{1-\eta(\widehat{q}_b)}{\eta(\widehat{q}_b)} + \alpha_i\right).$$

Comparing this with (41), it follows that $\widehat{q}_b < q_d$ if and only if

$$\frac{\alpha_i}{\eta(\widehat{q}_b)\widehat{q}_b f(\widehat{q}_b)(1-\alpha_i)\alpha_j} > 1.$$

A sufficient condition for this is $\alpha_i = \alpha_j$, which is the case here. Hence, $\widehat{q}_b < q_d$.

To verify that $\widehat{q}_b > q_c$, note that (39) implies that

$$\frac{k}{\eta(\widehat{q}_b)\widehat{q}_b f(\widehat{q}_b)(1-\alpha_i)\alpha_j} < \frac{y_h-y_l}{r+\delta},$$

which, together with the fact that $y_l > b$, implies that $\widehat{q}_b > q_c$. **QED**

If an interior non-revealing equilibrium exists, it is uniquely characterized by equations (12), (13), (15), (40) and (41). Recall that $w_e(s) \geq y_l$ if and only if $q_e(s) \geq \widehat{q}_a$ and $\tilde{W}(q_e(s)) \leq y_l$ if and only if $q_e(s) \leq \widehat{q}_b$, but we know only that $q' \in (q_c, q_d)$. Hence, we need to verify that the candidate interior solution for q' is such that $q' \in [\widehat{q}_a, \widehat{q}_b]$.

There are only three possible solutions to problems (P1) and (P2) with $\rho = 1-\alpha_i$. One is the interior allocation characterized above, provided that it is such that $q' \in [\widehat{q}_a, \widehat{q}_b]$. Another is the corner allocation that solves $q' \in \widehat{q}_b$, $\widetilde{W}(q') = y_l$, together with (12) and (41). In principle, a third possibility is the corner allocation such that $q' \in \widehat{q}_a$ and $w_e(s) = y_l$. However, it is easy to verify that there is a number $k_e > 0$ such that this case will never arise whenever $k \in (0, k_e)$, which is the relevant case below.

Therefore, in order to construct an equilibrium, consider the other two possible allocations and select the one that provides unemployed workers with the higher welfare. It is straightforward to characterize ψ. The unemployment rate is given by

$$\psi(s_u) = \frac{\delta}{\delta + f(q_u^*)},$$

where $q_u^* \equiv q_u(i, 1-\alpha_i)$. The wage distribution has two mass points: one wage w_u^* for workers who find jobs out of unemployment, and one wage w_e^* for workers who find jobs via on-the-job search. The mass of workers earning the wage w_u^* is

$$\psi(\{i, w_u^*, y_l\}) + \psi(\{i, w_u^*, y_h\}) = \left(\frac{f(q_u^*)}{\delta + (1-\delta)\alpha_j f(q_e^*)}\right)\psi(s_u),$$

where $w_u^* \equiv w_u^l(i, 1-\alpha_i) = w_u^h(i, 1-\alpha_i)$ and $q_e^* \equiv q_e(\{i, w_u^*, y_l\}, 1-\alpha_i) = q_e(\{i, w_u^*, y_h\}, 1-\alpha_i)$. The mass of workers earning the wage w_e^* is

$$\psi(\{i, w_e^*, y_h\}) + \psi(\{j, w_e^*, y_h\}) = \left(\frac{(1-\delta)\alpha_j f(q_e^*)}{\delta}\right)[\psi(\{i, w_u^*, y_l\}) + \psi(\{i, w_u^*, y_h\})],$$

where $w_e^* \equiv w_e(\{i, w_u^*, y_h\}, 1-\alpha_i) = w_e(\{j, w_u^*, y_h\}, 1-\alpha_i)$.

Furthermore, one can verify that $f(q_u^*)$ is an increasing function of y_l, y_h and α_1, and $f(q_e^*)$ is an increasing function of $(y_h - y_l)$ and α_1.

It remains to prove that there are mappings Q and μ that support the candidate equilibrium allocation. The construction of Q is standard. If $s \in S^*$, beliefs must be correct. If $s \notin S^*$, beliefs are arbitrary and we may assume that employers believe that $\mu(s\,|x) = 0$ for all $s \notin S^*$. It only remains to prove that a type-i employer posting a contract offering revealing wages will not attract any unemployed workers while making non-negative profits. We show that there is a number $\widehat{k}_d > 0$ such that this is the case for all $k \in \left(0, \widehat{k}_d\right)$. To see why, it is sufficient to consider the case where potential poachers will never hire workers with $s \notin S^*$. In this case, one can verify that the value of searching for a revealing contract to an unemployed worker, denoted by V_i^d is continuous in k with

