Future-biased intergenerational altruism*

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Abstract

We show that intergenerational altruism suffers from future bias if generations overlap and people's altruism concerns the well-being of immediate ancestors and descendants. Future bias involves preference reversals associated with increasing impatience, which can create a conflict of interest between current and future governments representing living generations. We explore the implications of this conflict for intergenerational redistribution when there is a sequence of utilitarian governments choosing policies independently over time. We show that future-biased governments can have an incentive to legislate and sustain a pay-as-you-go pension system, which can be understood, from the viewpoint of every government, as a self-enforcing commitment mechanism to increase future old-age transfers.

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1 Introduction

For an elected government, the relevant calculus of optimal intergenerational redistribution is such that the values of the present-day society are used to evaluate the distribution of consumption between present and future generations. Intergenerational disagreements create several problems. One is that governments have to strike a balance between the preferences of presently young and old people. Another is that the effect of current policies on future social outcomes can be undone by future policies. In this paper we consider these problems when people's altruism concerns the well-being of neighboring generations. We begin by showing that, in this familiar case, intergenerational altruism tends to be future biased when generations overlap.

Future bias, which is precisely defined below, involves preference reversals associated with increasing impatience: a future consumption allocation can be preferred to an earlier one, even though the earlier one is preferred when both allocations are equally delayed. The source of the future bias is the positive discounting of the consumption utility of others, which implies that the ranking of consumption allocations can be reversed with the passage of age. This is because young generations are more reluctant to transfer resources from themselves to the living old than from the young to the old at any future date.

To address the consequences of future bias for intergenerational redistribution, we consider a sequence of overlapping generations, each living for two periods, and a sequence of one-period governments seeking to maximize a weighted sum of the utilities of the two living generations. Others before us have noted that disagreements between coexisting generations render plausible social welfare functions time inconsistent, but they have explored neither the specific form of time inconsistency nor its implications for government intervention.[1] The difficulty lies in that today's young generations are tomorrow's old, so it is unclear how intergenerational disagreement at each point in time translates into intertemporal allocations.

We are able to make progress by tracking the source of time inconsistency — social preferences inherit the future bias of individual preferences — and by analyzing its equilibrium

[1] Burbidge (1983), Kimball (1987), Calvo and Obstfeld (1988) and Hori and Kanaya (1989) recognize the potential time-inconsistency and Bernheim (1989) emphasizes it as the source of Pareto inefficiency.

implications in the context of a tractable example. In particular, we restrict our attention to the case where consumption utilities exhibit a constant elasticity of marginal utility, which implies that the optimal allocation of consumption across generations within a period is independent of the level of aggregate consumption. In this setting, preferences can be expressed over aggregate consumption streams, while the sharing rules to allocate consumption across living generations determine the future bias. Moreover, conditional on a stationary sharing rule, preferences over aggregate consumption streams exhibit β-δ discounting: the discount factor between the current period and the next is $\beta\delta$ while the discount factor between any two future periods is δ. Here, however, the short-term discount factor β is endogenous and greater than one, reflecting an endogenous future bias, rather than the exogenous present bias of the quasi-hyperbolic preferences originally proposed by Phelps and Pollak (1968).

In the context of our example, we show that future bias implies that there can be both too little redistribution towards the old and too much saving at date $t+1$ from the viewpoint of the date-t government. Hence, the date-t government has an incentive to increase redistribution towards the old at date $t+1$. But if the current government cannot control future old-age transfers, it will have an incentive to influence future income instead, which can be achieved by strategically distorting current capital accumulation.

To gain some intuition, consider the limiting case where generations are non-altruistic. Moreover, suppose that the weight of the young in social preferences (i.e., their political weight) is relatively large. Then, not only do current governments not care about future generations, but their main concern is to make sure that the current young get enough consumption when they are old. While increasing current investment and growth is costly, governments will be willing to do so if they have a strong enough preference for consumption smoothing. The simplicity of our example allows us to show that current and future investments are strategic complements in this case. Consequently, from the viewpoint of the government at any given date, future equilibrium growth is excessive while future equilibrium old-age transfers are insufficient.

In this context, intergenerational redistribution can be understood as a second-best commitment designed to undo the equilibrium effects of future bias on discretionary policy. We emphasize the combination of the adverse effects of future bias when policymakers cannot commit future old-age transfers and the access to a commitment device, such as social security legislation, that allows policymakers to commit future old-age transfers. Such a commitment device lessens the strategic motive behind public investment and so it results in lower growth. Yet, future governments have an incentive to sustain the legislation, because each future government faces essentially the same problem as the government that introduced the original legislation, given the capital stock that it inherits.

Superficially, the above intuition seems to require that individuals do not care about future generations. However, we show that the incentive to legislate and sustain old-age transfers at the expense of growth stems from the presence of future bias and not from the absence of altruism. Moreover, this is so regardless of the relative political weight of the old and the young, as long as the young have positive weight.

Our paper contributes to the growing literature on time inconsistency.[2] Jackson and Yariv (2014) show that utilitarian aggregation of time-consistent preferences results in a present bias, involving preference reversals associated with diminishing impatience, when discount factors are heterogeneous. This is because, when aggregating time-consistent preferences, those with higher discount factors gain increasingly more weight when evaluating intertemporal rates of substitution further away in the future. By contrast, we show that a future bias can be expected in the intergenerational context, where time horizons are naturally heterogeneous and social preferences inherit the future bias of individual preferences.

Galperti and Strulovici (2017) consider non-overlapping generations and show that forward altruism exhibits a present bias whenever altruism is non-paternalistic and incorporates directly the utilities of, not just the immediate descendants, but *all* future descendants. In this case, grandparents care about their grandchildren directly as well as indirectly through their own children. However, parents care about their own children only directly, neglecting the grandparents' direct concern about their grandchildren. Consequently, parents care too little about their children from the viewpoint of grandparents. Like Galperti and Strulovici,

[2] See, for example, Laibson (1997), Caplin and Leahy (2004), Gollier and Zeckhauser (2005), Halevy (2008, 2015), Gollier and Weitzman (2010), Jackson and Yariv (2014) and Galperti and Strulovici (2017).

we challenge the common view that non-paternalistic altruism and time consistent preferences are two sides of the same coin (Phelps and Pollak 1968, Barro 1974). Our analysis complements theirs by showing that the likely bias depends on the demographic structure as well as the specific model of intergenerational altruism.

Our analysis offers a novel perspective on the widespread legislation of pay-as-you-go social security despite recognition of its negative effects on capital accumulation (Auerbach and Kotlikoff 1987). Our argument is clearly different from the idea that social security legislation is socially optimal. It is also different from the idea that social security legislation occurs because current generations do not internalize the costs to future generations. In our setting, social security legislation occurs even if current generations are altruistic and even though they understand that the legislation will harm future generations.

Grossman and Helpman (1998) consider a sequence of governments that cannot precommit future redistributive policy. Social security arises in equilibrium because the preferences of politicians are biased towards the old. By contrast, our theory does not depend on the relative political weight of young and old agents, as long as the young have some weight. We believe this is relevant because, in practice, neither the introduction of pay-as-you-go social security legislation nor its survival seems likely without support from the young.

Others have argued that government's self-control problems matter for fiscal policy. For instance, Phelps and Pollak (1968) and Krusell et al. (2002) consider the impact of policy on capital accumulation under the assumption that governments' preferences are present biased, favoring current over future consumption. This assumption seems natural in their non-overlapping generations setting, which is the standard setting in analyses of equilibrium growth with intergenerational disagreement.[3] However, our analysis warns that the direction of the bias depends crucially on the specification of altruism and the demographic structure.

To the best of our knowledge, we are the first to identify a role of future bias in redistributive politics. Veall (1986) and Hansson and Stuart (1989) presume that children place sufficiently large weight on their parents' consumption and, in the absence of social security, the current young would have an incentive to undersave in order to elicit old-age transfers

[3] Kohlberg (1976), Bernheim and Ray (1987), Ray (1987) and Barro (1999) are well known examples.

from the future young.[4] By contrast, we argue that policymakers have an incentive to over-save in order to increase future old-age transfers, precisely because the next generation will place insufficient weight on the parents' consumption.

In a similar spirit, the literature on strategic debt views government debt as a way to tie the hands of future governments that have different preferences from the current one (e.g., Persson and Svensson 1989 and Alesina and Tabellini 1990). Their focus is on discretionary policy when current voters have an incentive to lower strategically the size of the future pie to be distributed.[5] By contrast, we argue that policymakers would have a strategic incentive to increase the size of the future pie in the absence of social security legislation that commits future old-age transfers. Instead, it is the commitment to future old-age transfers that leads to a decrease in the size of the future pie.

The next section illustrates the logic of our argument in the simplest case, where individuals are non-altruistic. Section 3 presents the model with altruistic individuals and characterizes the future bias of both individual and social preferences. Section 4 considers the first-best allocation for an arbitrary government. In Section 5, we analyze the symmetric Markov perfect equilibrium in linear strategies. Section 6 considers the case where the current government can precommit future intergenerational transfers. Section 7 concludes. Technical proofs are in the Appendix.

2 An example with non-altruistic generations

Consider an economy with overlapping generations. A unit mass of individuals are born every period $t \geq 0$, each individual lives for two periods and individuals born at date t have preferences given by $U_t = u\left(c_t^y\right) + u\left(c_{t+1}^o\right)$, with

$$u\left(c\right) = \begin{cases} \frac{c^{1-\sigma}}{1-\sigma} & \text{if } \sigma \neq 1, \quad \sigma > 0 \\ \ln\left(c\right) & \text{if } \sigma = 1, \end{cases} \tag{1}$$

where c_t^y is the consumption the young at date t, and c_{t+1}^o is their consumption when old.

[4] Tabellini (2000) also presumes that children are sufficiently altruistic towards parents, but he stresses the fact that social security redistributes both across and within generations.

[5] Also see Azzimonti (2011) and Halac and Yared (2014).

