On the timing of non-renewable resource extraction with regime switching prices: an optimal stochastic control approach

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Abstract

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This paper develops a model of a profit maximizing firm with the option to exploit a non-renewable resource, choosing the timing and pace of development. The resource price is modelled as a regime switching process, which is calibrated to oil futures prices. A Hamilton-Jacobi-Bellman equation is specified that describes the profit maximization decision of the firm. The model is applied to a problem of optimal investment in a typical oils sands *in situ* operation, and solved for critical levels of oil prices that would motivate a firm to make the large scale investment needed for oil sands extraction, as well to operate, mothball or abandon the facility. The paper focuses on the impact of regime shifts on the optimal timing of investment and extraction compared with the case when the possibility of future regime shifts is ignored.

Keywords: non-renewable natural resources, oil sands, optimal control, HJB equation

JEL codes: Q30, Q40, C61, C63

1 Introduction

Commodity prices are typically highly volatile and characterized by cycles of boom and bust. Not surprisingly, investments in resource extraction tend to mirror these cycles. One example can be found in investment in the high cost reserves of Alberta's oil sands. Beginning in the 1970's, investment in extraction of the oil sands was an on-again off-again proposition depending on the strength of oil prices. World crude prices since 1986 and capital expenditure in the oil sands since 1973 are shown in Figures 1 and 2 respectively. It may be observed that buoyant oil prices in the past decade have been associated with unprecedented investment in oil sands extraction.

This run-up in oil prices and resultant strong investment in oil sands extraction have raised concerns nationally and internationally about the environmental impact of such large scale operations. The Alberta government has been criticized for not having adequate regulatory oversight in place to ensure that environmental impacts are kept at acceptable levels. Oil sands operators have felt the pressure of strong negative public opinion expressed around the world and there is a sentiment that they have lost their "social license to operate".[1] Oil firms and the Alberta government have sought to improve public perceptions through public relations campaigns as well as major investments in the development of technologies that will reduce environmental impacts. The Alberta government has also responded by tightening and rationalizing environmental regulations which raises costs for oil sands operators.

With increased costs and continuing environmental concerns, questions have been raised about the long run economics of oil sands investments. The difficulty in obtaining regulatory approvals for new pipeline capacity to get bitumen to market was blamed for an increase in the discount for the price paid for Alberta heavy crude in early 2013, which some referred to as the bitumen bubble. Figure 3 shows the evolution of the spread between the price of West Texas Intermediate (WTI) light crude in $Canadian/barrel and a heavy oil price in Alberta. Although there was an increase in this discount in early 2013, looking back over 10 years of data, it does not appear to be a huge anomaly. Nevertheless it is true

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[1]The environmental concerns raised by oil sands extraction are well documented by the Pembina Institute. *http://www.pembina.org/*
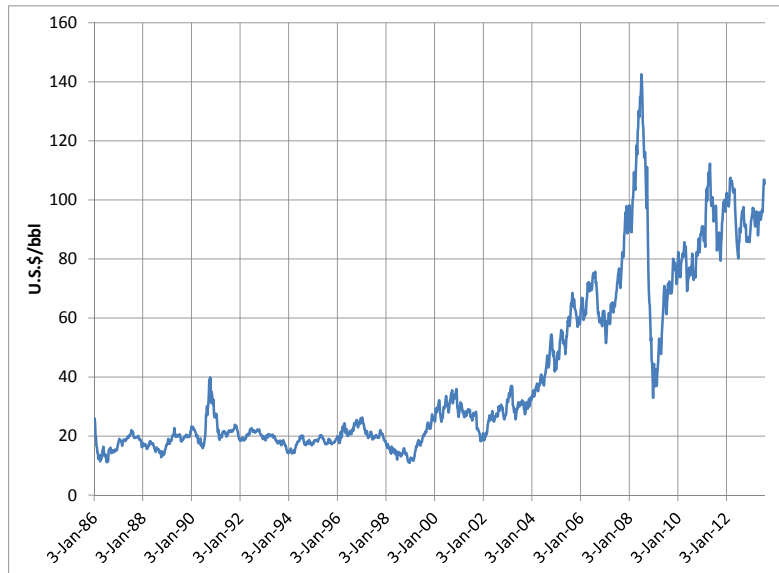
Figure 1: West Texas Intermediate Crude Oil Spot Price, U.S. $/barrel, Weekly, Jan 1986 - August 2013 Cushing, OK. Source: U.S. Energy Information Administration