$$\lim_{k\to 0} \frac{V_i^d}{1+r} = \alpha_i \frac{y_h}{r+\delta} + (1-\alpha_i)\frac{y_l}{r+\delta} + \left(\frac{r}{r+\delta}\right)\frac{\overline{V}(s_u)}{1+r},$$

whereas the candidate equilibrium allocation has

$$\lim_{k\to 0} \frac{\widehat{V}_i}{1+r} = \left(1 - \frac{r+\delta}{r+\delta+(1-\delta)\alpha_j}\right)\frac{y_h}{r+\delta} + \left(\frac{r+\delta}{r+\delta+(1-\delta)\alpha_j}\right)\frac{y_l}{r+\delta} + \left(\frac{r}{r+\delta}\right)\frac{\overline{V}(s_u)}{1+r}.$$

Hence, $\lim_{k\to 0}\widehat{V}_i > \lim_{k\to 0} V_i^d$ if and only if $(1-\alpha_i)\,\alpha_j/\alpha_i > (r+\delta)/(1-\delta)$. Since $\alpha_i = \alpha_j$, all that is needed is $(1-\alpha_i)(1-\delta) > (r+\delta)$ as assumed in the proposition. **QED**

Proof of Proposition 4

It follows from the arguments in Proposition 2 and Proposition 3. **QED**

Proof of Proposition 5

It follows from Definition 3, together with Proposition 2 and Proposition 4. **QED**

Proof of Proposition 6

Replicating the approach we followed in the proofs of propositions 2 and 3, one can verify that an interior equilibrium allocation where entry wages are revealing in type-2 matches, but not in type-1 matches, provided that it exists, can be constructed as follows.

Step 1. For a given value of $\widehat{c}$, find values of q_1, q_1', q_2 and q_2' such that q_2 solves

$$\begin{aligned}\frac{y_h - b}{r+\delta} - \frac{(1-\alpha_2)\,k}{\eta(q_b)\,q_b f(q_b)\,\alpha_1} \quad = \quad & \frac{F(\widehat{c})\,(\widehat{c} - \mathbb{E}(c\,|c \le \widehat{c})) - \widehat{c}}{r+\delta} \\ & + \frac{k}{\eta(q_2)\,q_2 f(q_2)} + \left(\frac{1-\eta(q_2)}{\eta(q_2)}\right)\frac{k}{(r+\delta)\,q_2},\end{aligned} \tag{42}$$

with $q_2' = q_b$, and (q_1, q_1') solve

$$\frac{y_h - b}{r+\delta} - \frac{k}{\eta(q_1')\, q_1' f(q_1')(1-\alpha_1)\alpha_1} = \frac{F(\widehat{c})(\widehat{c} - \mathbb{E}(c\,|c \le \widehat{c}))}{r+\delta} + \lambda_1 q_1 \left(\frac{1-\eta(q_1)}{\eta(q_1)}\right)\frac{k}{q_1}\left(\frac{1}{f(q_1)} + \frac{1}{r+\delta}\right), \tag{43}$$

where λ_1 is given by the analogue of (15), for $i = 1$, and

$$\frac{k}{q_1 f(q_1)} = -(1-\alpha_1)\left(\frac{r+\delta}{r+\delta+(1-\delta)\alpha_1 f(q_1')}\right)\left(\frac{y_h - y_l}{r+\delta}\right) + \left(\frac{k}{q_1' f(q_1')(1-\alpha_1)\alpha_1}\right)\left(\frac{(1-\alpha_1)(r+\delta)}{r+\delta+(1-\delta)\alpha_1 f(q_1')} + \frac{1-\eta(q_1')}{\eta(q_1')} + \alpha_1\right). \tag{44}$$

Step 2. Use those values of q_1, q_1', q_2 and q_2' to calculate the implied values of V_1 and V_2 as a function of $\widehat{c}$ and let

$$D(\widehat{c}) = V_2 - V_1. \tag{45}$$

Step 3. We are seeking to establish existence of a fixed point

$$D(c_0) = c_0 > 0. \tag{46}$$

Similarly, one can verify that a corner equilibrium allocation where entry wages are revealing in type-2 matches, but not in type-1 matches, with $\widetilde{W}(q_1') = y_l$, provided that it exists, can be constructed following the same steps, except that equation (43) is replaced with $q_1' = \widehat{q}_b$. As before, it is easy to verify that there is a number $k_f > 0$ such that these two cases exhaust all feasible cases whenever $k \in (0, k_f)$, which is the relevant case below.