Suppose there is a sequence of governments and the date t government seeks to maximize

$$V_t = U_{t-1} + aU_t, \tag{2}$$

with $a > 0$, for all $t \geq 0$. The utilitarian welfare objective captures in a simple manner the fact that democratic governments are unlikely to be immune to disagreement between coexisting generations. It can be interpreted as the outcome of political competition in a probabilistic voting model (Lindbeck and Weibull 1987, Grossman and Helpman 1998).

To focus on the equilibrium interaction between present and future governments, assume that the date-t government has sufficient instruments to implement its desired date-t allocation. Thus, one can treat the date-t government as if it can choose both the allocation of date-t aggregate resources between consumption and investment and the allocation of date-t aggregate consumption between the young and the old directly, without decentralizing the equilibrium allocations at date-t. Specifically, this implies that each government can commit current, but not future, old-age transfers and investment levels.

The aggregate resources constraint in the economy is given by

$$Ak_t \geq c_t^y + c_t^o + k_{t+1} - k_t, \tag{3}$$

where k_t units of capital produce Ak_t units of output, with $A > 0$, for all $t \geq 0$.

The symmetric Markov equilibrium in linear strategies is easy to construct. Letting $\tau_t = c_t^y / c_t$, with $c_t = c_t^y + c_t^o$, and disregarding past data, one has

$$V_t = u\left(\left(1 - \tau_t\right) c_t\right) + a\left[u\left(\tau_t c_t\right) + u\left(\left(1 - \tau_{t+1}\right) c_{t+1}\right)\right],$$

with $c_t = Ak_t - (k_{t+1} - k_t)$, for all $t \geq 0$. The optimal intergenerational allocation of consumption at date t solves the static problem

$$\max_{\tau_t} \left\{au\left(\tau_t c_t\right) + u\left(\left(1 - \tau_t\right) c_t\right)\right\},$$

and so the young's share of aggregate consumption is given by

$$\tau^* = \frac{1}{1 + a^{-1/\sigma}}, \tag{4}$$

for all t. If the date-t government takes as given that $i_{t+1} = \widehat{g} k_{t+1}$, the optimal investment decision at date t solves the problem

$$\max_{i_t} \alpha(\tau^*)\, u\left[A k_t - i_t\right] + u\left[(A - \widehat{g})(k_t + i_t)\right],$$

where $\alpha(\tau) = \frac{1}{a} + \left(\frac{\tau}{1-\tau}\right)^{1-\sigma}$. One can verify that the best investment response to the anticipation of $i_{t+1} = \widehat{g} k_{t+1}$ is given by $i_t = g k_t$, where

$$1 + g = \frac{A + 1}{1 + \left(\alpha(\tau^*) / (A - \widehat{g})^{1-\sigma}\right)^{1/\sigma}}. \tag{5}$$

The unique pair (τ^*, g^*) such that τ^* satisfies (4) and g^* is a fixed point of (5) characterizes a symmetric equilibrium. One can verify that g^* solves $\alpha(\tau^*)(1 + g^*)^\sigma = (A - g^*)$. Clearly, there is an equilibrium with $g^* \in (-1, A)$ and $g^* > 0$ if and only if $a^{-1}\left(1 + a^{1/\sigma}\right) < A$.

Now, suppose that the date-t government can precommit future transfers, provided that it treats current and future generations symmetrically, by making proportional transfers identical at all points in time. Furthermore, suppose that the date-t government can control *current,* but not future, investment. Thus, once transfers are legislated, consecutive governments will choose investment unilaterally, taking into account the investment strategies of future governments. Our previous analysis then implies that equilibrium investment is given by $\alpha(\tau)(1 + g)^\sigma = (A - g)$. Letting $g(\tau)$ be a solution to this equation, it also implies that the date-t government's optimal choice of τ solves the following problem:

$$\max_{\tau \in [0,1]} \left\{ \begin{array}{c} u\left((1-\tau)(A - g(\tau)) k_t\right) \\ +a\left[u\left(\tau(A - g(\tau)) k_t\right) + u\left((1-\tau)(A - g(\tau))(1 + g(\tau)) k_t\right)\right] \end{array} \right\}. \tag{6}$$

Let $\overline{\tau}$ denote a solution to this problem. We have the following result.

Proposition 1 *Suppose that $\sigma > 1$. The date-t government legislates old-age transfers ($\overline{\tau} < \tau^*$), even though the legislation will depress equilibrium growth ($g(\overline{\tau}) < g^*$).*

Proposition 1 rests on the fact that social preferences are time inconsistent, in the sense that preferences at date t are inconsistent with preferences at date $t+1$. In particular, the date-t government does not care about the generation born in period $t+1$, while the date-$t+1$ government does. This introduces dynamic preference reversals associated with a *future bias*: the date-t government would prefer that the date-$t+1$ government consume immediately, saving nothing, whereas when the time comes, the date-$t+1$ government will rather postpone consumption.

Intuitively, next-period old-age transfers are too low from the perspective of current policymakers. If they cannot influence future transfers, they will have an incentive to influence future income instead. If $\sigma > 1$, governments have an incentive to increase current growth in order to raise the young's future consumption, as income effects dominate substitution effects. Since current and future investments are strategic complements when $\sigma > 1$ (see (5)), the incentive to increase growth is self-enforcing and the economy's growth rate is too high from the viewpoint of the current government. Hence, if a pay-as-you-go system of intergenerational transfers could be legislated to redistribute from the young to the old every period, the current government would choose to do so. The strategic value of raising growth would be reduced and the new growth rate would be lower than before the legislation.

Although the equilibrium before the new legislation is not Pareto efficient, it is not the case that equilibrium investment is dynamically inefficient to begin with. As usual, we say that an investment allocation is dynamically efficient if there is no alternative allocation that provides more aggregate consumption in one period and at least the same consumption in every other period. It is not difficult to verify that investment in the Markov equilibrium is dynamically efficient. To see why, note that this is the case if the growth rate is lower than the *social* return to investment, that is, if $g^* < A$.[6] That this condition must hold, for all

[6]The following proof replicates the argument in Saint Paul (1992). Consider an allocation $\left\{\widetilde{k}_t\right\}$ with $\widetilde{k}_s < k_s$, for some s, with $\widetilde{c}_t \geq c_t$ for $t \geq s$. Since $\widetilde{k}_{t+1} = (A+1)\widetilde{k}_t - \widetilde{c}_t$ and $k_{t+1} = (A+1)k_t - c_t$, for $t \geq s$, it must be that $k_{t+1} - \widetilde{k}_{t+1} \geq (A+1)\left(k_t - \widetilde{k}_t\right)$, for $t \geq s$. In turn this implies that $k_{s+T} - \widetilde{k}_{s+T} \geq (A+1)^T\left(k_s - \widetilde{k}_s\right)$, and thus $\widetilde{k}_{s+T} \leq (1+g^*)^T k_s - (A+1)^T\left(k_s - \widetilde{k}_s\right)$, for any $T \geq 1$. Clearly, if $g^* < A$, the right side of the inequality becomes negative for T sufficiently large, contradicting the hypothesis that there is a feasible deviation $\widetilde{k}_s < k_s$, for some s, with $\widetilde{c}_t \geq c_t$ for $t \geq s$. This concludes the proof.

$c_t > 0$, follows immediately from the aggregate resources constraint: $Ak_t \geq c_t + k_{t+1} - k_t$.

The legislation hurts an infinite number of future generations. Yet, if a future government had the chance to legislate stationary intergenerational redistribution, it would choose to sustain the current legislation, because it would face essentially the same problem as the government that introduced the original legislation, given the capital stock that it inherits.

At first pass, Proposition 1 might seem to depend on the fact that individuals are not altruistic. If only they cared about the well-being of future generations, they might not ever have an incentive to sacrifice growth. However, we show below that the incentive to legislate and sustain old-age transfers at the expense of growth stems from the presence of future bias and not from the absence of altruism.

3 The model with altruistic generations

In what follows, we suppose that individuals care directly about the utility of their own parents and children. In this section, we show that both individual and social preferences are future biased in this context. In the following sections, we analyze the consequences of this bias for the allocation of consumption across generations. It will become clear that the example in Section 2 can be understood as a limiting case when altruism becomes negligible.

3.1 Intergenerational altruism

Suppose that individuals born at date t enjoy total utility U_t, where

$$U_t = u^y\left(c_t^y\right) + u^o\left(c_{t+1}^o\right) + \mu U_{t-1} + \lambda U_{t+1}, \tag{7}$$

for all $t \geq 0$, with $\mu > 0$ and $\lambda > 0$, where u^y and u^o denote the direct consumption utility when young and old, respectively. Further, suppose (i) preferences do not vary across generations; (ii) they put non-negative weight on the lifetime consumption utility of every generation; (iii) altruism is bounded, in the sense that concern for infinitely distant ancestors and descendants becomes negligible. Under these assumptions, Kimball (1987) and Hori and Kanaya (1989) show that $\mu + \lambda < 1$ is a necessary and sufficient condition for the existence of a utility function that satisfies (7). Moreover, the utility function is uniquely given by

$$U_t = \theta u^o\left(c_t^o\right) + \sum_{s=0}^{\infty} \delta^s \left[u^y\left(c_{t+s}^y\right) + u^o\left(c_{t+1+s}^o\right)\right] \text{ with } 0 < \theta < 1 \text{ and } 0 < \delta < 1, \tag{8}$$

$$\text{where } \theta = \frac{1-\sqrt{1-4\mu\lambda}}{2\lambda} \text{ and } \delta = \frac{1-\sqrt{1-4\mu\lambda}}{2\mu},$$

and where we have have disregarded past data irrelevant for the behavior of agents at date t. Kimball shows that non-negativity (assumption (ii) above) requires that $\mu\lambda < 1/4$, whereas bounded altruism (assumption (iii)) requires the stronger condition $\mu + \lambda < 1$. The latter implies positive discounting of the consumption of others ($\theta < 1 < \delta^{-1}$), where $\theta = \delta = 1$ is a limiting case as $\mu\lambda$ approaches 1/4. Importantly, the utility function given by (8) involves a geometric sequence backward, which we have omitted here, as well as a geometric sequence forward. However, a single geometric sequence forward and backward is impossible.[7]

3.2 Future bias

This section offers a novel perspective on the above familiar preferences by showing that they involve an inherent future bias. It also tracks a conflict of interest between current and future utilitarian governments representing living generations to the fact that each government inherits the future bias of individual preferences.