that the price paid for Alberta bitumen depends on the ability to transport oil to market. Several recent pipeline accidents as well as the disastrous train derailment in Lac-Megantic, Quebec, have raised serious questions about the safety oil transport, putting up significant roadblocks for oil sands producers as pipeline approvals are delayed. There are also threats coming from other sources of supply such as oil and natural gas from shale deposits which have been made accessible by newly developed technologies. Two 2012 Globe and Mail headlines highlight these concerns: "Canadian oil: a good choice for roller coaster fans," and "Economics biggest threat to embattled oil sands."[2] While the recent experience of oil sands development is particularly dramatic, parallels can be found in other resource extraction industries such as copper mining, potash, and gold. Industries ramp up investment when prices are buoyant, with resultant environmental impacts and public concern.

In this paper we investigate the economics of non-renewable natural resource extraction taking account the boom and bust cycle of commodity prices. In particular we examine

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[2]The former headline is from an article by Nathan VanderKlippethe in the Globe and Mail, August 24, 2012. The latter is from an article by Martin Mittelstaedt in the Globe and Mail, January 18, 2012.
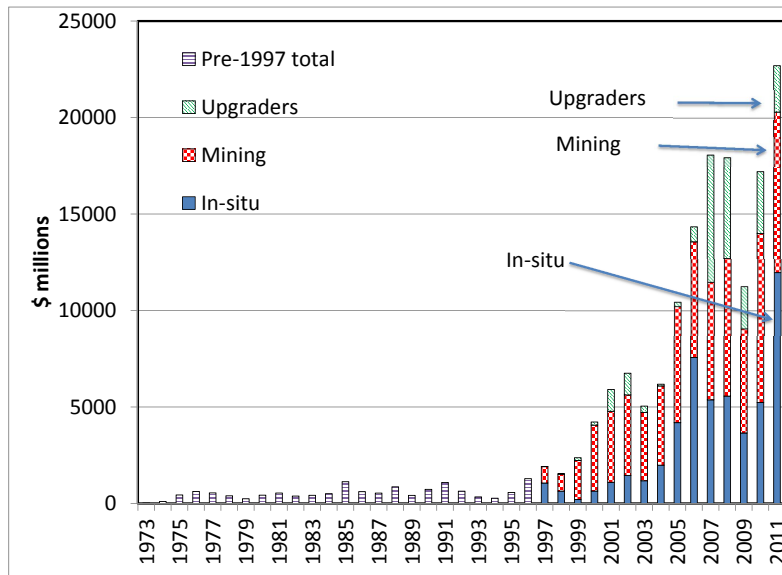
Figure 2: Alberta Oil Sands Capital Expenditures, 1973 - 2011. Data Source: Canadian Association of Petroleum Producers
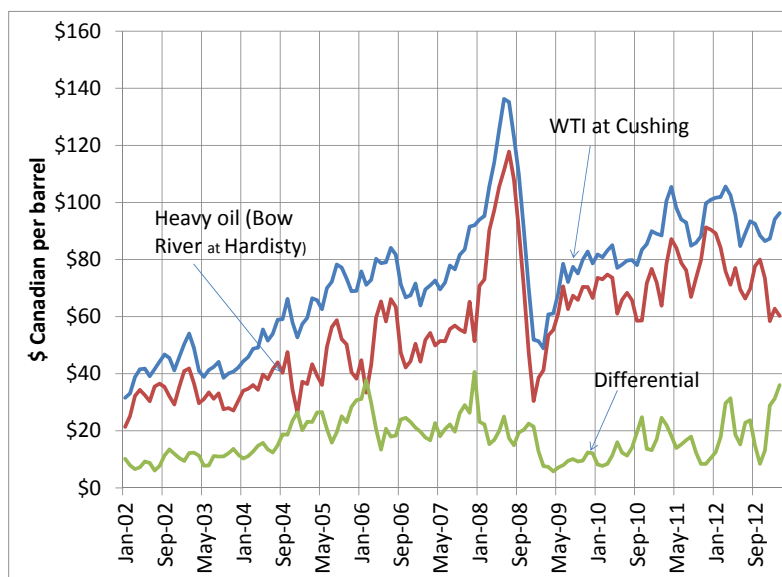