Suppose that $\widehat{c} = 0$, in which case all arguments in propositions 2 and 3 hold. First, suppose the solution at $\widehat{c} = 0$ is interior. Clearly there is a number $\alpha_a \in (0, \alpha_1)$ such that $D(0) > 0$ for all $\alpha_2 \in (\alpha_a, \alpha_1)$. Now start increasing the value of $\widehat{c}$. Following the same arguments we used in the proof of Proposition 3, one can verify that there is a number $c_a \in (0, \infty)$ such that there is a solution to the equations above with $q' \in (\tilde{q}_c(\widehat{c}), \tilde{q}_d)$ where $\tilde{q}_c(\widehat{c}) < \widehat{q}_b < \tilde{q}_d$, for all $\widehat{c} \in [0, c_a)$. There are only two possibilities. If there exists an interior equilibrium, then there is some value $c_0 \in (0, c_a)$ such that $D(c_0) = c_0$.

Otherwise, it must be that $q_1' = \widehat{q}_b$. Replicating the arguments in the proof of Proposition 2, one can verify that, for given $\widehat{c}$, there is a number $\widehat{k}_g > 0$ such that there is a unique value of $q_2 \in (0, \infty)$ that solves equation (42), with $q_2' = q_b$, for all $k \in \left(0, \widehat{k}_g\right)$. Moreover, $q_2 > 0$, and thus, V_2 is bounded, for all $\widehat{c} \ge 0$ since $\lim_{\widehat{c} \to \infty} [\widehat{c} - F(\widehat{c})(\widehat{c} - \mathbb{E}(c\,|c \le \widehat{c}))] = 1/\theta < \infty$. Similarly, the arguments in the proof of Proposition 3 imply that a non-revealing equilibrium allocation in the market for type-2 jobs can be supported for sufficiently small values of $k > 0$, provided that $(1-\alpha_1)(1-\delta) > (r+\delta)$, as assumed in the proposition. Since V_2 remains bounded as we increase $\widehat{c}$, there must exist a fixed point $D(c_0) = c_0$.

It is now straightforward to characterize ψ. The unemployment rate is given by

$$\psi(s_u) = \frac{\delta}{\delta + (1 - F(c_0)) f(q_{u1}^*) + F(c_0) f(q_{u2}^*)},$$

where $q_{ui}^* \equiv q_u(i, 1-\alpha_1, 1)$, for $i = 1, 2$. The wage distribution across type-2 jobs consists of two mass points. The mass of workers earning the wage $w_l^* \equiv w_u^l(2, 1-\alpha_1, 1) = y_l$ is

$$\psi(\{2, w_l^*, y_l\}) = \left(\frac{F(c_0)(1-\alpha_2) f(q_{u2}^*)}{\delta + (1-\delta)\alpha_1 f(q_{e2}^*)}\right)\psi(s_u),$$

where $q_{e2}^* \equiv q_e^l(2, 1-\alpha_1, 1)$; the mass of workers earning the wage $w_h^* \equiv w_u^h(2, 1-\alpha_1, 1)$ is

$$\psi(\{2, w_h^*, y_h\}) = \left(\frac{F(c_0)\alpha_2 f(q_{u2}^*)}{\delta}\right)\psi(s_u).$$

The wage distribution across type-1 jobs consists of three mass points. The mass of workers earning the wage $w_{e2}^* \equiv w_e(\{2, w_l^*, y_l\}, 1-\alpha_1, 1)$ is

$$\psi(\{1, w_{e2}^*, y_h\}) = \left(\frac{(1-\delta)\alpha_1 f(q_{e2}^*)}{\delta}\right)\psi(\{2, w_l^*, y_l\});$$

the mass of workers earning the wage $w_u^* \equiv w_u^l(1, 1-\alpha_1, 1) = w_u^h(1, 1-\alpha_1, 1)$ is

$$\psi(\{1, w_u^*, y_h\}) + \psi(\{1, w_u^*, y_l\}) = \left(\frac{(1-F(c_0)) f(q_{u1}^*)}{\delta + (1-\delta)\alpha_1 f(q_{e1}^*)}\right)\psi(s_u),$$

where $q_{e1}^* \equiv q_e^l(1, 1-\alpha_1, 1)$ and $(1-\alpha_1)\psi(\{1, w_u^*, y_h\}) = \alpha_1\psi(\{1, w_u^*, y_l\})$; the mass of workers earning $w_{e1}^* \equiv w_e(\{1, w_u^*, y_l\}) = w_e(\{1, w_u^*, y_h\})$ is

$$\psi(\{1, w_{e1}^*, y_h\}) = \frac{(1-\delta)\alpha_1 f(q_{e1}^*)}{\delta}\left(\frac{(1-F(c_0)) f(q_{u1}^*)}{\delta + (1-\delta)\alpha_1 f(q_{e1}^*)}\right)\psi(s_u).$$

This concludes the proof of Proposition 6. **QED**