The following definition builds on Jackson and Yariv's (2014) definition of present bias.

Definition 1 *Let* $c_t^y = \tau_t c_t > 0$ *and* $c_t^o = (1-\tau_t)\, c_t > 0$, *and let* $W_t = \sum_{s=0}^{\infty} w\left(\tau_{t+s}, c_{t+s}, s\right)$.
(i) W_t *is present biased if: (1) for any* (τ, c), (τ', c') *and* $s \geq 0$, $k \geq 1$, $w(\tau, c, s) \leq w(\tau', c', s+k)$ *implies* $w(\tau, c, s+1) \leq w(\tau', c', s+k+1)$ *and (2) for any* $s \geq 1$ *and* $k \geq 1$, *there exist* (τ, c) *and* (τ', c') *such that* $w(\tau, c, 0) > w(\tau', c', k)$ *and* $w(\tau, c, s) < w(\tau', c', s+k)$.
(ii) W_t *is future biased if: (1) for any* (τ, c), (τ', c') *and* $s \geq 0$, $k \geq 1$, $w(\tau, c, s) \geq w(\tau', c', s+k)$ *implies* $w(\tau, c, s+1) \geq w(\tau', c', s+k+1)$ *and (2) for any* $s \geq 1$ *and* $k \geq 1$, *there exist* (τ, c) *and* (τ', c') *such that* $w(\tau, c, 0) < w(\tau', c', k)$ *and* $w(\tau, c, s) >$

[7] See Bergstrom (1999) for further discussion of the relationship between (7) and (8).

$w(\tau', c', s+k)$.

The definition of present bias in Part (i) captures a notion of diminishing impatience. Part (1) states that if the allocation (τ', c') at some time $t+k$ is preferred to another allocation (τ, c) at an earlier time t, then the same preference ordering holds when both allocations are equally delayed. This implies that future preferences are at least as patient as current preferences, ruling out preference reversals associated with future bias (see below). By contrast, Part (2) emphasizes the possibility of preference reversals associated with diminishing impatience. When preferences are present biased, it is possible that as of date t, an allocation is preferred to another one at some later time $t+k$, but the reverse is true when both allocations are equally delayed.

Instead, the definition of future bias in Part (ii) captures a notion of *increasing* impatience. Part (1) implies that current preferences are at least as patient as future preferences: if the allocation (τ, c) at time t is preferred to another allocation (τ', c') at some later time $t+k$, then the same preference ordering holds when both allocations are equally delayed. Note that this condition rules out preference reversals associated with present bias. By contrast, Part (2) emphasizes the possibility of preference reversals associated with increasing impatience. When preferences are future biased, it is possible that an allocation at some time $t+k$ is preferred to another allocation at an earlier time t, but the reverse is true when both allocations are equally delayed.

In order to identify the bias inherent in U_t, it is useful to rewrite (8) as

$$U_t = \left(\theta - \delta^{-1}\right) u^o\left(c_t^o\right) + \sum_{s=0}^{\infty} \delta^s \left[u^y\left(c_{t+s}^y\right) + \delta^{-1} u^o\left(c_{t+s}^o\right)\right] \text{ with } 0 < \theta < 1 < \delta^{-1} < \infty.$$

Similarly, the utility of individuals born at date $t-1$ (from the viewpoint of date t) is

$$U_{t-1} = \delta \sum_{s=0}^{\infty} \delta^s \left[u^y\left(c_{t+s}^y\right) + \delta^{-1} u^o\left(c_{t+s}^o\right)\right] \text{ with } 0 < \delta < 1.$$

Now consider a sequence of governments, where the objective function of the date-t

government is given by (2). That is, $V_t = U_{t-1} + aU_t$. Therefore,

$$V_t = \left(\theta - \delta^{-1}\right) au^o\left(c_t^o\right) + (\delta + a)\sum_{s=0}^{\infty}\delta^s\left[u^y\left(c_{t+s}^y\right) + \delta^{-1}u^o\left(c_{t+s}^o\right)\right], \tag{9}$$

where $0 < \theta < 1 < \delta^{-1} < \infty$. This captures the idea that democratically elected governments care about future generations only to the extent that the current electorate does so.

Proposition 2 *For all $t \geq 0$, (i) U_t is future biased if and only if $\theta\delta < 1$ and (ii) V_t is future biased if and only if $\theta\delta < 1$ and $a > 0$.*

It is worth noting that future bias does not stem from an assumed asymmetry in the way ancestors and descendants are treated relative to each other. Thus, future bias in U_t arises whether $\mu > \lambda$, $\mu = \lambda$, or $\mu < \lambda$ in the preferences given by (7). It also arises whether $\theta < \delta$, $\theta = \delta$ or $\theta > \delta$ in the utility function given by (8).

Rather, future bias stems from the requirement that people care sufficiently more about themselves than they do about others ($\mu + \lambda < 1$), a necessary condition for altruism to be bounded, which translates into positive discounting of the consumption utility of others ($\theta < 1 < \delta^{-1}$). In turn, this implies that the ranking of consumption allocations can be reversed with the passage of age. This is because young individuals are more reluctant to transfer resources from themselves to the living old than from the young to the old at any future date. Note that this logic also applies to the limiting case in Section 2, where $\theta = \delta = 0$.[8]

Part (ii) of the proposition implies that the future bias of individual preferences is inherited by social preferences as long as they put positive weight on the young. This is because preferences from the viewpoint of old age exhibit no bias and the aggregation of the preferences of the young and the old is linear.

Importantly, future bias is the reason why social preferences are time inconsistent. Note that the two notions are logically distinct. The biases formalized in Definition 1 refer to the particular utility function W_t and so they are associated with static preference reversals: the

[8] For Definition 1 to apply in the non-altruistic case, it needs to be suitably modified in order to restrict attention to preference reversals over the individuals' life cycle.

ranking of consumption allocations depends on the distance from t (i.e., preferences are non-stationary). By contrast, time inconsistency is associated with dynamic preference reversals: the ranking of consumption allocations changes with the evaluation date (i.e., with t).

Formally, consider the sequence $\{W_t\}_{t\geq 0}$ and let $W_t = \sum_{s=0}^{\infty} w_t(\tau_{t+s}, c_{t+s}, s)$, for all t. Dynamic preference reversals consistent with static future bias imply that for any $s \geq 1$ and $k \geq 1$, there exist (τ, c) and (τ', c') such that $w_t(\tau, c, s) > w_t(\tau', c', s+k)$ and $w_{t+s}(\tau, c, s) < w_{t+s}(\tau', c', s+k)$. Thus, it is possible that as of date t, an allocation at some later date-$t+s$ is preferred to another one at an even later date-$t+s+k$, but the reverse is true when date $t+s$ arrives. When $W_t = V_t$, with $\theta\delta < 1$ and $a > 0$, the exact condition for such dynamic preference reversals is given by Part (2) of the definition of future bias. This is because $w_t = w$, for all t (i.e., social preferences are time invariant), hence $w_t(\tau, c, s+k) = w(\tau, c, s+k)$ and $w_{t+s}(\tau, c, s+k) = w(\tau, c, k)$, for all $s \geq 1$ and $k \geq 0$.[9]

Unlike social preferences, individual preferences are time varying. This is because the ranking of consumption allocations is a function of age, which also explains why preferences from the viewpoint of old age exhibit no bias — the old do not age any further, they care about the well-being of their children and all their ancestors are dead. Consequently, social preferences would be time consistent if the old were dictators.

3.3 Equilibrium

For the remainder of the paper, we consider the economy described in Section 2, except that the preferences of the generation born at date t are now given by (8), with $u^y(c) = u^o(c) = u(c)$, where u is given by (1). It is easy to see that the results stated in Proposition 2 hold regardless of the individuals' rate of time preference. Accordingly, we have assumed a zero time discount rate, for simplicity. The assumption of isoelastic utility (equation (1)) is restrictive, but it greatly facilitates the equilibrium analysis. Specifically, it implies that the optimal static allocation of consumption across generations within a period is independent of the level of aggregate consumption that period. It is also needed for the economy to converge

[9] Halevy (2015) shows that any two properties among stationarity, time invariance and time consistency imply the third.

to a balanced growth path. Furthermore, we assume that $\delta(A+1) > 1$ in order to ensure positive equilibrium growth rates and $\delta(A+1)^{1-\sigma} < 1$ in order to ensure that growth is not so fast that it leads to unbounded utility.

The time inconsistency of the sequence of social preferences implies disagreements between current and future governments. To address their interaction, we consider the following problem. Every period t the government chooses investment $(k_{t+1} - k_t)$, and consumption (c_t^y and c_t^o) in order to maximize (9), with $u^y(c) = u^o(c) = u(c)$, subject to (1) and (3), taking as given the strategies of all other governments. A Markov strategy of the date-t government consists of an investment policy $i^t(k_t)$ and consumption policies $c_y^t(k_t)$ and $c_o^t(k_t)$ that are only functions of the payoff-relevant state variable k_t. A sequence of Markov strategies $\left\{\left(i^t(k_t), c_y^t(k_t), c_o^t(k_t)\right)\right\}_{t=0}^{\infty}$ is a symmetric Markov perfect equilibrium if it is a subgame perfect equilibrium for every realization of the state variable k_t, and all governments follow the same strategy, that is, if $\left(i^t(k_t), c_y^t(k_t), c_o^t(k_t)\right) = (i(k_t), c_y(k_t), c_o(k_t))$, for all t.