Figure 3: Reference Crude Oil Prices in $Canadian/barrel. Light oil: West Texas Intermediate on NYMEX - Cushing; Heavy oil: Bow River at Hardisty; Heavy - light differential. January 2002 to February 2013. Data Source: Canadian Association of Petroleum Producers

the optimal investment strategy when there is an expectation that in the future, prices may switch to a regime with starkly different dynamics than those observed currently. As has happened in Alberta, a sudden ramp up in resource investment and extraction may have environmental consequences which the public expects that regulators will be able to address. We seek to deepen our understanding of the optimal response of resource investment to uncertain commodity prices and the implications for environmental regulation appropriate to the expected pace of development. To this end, we model the decisions of a profit maximizing firm with the option to develop a non-renewable resource deposit, choosing the timing and pace of development, as well as the decision to produce the resource or shut down if prices become weak. To capture the boom and bust cycle typical of many commodities, the resource price is modelled as a regime switching process. The model is applied to a typical oils sands *in situ* project, but the analysis and results are relevant for other types of resource extraction operations. The model is used to solve for critical price levels at which it is optimal for a firm to invest in extraction, begin production, or shut down operations. The paper focuses on the impact of the prospect of regime shifts in commodity prices on optimal decisions and the pace of development.

The paper uses a real options modelling approach in the spirit of Brennan and Schwartz [1985], which was one of the earlier papers showing how optimal policies for managing a natural resource can be derived using no-arbitrage arguments from the finance literature. A classic paper in this genre for valuing offshore petroleum leases is Paddock et al. [1988]. Since the 1980's the literature using a real options approach for analyzing natural resource decision-making has grown dramatically. Reviews are provided in Schwartz and Trigeorgis [2001] and Mezey and Conrad [2010]. Papers dealing specifically with optimal development of a non-renewable resource include Cortazar and Schwartz [1997] who use an options approach to value an undeveloped oil field. Slade [2001]contrasts the predictions of a real options model with decisions to open and close copper mines in Canada. Conrad and Kotani [2005] determine the optimal trigger price to begin drilling for oil in a wildlife preserve assuming stochastic oil prices, but also considering uncertainty in the amenity value that would be lost when drilling proceeds. Schwartz [1997] examines the impact of different models of the stochastic behaviour of commodity prices on the valuation and optimal decisions in resource

extraction projects. Mason [2001] extends Brennan and Schwartz [1985] by modelling the decision to suspend or reactivate the extraction of a non-renewable resource when the finite resource stock is accounted for explicitly as an additional state variable. Mason examines the impact of the costs of suspension and reactivation (so-called switching costs) and observes a hysteresis or tendency for firms to continue with the status quo, whether currently operating or suspended. This is in the spirit of the work by Dixit [1989a,b, 1992], Dixit and Pindyck [1994].