Our focus on Markov equilibria helps to understand how future bias distorts optimal intergenerational redistribution when the possibility of commitment is ruled out. This case is interesting on its own and we also build on it below to highlight the potential role of a pay-as-you-go pension system as a commitment mechanism to increase future old-age transfers.

Krusell and Smith (2003) show that there is a large set of equilibria for the type of game we consider (see below) even when attention is restricted to Markov equilibria.[10] Following Laibson (1997) and Krusell et al. (2002), we restrict our attention to the unique Markov equilibrium that is a limit of finite-horizon equilibria.

4 Benchmark commitment solution

In order to understand the consequences of future bias it will be useful to consider first a benchmark problem for an arbitrary government under the assumption that it can control future allocations. The following proposition characterizes the solution to this problem.

[10] Their indeterminacy result refers to decision makers with quasi-hyperbolic preferences, but it holds for the case of future as well as present bias. The problem is that disagreement between two consecutive decision makers can be used to support a continuum of Markov equilibria with discontinuous investment strategies.

Proposition 3 *If the date-t government could precommit future allocations, optimal allocations would be given by $i(k) = gk$, $c_y(k) = \tau (A-g) k$, $c_o(k) = (1-\tau)(A-g) k$, with*

$$(\tau, g) = \begin{cases} (\tau_c, g_c) & \text{in the first period} \\ (\overline{\tau}_c, \overline{g}_c) & \text{in every future period,} \end{cases}$$

$$\tau_c = \frac{1}{1+\left(\frac{1+\theta a}{\delta+a}\right)^{1/\sigma}} > \overline{\tau}_c = \frac{1}{1+\delta^{-1/\sigma}},$$

$$1+g_c = \frac{A+1}{1+\left(\frac{q(\tau_c,a)}{q(\overline{\tau}_c,0)}\frac{\delta^{-1}-(1+\overline{g}_c)^{1-\sigma}}{(A-\overline{g}_c)^{1-\sigma}}\right)^{1/\sigma}} > 1+\overline{g}_c = \left[\delta\left(A+1\right)\right]^{1/\sigma},$$

where $q(\tau, a) = \tau^{1-\sigma} + \frac{1+\theta a}{\delta+a}(1-\tau)^{1-\sigma}$.

The nature of this first-best solution is clarified by formulating the date-t government problem recursively. We will simplify notation by avoiding time subscripts and using primes to denote next-period values whenever possible.

First, consider the static intergenerational allocation of consumption every period from the viewpoint of the date-t government. At date t, the optimal intergenerational allocation of consumption solves the static problem

$$\max_{c^y, c^o} \left\{ (\delta+a)\, u(c^y) + (1+\theta a)\, u(c^o) \right\} \text{ subject to } c^y + c^o \leq c, \text{ with } c^y, c^o \geq 0, \tag{10}$$

and so the young's share of aggregate consumption is given by τ_c. By contrast, from date $t+1$ onwards, the date-t government would choose intergenerational consumption allocations differently than future governments would actually do. Instead, the date-t government's optimal allocation would solve the static problem

$$\max_{c^y, c^o} \left\{ u(c^y) + \delta^{-1} u(c^o) \right\} \text{ subject to } c^y + c^o \leq c, \text{ with } c^y, c^o \geq 0. \tag{11}$$

Accordingly, the young's share of aggregate consumption at every future date would be $\overline{\tau}_c$. It is easy to see that $\tau_c > \overline{\tau}_c$ if and only if $\theta < \delta^{-1}$. Thus, the date-t government prefers to allocate a larger share of aggregate consumption to the current young than the share he would like to allocate to the young in every future period.

Conveniently, with isoelastic utility, one can express the relevant preferences for the date-t government in terms of aggregate consumption levels as

$$\widetilde{V}_t = q\left(\tau_c, a\right) u\left(c_t\right) + q\left(\overline{\tau}_c, 0\right) \sum_{s=1}^{\infty} \delta^s u\left(c_{t+s}\right), \tag{12}$$

where $q(\tau, a)$ is given by Proposition 3. Note that $\widetilde{V}_t$ is simply a positive linear transformation of the utility function V_t given in equation (9). In order to apply Definition 1, let $w\left(\tau_{t+s}, c_{t+s}, s\right) = d(s)\, u\left(c_{t+s}\right)$, with

$$d(s) = \begin{cases} 1 & \text{if} \quad s = 0 \\ \beta\delta^s & \text{if} \quad s \geq 1 \end{cases}$$

where

$$\beta \equiv \frac{q\left(\overline{\tau}_c, 0\right)}{q\left(\tau_c, a\right)},$$

in which case each government discounts consumption streams starting then according to the sequence 1, $\beta\delta$, $\beta\delta^2$, $\beta\delta^3$, ... and so the discount factor between the current period and the next is $\beta\delta$ whereas the discount factor between any two future periods is δ.

The social preferences given in (12) would be present biased if $\beta < 1$, in which case governments would have quasi-hyperbolic preferences over aggregate consumption streams of the form proposed by Phelps and Pollak (1968). Here, however, not only is β endogenous, but also greater than one, because $q\left(\tau_c, a\right) < q\left(\overline{\tau}_c, 0\right)$ if and only if $\theta\delta < 1$ (see Appendix).

To understand the source of β-δ discounting, note that equation (12) characterizes individual preferences in terms of only two objects: the flow utility from current aggregate consumption and the discounted sum of utilities from the stream of future aggregate consumption using the constant discount factor δ. Moreover, the trade-off between these two objects is characterized by the distribution of consumption across living generations today relative to the distribution at any other future date. The future bias emerges because the current government is more reluctant to transfer resources from the young to the old today than at any future date.

Now consider the date-t government's investment problem. From date $t+1$ onwards, the date-t government would solve the following problem:

$$W(k) = \max_{0 \leq k' \leq Ak} \{q(\overline{\tau}_c, 0)\, u(Ak - k' + k) + \delta W(k')\}. \tag{13}$$

It is easy to verify that the solution to the above standard dynamic programming problem implies that investment from date $t+1$ onwards is given by $k' - k = \overline{g}_c k$, where $\overline{g}_c$ is given by Proposition 3.

Finally, the investment problem at date t can be formulated as

$$W_0(k) = \max_{0 \leq k' \leq Ak} \{q(\tau_c, a)\, u(Ak - k' + k) + \delta W(k')\}. \tag{14}$$

One can verify that the solution to problem (14) implies that investment at date t is given by $i(k) = g_c k$, where g_c is given by Proposition 3.

In the Appendix we show that $g_c > \overline{g}_c$, for all $\sigma > 0$. If the date-t government could commit future allocations, it would choose a current growth rate that is larger than the growth rate it would dictate to future generations. Thus, there is a "pro-growth" bias associated with the future bias in the government's preferences. Indeed, it can be verified that g_c is equal to $\overline{g}_c$ if and only if $\theta\delta = 1$.

Of course, the problem with the solution characterized in Proposition 3 is that it is time inconsistent. In the following section we consider equilibrium behavior when current governments recognize that future allocations will be chosen optimally by future governments.

5 Markov perfect equilibrium

In equilibrium, each government recognizes that every future government will choose the same optimal intergenerational allocation of consumption each period as the one chosen in the current period by the current government. This is the allocation that solves the static problem given by equation (10) and so the young's share of aggregate consumption is now given by $\tau^* = \tau_c$ every period, where τ_c is given by Proposition 3.

Our previous arguments imply that the relevant date-t government's preferences can be expressed in terms of aggregate consumption levels as

$$\widehat{V}_t = q(\tau^*, a) u(c_t) + q(\tau^*, 0) \sum_{s=1}^{\infty} \delta^s u(c_{t+s}), \tag{15}$$

where $q(\tau, a)$ is given by Proposition 3. Given τ^*, our problem has the same structure as the one originally analyzed by Phelps and Pollak (1968) and more recently Laibson (1997), Krusell et al. (2002) and Krusell and Smith (2003). Accordingly, there is a unique Markov perfect equilibrium that is the limit of finite-horizon equilibria. Here, one can verify by explicit backward induction that the corresponding investment strategies are linear.

In our case, however, the short-term discount factor (i.e., the β in the β-δ discounting function) is endogenous and greater than one, reflecting an endogenous future bias, rather than an exogenous present bias. Here, one can verify that $q(\tau^*, 0) > q(\tau^*, a)$ if and only if $\theta\delta < 1$ (see Appendix). By comparing (12) and (15), one can also verify that the strength of the future bias, as given by the short-term discount factor, in the Markov equilibrium is different from that in the commitment solution.

To construct such an equilibrium, suppose that the current government anticipates that every future government follows the linear investment policy $i' = \widehat{g}k'$, with $\delta(1+\widehat{g})^{1-\sigma} < 1$. Then, the current investment decision solves the following problem:

$$V_0(k) = \max_{0 \le k' \le Ak} \{q(\tau^*, a) u(Ak - k' + k) + \delta V(k')\}, \tag{16}$$

with

$$V(k) = q(\tau^*, 0) u(Ak - (1+\widehat{g}) k + k) + \delta V((1+\widehat{g}) k), \tag{17}$$

where $q(\tau, a)$ is given by Proposition 3. An investment policy $i(k) = gk$ that is part of a symmetric Markov perfect equilibrium must have $g = \widehat{g}$.

In the Appendix, we show that the above dynamic programming problem implies

$$1 + g = \frac{A+1}{1 + \left(\frac{q(\tau^*, a)}{q(\tau^*, 0)} \frac{\delta^{-1} - (1+\widehat{g})^{1-\sigma}}{(A-\widehat{g})^{1-\sigma}} \right)^{1/\sigma}} \equiv 1 + B(\tau^*, \tau^*, \widehat{g}). \tag{18}$$

The best response mapping $g = B(\tau, \tau', \widehat{g})$ characterizes the best investment response by a government that allocates a share τ of current consumption to the current young and anticipates that future governments will allocate a share τ' of consumption to the young and invest according to $i(k') = \widehat{g}k'$. Note that the above commitment solution has $g_c = B(\tau_c, \overline{\tau}_c, \overline{g}_c)$, whereas the Markov equilibrium has $g^* = B(\tau^*, \tau^*, g^*)$.