This paper contributes to the literature by solving for optimal resource investment and extraction decisions assuming that price uncertainty can be characterized by a Markov-switching process - something not done previously in the literature to the best of our knowledge. With price and resource stock as state variables, we consider a multistage investment decision in which the owner must choose when to proceed through several phases of construction in addition to whether to temporarily mothball a producing facility or permanently abandon it. The problem is specified as a Hamilton-Jacobi-Bellman (HJB) partial differential equation. An analytical solution is not available, hence a finite difference numerical approach is used to obtain the solution for a prototype oil sands investment problem. The HJB equation is solved for the case where there are two price regimes, and in each regime price follows a different mean reverting stochastic process. Parameter estimates for the price process are determined though a calibration procedure using oil futures prices. The paper does not focus on the econometric issues involved in obtaining the best parameter estimates. Rather the focus is on examining the impact of regime switching on the optimal decision.

Our findings show that decision makers who anticipate random regime shifts in resource prices will behave differently than those who expect the pricing environment to remain in the current cycle for many years. In particular the timing of investment and extraction decisions are different, implying that regime shifts are important in analyzing optimal extraction decisions. We also observe an interesting pattern of critical prices as project construction proceeds through various stages. Each stage of construction may be viewed as an option to take the next step towards having a producing property. We find that critical prices to exercise the option to move to the next construction phase may be rising or falling through time, or may be non-monotonic. Rising critical prices imply a lower optimal price to begin

construction than to complete the project. This will be of interest to environmental regulators to the extent that different phases of the project have more or less severe environmental consequences. In the case of oil sands and other large mining projects, the early construction phases may have significant environmental consequences as sites are made ready for extraction. This underlines the need to have the regulatory framework in place to deal with a surge in interest by firms to undertake resource extraction projects.

The next section of the paper describes alternate ways of modelling oil prices and presents the regime switching model. The resource valuation model including the solution approach is described in Section 3. Section 4 explains the methodology used to calibrate the parameters of price process. A description of the oil sands example and analysis of the results is presented in Section 5. A summary and concluding remarks are given in Section 6.

2   The stochastic oil price process

Considerable effort has been made in the literature to determine the best models of commodity prices. The criteria for judging what is "best" depends on the goal - whether pricing commodity based derivatives, matching the term structure of futures prices, valuing long term investments, or other objectives. In this paper we are examining the optimal control of resource extraction over the long term. In this context, the price model should capture the long run dynamics of oil prices, but should also be parsimonious so that interpretation of the optimal control is not problematic.

For convenience, many papers adopt a simple process geometric Brownian motion (GBM) to describe uncertain commodity prices such as in the much cited paper by Brennan and Schwartz [1985] who used GBM model for oil prices. However as is noted by Schwartz [1997] among others, economic logic suggests that commodity prices should tend to some long run mean determined by the marginal cost of new production and the price of substitutes. In addition the volatility of futures prices tends to decrease with maturity, whereas a simple GBM process implies that futures prices will have constant volatility.[3] Two mean reverting

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[3]See Chen and Insley [2012] for further discussion and references.

processes which have been used in the literature include:

$$dP = \eta(\bar{P} - P)dt + \sigma P dz \qquad (1)$$

and

$$dP = \eta(\mu - log(P))Pdt + \sigma P dz \qquad (2)$$

where P denotes price; η and σ represent the speed of mean reversion and price volatility, respectively; and $\bar{P}$ and μ are the long run equilibrium levels of price and the log of price, respectively. Equation (1) has been used in various contexts such as in Insley and Rollins [2005] to model timber prices and in Chen and Forsyth [2007] to model natural gas prices. Schwartz [1997] uses Equation (2) to model oil, copper and gold prices. Neither of these models is fully satisfactory in terms of their ability to describe the behaviour of commodity futures prices. Although the implied volatility of futures prices decreases with maturity, which is desirable, volatility tends to zero for very long maturities, which is not a phenomenon observed in practice [Chen and Insley, 2012]. A better description of commodity prices can be obtained by including additional stochastic factors. Schwartz [1997] compared two and three factor models with the one factor model of Equation (2). The two and three factor models clearly outperformed the one factor model in terms of modelling the term structure of futures prices as well as the term structure of volatilities for copper and oil.

Another strand of the literature allows the variance of the stochastic process to change at discrete points in time or continuously. For example Larsson and Nossman [2011] model oil prices with volatility as a stochastic process with jumps and use a Markov chain Monte Carlo method to estimate parameters using WTI crude spot prices. The estimates obtained are consistent with the spot price under the $\mathbb{P}$-measure. Note that if the goal is to price options or analyze investment decisions, it is desirable to estimate risk adjusted parameters under the $\mathbb{Q}$ measure.

A regime switching model provides an alternate approach to capturing non-constant drift and volatility terms for the stochastic process followed by oil prices. First described by Hamilton [1989], it has intuitive appeal in that the boom and bust periods of commodity

prices may be thought of as different price regimes each characterized by a unique stochastic process. Regime switching has been considered in the context of macroeconomic cycles such as in Hamilton [1989] and Lam [1990]. Guo et al. [2005] notes that macroeconomic business cycle regimes may potentially have significant impacts on firms' choices, and that "... despite these potential effects we still know very little about the relation between regime shifts and investment decisions."

Regime switching models have been used by several authors to capture the dynamics of electricity prices. An overview can be found in Janczura and Weron [2010] and Niu and Insley [2013]. Chen and Forsyth [2010] use a regime switching model of natural gas prices to examine optimal decisions in a natural gas storage operation. Chen and Insley [2012] model lumber prices as a regime switching process to examine optimal tree harvesting decisions.

In a regime switching model, different regimes are defined which can accommodate different specifications of price behaviour. A general regime switching process is given as:

$$dP = a^j(P,t)dt + b^j(P,t)dz + \sum_{l=1,l\neq j}^{J} P(\xi^{jl} - 1)dX_{jl} \tag{3}$$
$$j = 1,...,J,\ l = 1,...,J$$

j and l refer to regimes and there are J regimes with j being the current regime. $a(P,t)$ and $b(P,t)$ represent known functions and dz is the increment of a Wiener process. When a regime switch occurs, the price level jumps from P to $\xi_{jl}P$. The term dX_{jl} governs the transition between j and l:

$$dX_{jl} = \begin{cases} 1 & \text{with probability } \lambda^{jl}dt \\ 0 & \text{with probability } 1 - \lambda^{jl}dt \end{cases} \tag{4}$$

For simplicity in this paper we make the assumption that there are only two possible regimes, and in each regime price follows an independent stochastic process as follows:

$$dP = \eta^j(\bar{P}^j - P)dt + \sigma^j Pdz \tag{5}$$
$$j = 1,2;$$

where η^j is the speed of mean reversion in regime j, $\bar{P}^j$ is the long run price level in regime j, and σ^j is the volatility in regime j. Regime switching is governed by Poisson process dX_{jl} specified in Equation (4). It will be noted that we do not include a jump term which allows price to jump suddenly to a new level when a regime change occurs. This is done to simplify parameter calibration. The transition to a new regime entails only new drift and volatility terms. However if the speed of mean reversion is quite high, then the switch to a new regime will cause a change in the price level, as price is pulled towards its new long run mean.

The regime switching price model chosen here is similar to the one used by Chen and Forsyth [2010] to analyze a natural gas storage problem. In that paper, natural gas prices were assumed to follow a process similar Equation (5), except that a seasonality component was also included. Note that seasonality has not typically been included in models of oil prices [Schwartz, 1997, Borovkova, 2006]. This paper is concerned with long run investment decisions in oil sands, and seasonality would not be important for such decisions.

The parameters of Equations (4) and (5) are estimated by calibrating to oil futures prices. This allows the estimation of risk adjusted parameter values which reflect market expectations about future prices. The calibration procedure and estimated parameter values are presented in Section 4.