Proposition 4 *(i) There is a unique symmetric, interior, Markov perfect equilibrium in linear strategies. The equilibrium is characterized by* $i(k) = g^*k$, $c_y(k) = \tau^*(A - g^*)k$, *and* $c_o(k) = (1-\tau^*)(A-g^*)k$, *with* $\tau^* = \tau_c$ *and* $g^* = B(\tau^*, \tau^*, g^*) \in (\overline{g}_c, A)$, *where* B *is given by equation (18) and* τ_c *and* $\overline{g}_c$ *are given by Proposition 3. (ii) For all* $\widehat{g} \in (\overline{g}_c, A)$, $\partial B(\tau^*, \tau^*, \widehat{g}) / \partial \widehat{g} \geq 0$ *if and only if* $\sigma \geq 1$, *with equality if and only if* $\sigma = 1$.

This proposition provides additional insight into the role of commitment problems. Note that the disagreement between governments about investment decisions takes the particular form that the date-$(t+1)$ government invests too much from the viewpoint of the date-t government. The best response mapping (18) indicates how each government will attempt to manipulate investment next period. Part (ii) of the proposition implies that locally around the equilibrium current and next-period investments are strategic complements if $\sigma > 1$ and strategic substitutes if $\sigma < 1$. The panels in Figure 1 plot the different types of best responses.

[FIGURE 1]

Panel (1) shows that $B(\tau^*, \tau^*, \widehat{g})$ decreases at first, reaches a minimum at $\overline{g}_c$, then increases, when $\sigma > 1$. Panel (2) shows that the best response is flat when $\sigma = 1$. In this case, $g^* = B(\tau^*, \tau^*, g^*)$ has a closed-form solution and the equilibrium growth rate is given by

$$1 + g^* = \frac{A+1}{1 + \left(1 + \frac{1+\theta a}{\delta + a}\right)\left(\frac{1-\delta}{1+\delta}\right)}.$$

Panel (3) in the above figure illustrates that $B(\tau^*, \tau^*, \widehat{g})$ increases at first, peaking at $\overline{g}_c$, and then decreases, when $\sigma < 1$.

The role of the elasticity of intertemporal substitution, given by $1/\sigma$, is worth noting. With respect to a generation's lifetime, higher values of σ indicate greater aversion to differences in consumption over the life cycle. However, since individuals are altruistic, higher values of σ also indicate greater aversion to unequal consumption across generations. With balanced growth, the higher the value of σ, the less individuals are willing to tolerate larger positive, or smaller negative, growth rates.

Next, we compare the equilibrium growth rate (Proposition 4) and the growth rate that governments would choose if they could commit all future allocations (Proposition 3).

Proposition 5 *$g^* > g_c$ if and only if $\sigma > 1$, that is, whenever the elasticity of intertemporal substitution is less than one ($1/\sigma < 1$), the equilibrium growth rate is higher than the growth rate every generation would set if they were able to control future allocations.*

Proposition 5 is a striking result for two reasons. First, note that $g_c > \overline{g}_c$, where $\overline{g}_c$ is the first-best growth rate from the viewpoint of the old, and it is also the growth rate that every young generation, and every government, would dictate on every future generation, if they could do so. In this sense, commitment problems lead to equilibrium growth that is too high, relative to the preferences of all generations. Second, the private return to investment is lower than the social return to investment. The latter is given by the constant marginal product of capital A, whereas the former is given by $A - \partial\left(g^* k\right)/\partial k = A - g^*$.

In order to understand the source of the result stated in Proposition 5, it is useful to consider the relationship between the current investment decisions of a government with and without commitment. Note that the investment problem of the date-t government at date t, given by equation (16), can be written as

$$V_0\left(k\right) = \max_{0 \leq k' \leq Ak} \left\{ \widetilde{W}_0\left(k, k'\right) - \delta\left(W\left(k'\right) - V\left(k'\right)\right) \right\} \tag{19}$$

where W and V are given by equation (13) and equation (17), respectively, and where $\widetilde{W}_0\left(k, k'\right)$ is precisely the objective to be maximized at date t under the assumption that the date-t government can commit future allocations (see equation (14)), that is,

$$W_0(k) = \max_{0 \leq k' \leq Ak} \widetilde{W}_0(k, k'). \tag{20}$$

Clearly, it must be that $V_0(k) < W_0(k)$, since the commitment solution from date $t+1$ onwards is the date-t government's first-best solution (i.e., since $W(k') - V(k') > 0$). Thus, whenever the current government anticipates $(\tau^*, \widehat{g})$ to deviate from $(\overline{\tau}_c, \overline{g}_c)$ in the future, it anticipates a welfare loss. Accordingly it has an incentive to invest strategically to compensate for this loss. Formally, the marginal effect of additional current investment on the welfare loss associated with the difference between $\widehat{g}$ and $\overline{g}_c$ in the future is given by $\partial W(k')/\partial k' - \partial V(k')/\partial k'$, where $\partial W(k')/\partial k'$ is given by equation (23) and $\partial V(k')/\partial k'$ is given by equation (26). It is easy to verify that $\partial W(k')/\partial k' - \partial V(k')/\partial k' \leq 0$ if and only if $B(\tau^*, \tau^*, \widehat{g}) \geq B(\tau_c, \overline{\tau}_c, \overline{g}_c)$, where recall that the best-response mapping $B(\tau^*, \tau^*, \widehat{g})$ is given by equation (18), with $B(\tau^*, \tau^*, g^*) = g^*$ and $B(\tau_c, \overline{\tau}_c, \overline{g}_c) = g_c$.

To understand the above "strategic-compensation effect", note that the anticipated discrepancy between $(\tau^*, \widehat{g})$ and $(\overline{\tau}_c, \overline{g}_c)$ gives rise to two opposing effects. The problem arises because future governments weigh future consumption too little relative to the current government, that is, $q(\tau^*, a) < q(\tau^*, 0)$. On the one hand, for given next-period consumption, next-period utility is anticipated to be lower because next-period's government will misallocate consumption over the two generations. The current government can compensate for this loss by strategically raising investment in order to increase next-period aggregate consumption. On the other hand, transferring wealth to the future has a lower return, because the increase in production is misallocated over the two coexisting generations: by strategically lowering investment the current government can substitute intertemporally away from misallocated future investment.

With log utility, the current government is in effect unable to use current investment strategically to its advantage, as the welfare loss from misallocation of future investment exactly offsets the welfare gain from additional future consumption. Consequently, the current government's best response to any future growth rate $\widehat{g} \geq \overline{g}_c$ is given by $i(k) = g_c k$ when $\sigma = 1$. Since every future government faces the same problem, each government will choose the growth rate that it would be chosen in the first period if future allocations could be controlled. Thus, the resulting equilibrium growth rate g^* must be equal to g_c. This outcome reflects the pro-growth bias inherent to the social preferences that aggregate the preferences of coexisting generations that disagree about current investment: the young would like faster current growth than the old. This is the “future-bias effect” that underlaid the relatively high *short-run* growth rate in the benchmark commitment solution. Now, however, this effect translates into higher *long-run* equilibrium growth, relative to the commitment solution.

If inequality aversion is large enough (i.e., if $\sigma > 1$), the welfare loss from misallocation of future investment cannot offset the welfare gain from additional future consumption and thus, each government has a strategic incentive to overinvest, relative to $i(k) = g_c k$. This explains the strategic complementarity between current and next-period investments (see Proposition 4), which leads to a long-run equilibrium growth rate g^* that is not only higher than $\overline{g}_c$, but also higher than g_c.

6 Intergenerational redistribution

In this section we consider the analogue of Proposition 1 for the model with altruism. To that end, suppose that the date-t government can precommit future transfers, provided that it treats current and future generations symmetrically, by making proportional transfers identical at all points in time. Further, suppose that the date-t government can control *current,* but not future, investment. Thus, once transfers are legislated, consecutive governments will choose investment unilaterally, taking into account the investment strategies of future governments.

Conditional on the share τ, our previous analysis implies that there is a symmetric Markov perfect equilibrium in linear strategies such that equilibrium growth solves $g = B(\tau, \tau, g)$, where B is given by (18). Letting $g(\tau)$ be a solution to this equation, one can easily verify that our previous analysis implies that the date-t government's optimal choice of τ solves the following problem:

$$\max_{\tau\in[0,1]} \left\{ q(\tau,a)\, u\left(\left(A-g(\tau)\right)k\right) + \frac{\delta q(\tau,0)}{1-\delta\left(1+g(\tau)\right)^{1-\sigma}} u\left(\left(A-g(\tau)\right)\left(1+g(\tau)\right)k\right) \right\}. \quad (21)$$

Let $\overline{\tau}$ denote a solution to this problem. We have the following analogue of Proposition 1.

Proposition 6 *Suppose that $\sigma > 1$. The date-t government legislates old-age transfers ($\overline{\tau} < \tau^*$), even though the legislation will depress equilibrium growth ($g(\overline{\tau}) < g^*$) and so it will hurt an infinite number of future generations.*

Proposition 6 provides a positive theory of intergenerational redistribution. The main insight here is that institutions that will necessarily harm future generations are supported because future old-age transfers are too low and future growth is too high *from the perspective of currently living generations*, not because equilibrium investment is dynamically inefficient. The same arguments we used in the case of non-altruistic generations continue to apply here, implying that equilibrium investment is dynamically efficient when generations are altruistic.

Moreover, it should be noted that the Markov perfect equilibrium, both before and after the legislation, is not Pareto efficient, because the private and the social return to investment are different. A Pareto improvement would result from investing optimally from the viewpoint of the currently young generation at the socially optimal rate of return, without changing the allocation for any other generation. This is in contrast with the common perception that non-paternalistic altruism towards the following generation must lead to Pareto efficiency (Streufert, 1993). This is the case in the non-overlapping generations models studied in the literature, because non-paternalistic altruism then amounts to time-consistent preferences. However, with time-inconsistent social preferences, as is the case here, the private return to investment is necessarily lower than the social return, because the incentive to manipulate future investment does not disappear.

Proposition 6 rests on the fact that every government has future biased preferences. Consequently, future old-age transfers are too low from the perspective of the current government (and that of both living generations). Accordingly, the current government has an incentive to legislate intergenerational redistribution to increase future old-age transfers at the expense of growth. When $\sigma > 1$, current and future investments are strategic complements and so the legislation ends up lowering investment and growth. Note that the relationship between transfers and growth is such that $g'(\tau) > 0$ if and only if $\sigma > 1$. Thus, when income effects dominate substitution effects, old-age transfers, given by $1 - \tau$, and growth are negatively related, as one may expect.

Recall that Proposition 5 shows that even if the current government could control all future allocations, instead of just being able to control old-age transfers, it would choose to lower growth below the equilibrium rate. This shows that Proposition 6 does not rely on the fact that the government has limited instruments of intergenerational redistribution.

Clearly, the above legislation hurts future generations. Yet, it is self-enforcing in the following sense.

Corollary 1 Once stationary old-age transfers given by $\overline{\tau}$ are legislated, no government will ever support a proposal to switch to an alternative stationary sequence of old-age transfers.

Each future government chooses to sustain the current legislation because it faces essentially the same problem as the government that enacted the original legislation, given the capital stock that it inherits. The restriction to stationary sequences of transfers allows for a simple commitment device. For instance, this may be the case if the date-t government can legislate a transfer at date t that is sufficiently costly to change in the future (Boadway and Wildasin 1989), or if the stationary sequence of transfers is supported by the threat of collapse of the system if any government repeals the legislation (Cooley and Soares 1999).[11]

7 Conclusion

We have shown that intergenerational altruism tends to be future biased if generations overlap and people's altruism concerns the well-being of immediate ancestors and descendants. The source of the future bias is the positive discounting of the consumption utility of others,

[11] Azariadis and Galasso (2002) argue that giving current voters or policymakers some veto power over changes in future policies acts like a commitment device.

which implies that the ranking of consumption allocations can be reversed with the passage of age. This is because young individuals are more reluctant to transfer resources from themselves to the living old than from the young to the old at any future date.

We have tracked a conflict of interest between current and future utilitarian governments representing living generations to the fact that every government inherits the future bias of individual preferences and so it is more reluctant to transfer resources from the young to the old currently than at any future date. We have analyzed the implications of this conflict for intergenerational redistribution in the context of a tractable example, where consumption utilities are isoelastic and, consequently, the optimal sharing rule to allocate consumption between coexisting generations every date is independent of aggregate consumption. Conditional on a stationary sharing rule, preferences over aggregate consumption streams exhibit β-δ discounting with $\beta > 1$, where the value of the short-term discount factor β, hence the strength of the future bias, is a function of the consumption sharing rule.

Here, β-δ discounting is a reflection of indirect, non-paternalistic altruism. In Galperti and Strulovici's (2017) non-overlapping generations model, β-δ discounting reflects direct, as opposed to indirect, non-paternalistic altruism. These results challenge the common view that non-paternalistic altruism and time consistency are two sides of the same coin (e.g., Phelps and Pollak 1968, Barro 1974). Yet, whether altruism is present biased or future biased depends on the demographic structure and the specific model of altruism.

Our example illustrates how a pay-as-you-go pension plan can be understood, from the viewpoint of every successive government, as a commitment device to increase future old-age transfers. This sheds new light on the widespread legislation of pay-as-you-go social security despite recognition of its negative effects on capital accumulation. Our analysis implies that social security legislation systematically favors current generations at the expense of future generations. Yet, future governments do not have an incentive to repeal the legislation, because each future government faces essentially the same problem as the government that introduced the original legislation, given the capital stock that it inherits.

Strotz's (1956) seminal work and more recently Laibson's (1997) demonstrate the general relevance of economic agents' time inconsistency for the design of institutions that can cope with intertemporal disagreement by facilitating commitments. With respect to this, an implication of our analysis is that the availability of commitment mechanisms to cope with intergenerational disagreement can harm future generations.

Appendix

Proof of Proposition 1

Assume that $\delta = 0$ and $\sigma > 1$. Let $U(\tau, g(\tau))$ denote the objective function in (6). Since $U(\tau, g(\tau))$ is a continuous function of τ on $[0,1]$, it must have a maximum. Moreover, U is differentiable on $(0,1)$ with

$$\frac{dU}{d\tau} = \frac{\partial U}{\partial \tau} + \frac{\partial U}{\partial g} g'(\tau).$$

Now we prove that $dU/d\tau < 0$ for all $\tau \geq \tau^*$. First, note that $\frac{\partial U}{\partial \tau} < 0$ for all $\tau \geq \tau^*$, because $\tau^{-\sigma} - \frac{1}{a}(1-\tau)^{-\sigma} \leq 0$ for all $\tau \geq \tau^*$ and

$$\frac{\partial U}{\partial \tau} = ((A-g)k_t)^{1-\sigma} a \left(\tau^{-\sigma} - \frac{1}{a}(1-\tau)^{-\sigma} - (1+g)^{1-\sigma}(1-\tau)^{-\sigma} \right).$$

Then, note that $\frac{\partial U(\tau,g)}{\partial g} < 0$, for $g = g(\tau)$ and for all $\tau \in [0,1]$, because

$$\begin{aligned} \frac{\partial U(\tau,g)}{\partial g} &= ((A-g)k_t)^{-\sigma} k_t a (1-\tau)^{1-\sigma} (1+g)^{-\sigma} \left[-\alpha(\tau)(1+g)^{\sigma} + A - g - (1+g) \right] \\ &= -((A-g)k_t)^{-\sigma} k_t a (1-\tau)^{1-\sigma} (1+g)^{1-\sigma}, \end{aligned}$$

where the second equality follows from the fact that $\alpha(\tau)(1+g(\tau))^{\sigma} = A - g(\tau)$. Differentiating this last equation with respect to τ and g, we have that $g'(\tau) > 0$ if and only if $\sigma > 1$. Hence, $dU/d\tau < 0$ for all $\tau \geq \tau^*$, which implies that $\overline{\tau} < \tau^*$.

From (6), it is easy to verify that $\lim_{\tau \to 0} U(\tau, g(\tau)) = -\infty$. Therefore, we have $\overline{\tau} \in (0, \tau^*)$, which implies $g(\overline{\tau}) \in (-1, g^*)$. This concludes the proof. **QED**

Proof of Proposition 2

Applying Definition 1 with

$$w(\tau_{t+s}, c_{t+s}, s) = \begin{cases} u^y(\tau_t c_t) + \theta u^o((1-\tau_t)c_t) & \text{if } \; s = 0 \\ \delta^s \left[u^y(\tau_{t+s} c_{t+s}) + \delta^{-1} u^o((1-\tau_{t+s})c_{t+s}) \right] & \text{if } \; s \geq 1 \end{cases}$$

one can easily verify Part (i) of the proposition. Similarly, applying Definition 1 with

$$w(\tau_{t+s}, c_{t+s}, s) = \begin{cases} u^y(\tau_t c_t) + \left(\frac{1+\theta a}{\delta + a}\right) u^o((1-\tau_t)c_t) & \text{if } \; s = 0 \\ \delta^s \left[u^y(\tau_{t+s} c_{t+s}) + \delta^{-1} u^o((1-\tau_{t+s})c_{t+s}) \right] & \text{if } \; s \geq 1, \end{cases}$$

Part (ii) of the proposition can be readily verified. **QED**

Proof of Proposition 3

The structure of the commitment solution follows from the discussion in the main text. The consumption shares τ_c and $\overline{\tau}_c$ are the unique solutions to problems (10) and (11), respectively. In order to derive g_c and $\bar{g}_c$, first consider the problem given by (13). It is straightforward to derive the Euler equation

$$\frac{\partial u(c)/\partial c}{\delta\left(\partial u(c')/\partial c'\right)} = \frac{-\partial c'/\partial k'}{\partial c/\partial k'}.$$

Taking derivatives and noting that consumption and capital grow at the common rate $\overline{g}_c$, it can be verified that the solution to the above standard dynamic programming problem implies that investment from date $t+1$ onwards is given by $k' - k = \overline{g}_c k$, where $\overline{g}_c$ is given by Proposition 3.

Now, consider the investment problem at date t, given by (14). The first-order condition for an interior solution is

$$-q(\tau_c, a)\frac{\partial u(c)}{\partial c}\frac{\partial c}{\partial k'} = \delta\frac{\partial W(k')}{\partial k'}. \tag{22}$$

Noting that

$$\delta W(k_{t+1}) = \sum_{s=1}^{\infty}\delta^s q(\overline{\tau}_c, 0)\, u(c_{t+s}),$$

and noting that consumption from date $t+1$ onwards grows at the constant growth rate $\overline{g}_c$, one can verify that the value of future capital is such that

$$W(k') = \left(\frac{q(\overline{\tau}_c, 0)}{1-\delta(1+\overline{g}_c)^{1-\sigma}}\right) u(c'),$$

therefore, the marginal value of additional capital next period is given by

$$\frac{\partial W(k')}{\partial k'} = \left(\frac{q(\overline{\tau}_c, 0)}{1-\delta(1+\overline{g}_c)^{1-\sigma}}\right)\frac{\partial u(c')}{\partial c'}\frac{\partial c'}{\partial k'}. \tag{23}$$

Combining equations (22) and (23), it is easy to derive the Euler equation

$$\frac{\partial u(c)/\partial c}{\delta\left(\partial u(c')/\partial c'\right)} = \left(\frac{q(\overline{\tau}_c, 0)/q(\tau_c, a)}{1-\delta(1+\overline{g}_c)^{1-\sigma}}\right)\frac{-\partial c'/\partial k'}{\partial c/\partial k'}. \tag{24}$$

Using the facts that instantaneous utility functions are isoelastic, the aggregate resources constraint holds with equality and investment from date $t+1$ onwards is given by $i(k) = \overline{g}_c k$, to evaluate equation (24), one can verify that the solution to problem (14) implies that investment at date t is given by $i(k) = g_c k$, where g_c is given by Proposition 3.