3 Resource Valuation Model

3.1 Specifying the Decision Problem

We model the optimal decision of a firm considering when to invest in the extraction of a non-renewable resource, which is an oil sands project for the purposes of this paper. The project has significant capital costs and construction takes several years. The firm's decision is taken in the context of uncertain prices characterized by Equations (4) and (5). The firm's objective is to maximize the value of the resource asset by optimally choosing an extraction path over time, as well as determining the optimal timing for construction, production, temporarily mothballing the operation, reactivating from a mothballed state, and finally abandoning the property. Let $V_m^j(P, S, t) \equiv V(P, S, t, \delta_m, j)$ be the risk neutral value of cash

flows from the resource extraction project where:

- P is the resource price, $P \in [0, \infty]$

- S is the size of the resource stock, $S \in [0, S_0]$, where S_0 is the original size of the reserve

- t is time, $t \in [t_0, T]$

- δ_m represents the project stage, $m = 1, ..., M$, i.e. under construction, producing, mothballed, or abandoned

- j is the regime, $j = 1, ..., J$.

The firm chooses the extraction rate, R_m^j, which depends on the current regime j and operating stage m, and at certain discrete points in time is able to switch to a different operating stage by incurring a cost. Let $Z(S)$ represent the admissible set for R_m^j.

$$Z(S) \in [0, R_{\max}], \ S \neq 0$$
$$Z(S) = 0, \ S = 0 \tag{6}$$

The change in the stock of the resource is then $dS = -R_m^j dt$.

Let t_n, $n = 1, ..., N$ be discrete decision dates and the admissible set for the project stage, δ_m, be $Y = [\delta_1^j, ..., \delta_M^j]$ where changes in δ_m^j can only occur at discrete times t_n. In other words, if δ_m^j is the optimal choice at $t = t_n$, then $\delta^j = \delta_m^j$ for $t_{n-1} \leq t \leq t_n$.

When the project is operational a cash flow, $\pi_m^j(t)$, is earned as follows:

$$\pi_m^j(t) = R_m^j(P, S, t)\left(P(t) - c_v\right) - c_f - \text{taxes} \tag{7}$$

c_v is per unit variable cost and c_f is per unit fixed cost.

The value of the asset, V, is determined by maximizing the expected present value of profits from the initial period t_0 before construction has begun through to the permanent shut down of the project at time $t = T$. The permanent closure of the project may be due to the exhaustion of reserves or some other reason such as the end of a lease. We let p and s denote the price and stock respectively at a particular moment in time. The risk neutral

value of the project in rejime j and stage m is $V_m^j(p, s, t)$ where

$$V_m^j(p, s, t) = \max_{R, \delta_m} \mathrm{E}^Q \left\{ \int_{t_0}^{T} e^{-rt'} \left[\pi_m^j \right] dt \mid P(t) = p, S(t) = s \right\},$$

$$m = 1, ..., M; \ j = 1, ..., J$$

$$\text{subject to } \int_{t_0}^{T} R(:, t) dt \le S(t).$$

The constraint states that total production of the resource cannot exceed the initial stock in place.

We use standard contingent claims arguments to derive a system of PDE's describing V in project stage δ_m between decision dates. Let $t_n^+ = t_n + \epsilon$ and $t_n^- = t_n - \epsilon$ where $\epsilon > 0$, $\epsilon \to 0$. Then between decision dates we have:

$$\frac{\partial V_m^j}{\partial t} = \max_{R \in Z(S)} \left\{ -\frac{1}{2} b^j(p, t)^2 \frac{\partial^2 V_m^j}{\partial p^2} - a^j(p, t) \frac{\partial V_m^j}{\partial p} + R_m^j \frac{\partial V_m^j}{\partial s} - \pi_m^j(t) + \sum_{l=1, l \ne j}^{J} \lambda^{jl}(V_m^l - V_m^j) - rV_m^j \right\}$$

(8)

$$j = 1, ..., J; \ m = 1, ..., M$$

where $a^j(p, t)$ is the risk adjusted drift rate conditional on $P(t) = p$ and λ^{jl} is the risk adjusted transition j to regime l from regime . For our chosen price process $a^j(p, t) \equiv \eta^j(\bar{P}^j - p)$ and $b^j(p, t) \equiv \sigma_j p$.

At decision dates, the decision maker will check to see if it is optimal to switch to a different operating stage. There are M operating stages. Let $C_{\bar{m}m}$ denote the cost of switching from the current stage $\bar{m}$ to another stage m. Let $t = t^-$ denote the moment before a decision is taken and $t = t^+$ the moment after a decision. At the moment the decision is taken, the decision maker is just indifferent to staying in the current stage or moving to the next best stage. This implies the following:

$$V(t^-, \delta_{\bar{m}}) = \max \left\{ V(t^+, \delta_1) - C_{\bar{m}1}, \ ... \ , V(t^+, \delta_{\bar{m}}) - C_{\bar{m}\bar{m}}, \ ... \ , V(t^+, \delta_M) - C_{\bar{m}M} \right\} \quad (9)$$

Note that the firm can choose to remain in the current stage $\bar{m}$ but will incur a cost to do so. If resource is currently under production then the cost is just the fixed cost of production, c_f. However costs would also be incurred in the moth balled state or if one stage of construction has happened and the firm chooses to delay going on to the next stage.

For computational purposes Equation (8) is solved over the finite domains $P \in [0, P_{\max}]$ and $S \in [0, S_0]$. For convenience we write out Equation (8) substituting for a^j and b^j. In addition, using the usual dynamic programming technique we must solve backwards from the final time $t = T$ to the initial period $t = t_0$. It is convenient to define $\tau = T - t$ as the time remaining in the life of the asset. We then solve from $\tau = 0$ to $\tau = T$. Below Equation (8) is specified in terms of τ. Note that we also show the maximization with respect to R only for those terms that contain R.

$$\frac{\partial V_m^j}{\partial \tau} = \frac{1}{2}\sigma_j p^2 \frac{\partial^2 V_m^j}{\partial p^2} + \eta^j(\bar{P}^j - p)\frac{\partial V_m^j}{\partial p} + \max_{R \in Z(S)} \left\{ \pi_m^j - R_m^j \frac{\partial V_m^j}{\partial s} \right\} + \sum_{l=1, l \neq j}^{J} \lambda^{jl}(V_m^l - V_m^j) - rV_m^j \tag{10}$$

$$j = 1, ..., J; \ m = 1, ..., M.$$

3.2 Boundary Conditions

Boundary conditions must be specified to fully characterize the resource valuation problem. Taking the limit of Equation (10) as $P \to 0$ gives:

$$\frac{\partial V_m^j}{\partial \tau} = \eta^j \bar{P}^j \frac{\partial V_m^j}{\partial p} + \max_{R \in Z(S)} \left\{ \pi_m^j - R_m^j \frac{\partial V_m^j}{\partial s} \right\} + \sum_{l=1, l \neq j}^{J} \lambda^{jl}(V_m^l - V_m^j) - rV_m^j \tag{11}$$

$$j = 1, ..., J; m = 1, ..., M.$$

Equation (11) is solved at $P = 0$ and no special boundary condition is needed. Along the boundary there are outgoing characteristics in the P direction, which means that no information is required from outside the domain of P.[4]

As $P \to Pmax$ we assume that $\frac{\partial^2 V_m^j}{\partial P^2} \to 0$, which is a common assumption used in the

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[4]See Duffy [2006] for a discussion of boundary conditions and finance difference methods.