It remains to show that $g_c > \overline{g}_c$. To prove this, let

$$1 + \tilde{B}(\hat{g}, Q) \equiv \frac{A+1}{1 + Q\left(\frac{\delta^{-1} - (1+\widehat{g})^{1-\sigma}}{(A-\widehat{g})^{1-\sigma}}\right)^{1/\sigma}},$$

and note that $\tilde{B}(\overline{g}_c, Q_c) = g_c$, where

$$Q_c \equiv \left(\frac{q(\tau_c, a)}{q(\overline{\tau}_c, 0)}\right)^{1/\sigma}.$$

Note that $Q_c = \overline{\tau}_c/\tau_c$, thus $Q_c < 1$ if and only if $\theta\delta < 1$. Also note that $\tilde{B}(\overline{g}_c, 1) = \overline{g}_c$. Since $\partial\tilde{B}(\hat{g}, Q)/\partial Q < 0$, it follows that $\tilde{B}(\overline{g}_c, Q_c) = g_c > \tilde{B}(\overline{g}_c, 1) = \overline{g}_c$, as required. **QED**

Proof of Proposition 4

To derive the best response mapping (18), first note that the first-order condition with respect to k' at date t is given by

$$-q(\tau^*, a)\frac{\partial u(c)}{\partial c}\frac{\partial c}{\partial k'} = \delta\frac{\partial V(k')}{\partial k'}. \tag{25}$$

Solving the recursion in equation (17) it can be verified that

$$V(k') = \left(\frac{q(\tau^*, 0)}{1 - \delta(1+\widehat{g})^{1-\sigma}}\right) u(c')$$

and so we have

$$\frac{\partial V(k')}{\partial k'} = \left(\frac{q(\tau^*, 0)}{1 - \delta(1+\widehat{g})^{1-\sigma}}\right)\frac{\partial u(c')}{\partial c'}\frac{\partial c'}{\partial k'}. \tag{26}$$

Combining equations (25) and (26), the relevant Euler equation is given by

$$\frac{\partial u(c)/\partial c}{\delta(\partial u(c')/\partial c')} = \left(\frac{q(\tau^*, 0)/q(\tau^*, a)}{1 - \delta(1+\widehat{g})^{1-\sigma}}\right)\frac{-\partial c'/\partial k'}{\partial c/\partial k'}. \tag{27}$$

Recognizing that

$$\frac{\partial c'}{\partial k'} = A - \frac{\partial i\,(k')}{\partial k'} = A - \widehat{g},$$

since $c' = Ak' - i\,(k')$ and $i\,(k') = \widehat{g}k'$, and using the facts that instantaneous utility functions are isoelastic and the aggregate resources constraint holds with equality, it is straightforward to write the above Euler equation as

$$\left(\frac{k'}{Ak - k' + k}\right)^{\sigma} = \frac{q\,(\tau^*, 0)\,(A - \widehat{g})^{1-\sigma}}{q\,(\tau^*, a)\left(\delta^{-1} - (1 + \widehat{g})^{1-\sigma}\right)},$$

which describes the best response k' to the anticipation of $\widehat{g}$, for given k. Clearly, the best response to any given $\widehat{g}$ is linear in k and we obtain the best-response mapping (18).

Consider Part (ii) of the proposition. From equation (18), $B(\tau^*, \tau^*, \hat{g})$ can be written as $\tilde{B}(\hat{g}, Q^*)$, with

$$Q^* \equiv \left(\frac{q\,(\tau^*, a)}{q\,(\tau^*, 0)}\right)^{1/\sigma},$$

where $\tilde{B}$ is defined in the proof of Proposition 3. One can verify that

$$Q^* = \left(\frac{1 + \left(\frac{1+\theta a}{\delta + a}\right)^{1/\sigma}}{1 + \delta^{-1}\left(\frac{1+\theta a}{\delta + a}\right)^{-1}\left(\frac{1+\theta a}{\delta + a}\right)^{1/\sigma}}\right)^{1/\sigma},$$

hence, $Q^* < 1$ if and only if $\theta\delta < 1$. Next, note that the sign of $\partial\tilde{B}(\hat{g}, Q)/\partial\hat{g}$ is given by the sign of $(\sigma - 1)\left[(1 + \widehat{g})^{\sigma} - \delta(A + 1)\right]$. It is easy to verify that, for given Q^*, $\tilde{B}(\hat{g}, Q^*)$ has a global minimum at $\widehat{g} = \overline{g}_c$ if $\sigma > 1$; it has a global maximum at $\widehat{g} = \overline{g}_c$ if $\sigma < 1$; and it is flat at $\widehat{g} = g_c$ if $\sigma = 1$. This proves Part (ii) of the proposition.

Now consider Part (i). To prove existence of a unique fixed point $g^* \in (\overline{g}_c, A)$, evaluate $g = \tilde{B}(\hat{g}, Q)$ at $\hat{g} = g$ and rewrite it as

$$\delta^{-1}Q^{\sigma}(1 + g)^{\sigma} + (1 - Q^{\sigma})(1 + g) - (A + 1) = 0. \tag{28}$$

As long as $Q \leq 1$, the left side of the equation is increasing in g. Moreover, it is negative when $g = -1$ and positive when $g = A$. Hence, there is exactly one fixed point, $\tilde{g}(Q) < A$, where $g^* = \tilde{g}(Q^*)$. Differentiating equation (28), one can verify that $\partial\tilde{g}(Q)/\partial Q < 0$ if and only if $1 > \delta\,(1 + \tilde{g}(Q))^{1-\sigma}$. For $\sigma \leq 1$, the latter inequality holds since $g \leq A$ and we have assumed $1 > (A+1)^{1-\sigma}\delta$. For $\sigma > 1$, first note that $\tilde{g}(1) = \bar{g}_c$ and $1 > \delta\left(1+\tilde{g}(1)\right)^{1-\sigma}$, so $\partial\tilde{g}(1)/\partial Q < 0$; hence for all $Q \leq 1$ the inequality $1 > \delta\left(1+\tilde{g}(Q)\right)^{1-\sigma}$ holds. Since $Q^* < 1$, for all $\theta\delta < 1$, we have $\tilde{g}(Q^*) = g^* > \tilde{g}(1) = \bar{g}_c$. Therefore, $A > g^* > \bar{g}_c$. All other statements in the proposition are proven in the main text. **QED**

Proof of Proposition 5

First note that

$$\frac{Q_c}{Q^*} = \left(\frac{q\left(\tau_c, 0\right)}{q\left(\overline{\tau}_c, 0\right)}\right)^{1/\sigma} = \left(\frac{\tau_c^{1-\sigma} + \delta^{-1}(1-\tau_c)^{1-\sigma}}{\overline{\tau}_c^{1-\sigma} + \delta^{-1}(1-\overline{\tau}_c)^{1-\sigma}}\right)^{1/\sigma},$$

where the first equality follows from the definitions of Q_c and Q^* and the fact that $\tau^* = \tau_c$, and the second equality follows from the definition of $q\left(\tau, a\right)$. Next, note that the right side of the second equality above is increasing in τ_c for $\sigma > 1$ and is decreasing in τ_c for $\sigma < 1$. Since $\tau_c > (1+\delta^{-1/\sigma})^{-1} = \overline{\tau}_c$, it follows that $Q_c > Q^*$ if $\sigma > 1$ and $Q_c < Q^*$ if $\sigma < 1$. Hence, we have the following: (1) If $\sigma > 1$, then $\tilde{B}(g^*, Q^*) = g^* > \tilde{B}(g^*, Q_c) > \tilde{B}(\bar{g}_c, Q_c) = g_c$, where the first inequality follows from the fact that $\partial\tilde{B}(\hat{g}, Q)/\partial Q < 0$, and the second one from the fact that $g^* > \bar{g}_c$ and $\partial\tilde{B}(g, Q)/\partial g > 0$ for $g > \bar{g}_c$. (2) If $\sigma < 1$, then $\tilde{B}(g^*, Q^*) = g^* < \tilde{B}(g^*, Q_c) < \tilde{B}(\bar{g}_c, Q_c) = g_c$, where the first inequality follows from the fact that $\partial\tilde{B}(\hat{g}, Q)/\partial Q < 0$, and the second one from the fact that $g^* > \bar{g}_c$ and $\partial\tilde{B}(g, Q)/\partial g < 0$ for $g > \bar{g}_c$. It follows that $g^* > g_c$ if and only if $\sigma > 1$, as required. **QED**

Proof of Proposition 6

Assume that $\delta \in (0,1)$ and $\sigma > 1$. Let $U\left(\tau, g\left(\tau\right)\right)$ denote the objective function in (21). Since $U\left(\tau, g\left(\tau\right)\right)$ is a continuous function of τ on $[0,1]$, it must have a maximum. Moreover, U is differentiable on $(0,1)$ with

$$\frac{dU}{d\tau} = \frac{\partial U}{\partial \tau} + \frac{\partial U}{\partial g} g'\left(\tau\right).$$

Now we prove that $dU/d\tau < 0$ for all $\tau \geq \tau^*$. First, note that $\frac{\partial U}{\partial \tau} < 0$ for all $\tau \geq \tau^*$, because $\tau^{-\sigma} - \frac{1+\theta a}{\delta+a}\left(1-\tau\right)^{-\sigma} \leq 0$ and $\tau^{-\sigma} - \frac{1}{\delta}\left(1-\tau\right)^{-\sigma} < 0$ for all $\tau \geq \tau^*$ and

$$\frac{\partial U}{\partial \tau} = \left(\left(A-g\right)k\right)^{1-\sigma}\left(\tau^{-\sigma} - \frac{1+\theta a}{\delta+a}\left(1-\tau\right)^{-\sigma} + \left(\tau^{-\sigma} - \frac{1}{\delta}\left(1-\tau\right)^{-\sigma}\right)\left(\frac{\delta\left(1+g\right)^{1-\sigma}}{1-\delta\left(1+g\right)^{1-\sigma}}\right)\right).$$

Then, note that $\frac{\partial U(\tau,g)}{\partial g} < 0$, for $g = g(\tau)$ and for all $\tau \in [0,1]$, because $q(\tau,a) < q(\tau,0)$ and

$$\begin{aligned}\frac{\partial U(\tau,g)}{\partial g} &= k^{1-\sigma}(A-g)^{-\sigma}\left[\begin{array}{c}-q(\tau,a)\\ +\left(\frac{\delta q(\tau,0)(1+g)^{-\sigma}}{1-\delta(1+g)^{1-\sigma}}\right)\left(A-g-(1+g)+\frac{(A-g)\delta(1+g)^{1-\sigma}}{1-\delta(1+g)^{1-\sigma}}\right)\end{array}\right]\\ &= k^{1-\sigma}(A-g)^{-\sigma}\left(\frac{\delta q(\tau,0)(1+g)^{1-\sigma}}{1-\delta(1+g)^{1-\sigma}}\right)\left(\frac{q(\tau,a)}{q(\tau,0)}-1\right),\end{aligned}$$

where the second equality follows from the fact that $g(\tau)$ satisfies $g = B(\tau,\tau,g)$, hence

$$\frac{q(\tau,a)}{q(\tau,0)} = \frac{\delta(1+g)^{-\sigma}}{1-\delta(1+g)^{1-\sigma}}(A-g).$$

Differentiating this last equation with respect to τ and g, we have that $g'(\tau) > 0$ if and only if $\sigma > 1$. Hence, $dU/d\tau < 0$ for all $\tau \geq \tau^*$, which implies that $\overline{\tau} < \tau^*$.

It is easy to verify that the proof of Proposition 4 implies that $g(\overline{\tau}) > \bar{g}_c$, with $\overline{g}_c = [\delta(A+1)]^{1/\sigma} - 1$. Therefore, we have $\overline{\tau} \in (0,\tau^*)$ and $g(\overline{\tau}) \in (\bar{g}_c, g^*)$, as required. **QED**

References

Alesina, A. and G. Tabellini (1990): "A positive theory of fiscal deficits and government debt", *Review of Economic Studies* 57, 403-414.

Auerbach, A.J. and L.J. Kotlikoff (1987): *Dynamic Fiscal Policy*, Cambridge: Cambridge University Press.

Azzimonti, M. (2011): "Barriers to investment in polarized societies", *American Economic Review* 101, 2182-2204.

Azariadis C. and V. Galasso (2002): "Fiscal Constitutions", *Journal of Economic Theory* 103, 255-281.

Barro, R.J. (1974): "Are government bonds net wealth?", *Journal of Political Economy* 82, 1095-1117.

Barro, R.J. (1999): "Ramsey meets Laibson in the Neoclassical growth model", *Quarterly Journal of Economics* 114, 1125-1152.

Bergstrom, T.C. (1999): "Systems of benevolent utility functions", *Journal of Public Economic Theory* 1, 71-100.

Bernheim, B.D. (1989): "Intergenerational altruism, dynastic equilibria and social welfare", *Review of Economic Studies* 56, 119-128.

Bernheim, B.D. and D. Ray (1987): "Economic growth with intergenerational altruism", *Review of Economic Studies* 54, 227-241.

Boadway, R.W. and D.E. Wildasin (1989): "A median voter model of social security", *International Economic Review* 30, 307-328.

Burbidge, J.B. (1983): "Government debt in an overlapping-generations model with bequests and gifts", *American Economic Review* 73, 222-227.

Calvo, G.A. and M. Obstfeld (1988): "Optimal time-consistent fiscal policy with finite lifetimes", *Econometrica*, 56, 411-432.

Caplin, A. and J. Leahy (2004): "The social discount rate", *Journal of Political Economy* 112, 1257-1268.

Cooley, T.F. and J. Soares (1999): "A positive theory of social security based on reputation", *Journal of Political Economy* 107, 135-160.

Galperti, S. and B. Strulovici (2017): "A theory of intergenerational altruism", *Econometrica*, forthcoming.

Gollier, C. and R. Zeckhauser (2005): "Aggregation of heterogeneous time preferences", *Journal of Political Economy* 113, 878-896.

Gollier, C. and M. Weitzman (2010): "How should the distant future be discounted when discount rates are uncertain?" *Economic Letters* 107, 350-353.

Grossman, G.M., Helpman, E. (1998): "Intergenerational redistribution with short-lived governments", *Economic Journal* 108, 1299-1329.

Halac M. and P. Yared (2014): "Fiscal rules and discretion under persistent shocks", *Econometrica* 82, 1557-1614.

Halevy, Y. (2008): "Strotz meets Allais: diminishing impatience and the certainty effect", *American Economic Review* 98, 1145-1162.

Halevy, Y. (2015): "Time consistency: stationarity and time invariance", *Econometrica* 83, 335-352.

Hansson, I. and C. Stuart (1989): "Social security as trade among living generations", *American Economic Review* 79, 1182-1195.

Hori, H. and S. Kanaya (1989): "Utility functionals with nonpaternalistic intergenerational altruism", *Journal of Economic Theory* 49, 241-265.

Jackson, M.O. and L. Yariv (2014): "Present bias and collective dynamic choice in the lab", *American Economic Review* 104, 4184-4204.

Kimball, M.S. (1987): "Making sense of two-sided altruism", *Journal of Monetary Economics* 20, 301-326.

Kohlberg, E. (1976): "A model of economic growth with altruism between generations", *Journal of Economic Theory* 13, 1-13.

Krusell, P., B. Kuruşçu and A.A. Smith (2002): "Equilibrium welfare and government policy with quasi-geometric discounting", *Journal of Economic Theory* 105, 42-72.

Krusell, P. and A.A. Smith (2003): "Consumption-savings decisions with quasi-geometric discounting", Econometrica 71, 365-375.

Laibson, D. (1997): "Golden eggs and hyperbolic discounting", *Quarterly Journal of Economics* 112, 443-447.

Lindbeck, A. and J.W. Weibull (1987): "Balanced-budget redistribution as the outcome of political competition", *Public Choice* 52, 273-297.

Persson, T. and L.E.O. Svensson (1989): "Why a stubborn conservative would run a deficit: policy with time-inconsistent preferences", *Quarterly Journal of Economics* 104, 325-345.

Phelps, E. and R. Pollak (1968): "On second-best national saving and game-equilibrium growth", *Review of Economic Studies* 35, 185-199.

Ray, D. (1987): "Nonpaternalistic intergenerational altruism", *Journal of Economic Theory* 41, 112-132.

Saint Paul, G. (1992): “Fiscal policy in an endogenous growth model”, *Quarterly Journal of Economics* 107, 1243-1259.

Streufert, P.A. (1993): “Consistent preferences and intergenerational equilibria in Markov strategies,” in B. Dutta, S. Gangopadhyay, D. Mookerjee, and D. Ray (eds.), *Theoretical Issues in Economic Development*, 263-287, Bombay: Oxford University Press.

Strotz, R.H. (1956): “Myopia and inconsistency in dynamic utility maximization”, *Review of Economic Studies* 23, 165-180.

Tabellini, G. (2000): “A positive theory of social security”, *Scandinavian Journal of Economics* 102, 523-545.

Veall, M.R. (1986): “Public pensions as optimal social contracts”, *Journal of Public Economics* 31, 237-251.

(1) $\sigma > 1$ (2) $\sigma = 1$ (3) $\sigma < 1$

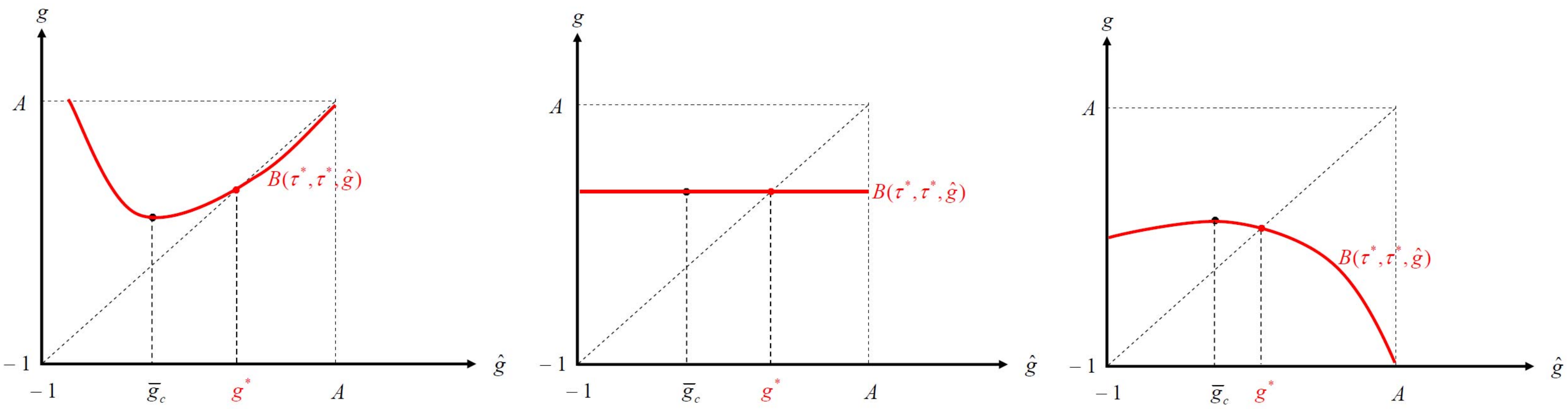

Figure 1