literature[5]. Equation (10) then becomes

$$\frac{\partial V_m^j}{\partial \tau} = [\eta^j(\bar{P}^j - p)]\bigg|_{p \to Pmax} \frac{\partial V_m^j}{\partial p} + \max_{R \in Z(S)} \left\{\pi_m^j - R_m^j \frac{\partial V_m^j}{\partial s}\right\} + \sum_{l=1, l \neq j}^{J} \lambda^{jl}(V_m^l - V_m^j) - rV_m^j \tag{12}$$

$j = 1, ..., J; m = 1, ..., M.$

No further specifications are needed as we will always have $Pmax > \bar{P}$. This implies that along the boundary $P = Pmax$ there are outgoing characteristics and no information outside the domain of P is required to compute the solution.

The domain of the resource stock is $S \in [0, S_0]$ and S is depleted by production, R_m^j:

$$S(t) = S_0 - \int_{t_0}^{t_n} R_m^j(t)dt \tag{13}$$

Z is the admissible set of R defined in Equation (6). As $S(t) \to 0$, the admissible set of R collapses to 0. We set $V = -D$ where D represents the present value of required restoration costs. Once reserves are depleted it is a regulatory requirement that restoration of the site must be undertaken.

For $S = S_0$, we solve Equation (10) at this boundary. No special boundary condition is needed as there are outgoing characteristics in the S direction.

When $\tau = 0$ $(t = T)$, we assume $V(P, S, \tau = 0) = 0$.

3.3   Solution approach

Equations (8) and (9) represent a stochastic optimal control problem which must be solved using numerical methods. Define $\mathcal{L}V$ as a differential operator where:

$$\mathcal{L}V_m^j = \frac{1}{2}\sigma^j p^2 \frac{\partial^2 V_m^j}{\partial p^2} + \eta^j(\bar{P}^j - p)\frac{\partial V_m^j}{\partial p} + \sum_{l=1, l \neq j}^{J} \lambda^{jl}(V_m^l - V_m^j) - rV_m^j \tag{14}$$

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[5]See for example [Wilmott, 1998, chapter 46]

Using $\mathcal{L}V_m^j$ as defined above, the partial differential equation, Equation (10), can be written as:

$$\frac{\partial V_m^j}{\partial \tau} - \max_{R \in Z(S)} \left[\pi_m^j - R_m^j \frac{\partial V_m^j}{\partial s} \right] - \mathcal{L}V = 0; \; j = 1, ..., J; \; m = 1, ..., M. \tag{15}$$

LV in Equation (15) is discretized using a standard finite difference approach. The other terms in the equation are discretized using a semi-Lagrangian scheme. Consider the path, $\mathcal{S}$ defined by the ordinary differential equation:

$$\frac{ds}{d\tau} = -R \tag{16}$$

Use Equation (16) to write two terms from Equation (15), $\frac{\partial V_m^j}{\partial \tau} + R_m^j \frac{\partial V_m^j}{\partial s}$, as a Lagrangian directional derivative:

$$\frac{DV_m^j}{D\tau} = \frac{\partial V_m^j}{\partial \tau} - \frac{\partial V_m^j}{\partial s} \frac{ds}{d\tau}. \tag{17}$$

Equation (15) can then be rewritten as

$$\max_{R \in Z(S)} \left[\frac{DV_m^j}{D\tau} - \pi_m^j \right] - \mathcal{L}V_m^j = 0; \; j = 1, ..., J; \; m = 1, ..., M. \tag{18}$$

A semi-Lagrangian discretization is implemented for Equation (18) as described in Chen and Forsyth [2007, 2010], and will not be described further here.

Within the admissible set $R \in Z(S)$ we define a grid $[0, ..., R_{\max}]$ over which we check for the choice of R that maximizes Equation (18) at each time step. Given the nature of the revenue and cost functions used in this example, the optimal choice of R turns out to be a bang-bang solution - either 0 or $Rmax$.

4   Calibrating the parameters of the price process

4.1   Methodology

We use oil futures prices to calibrate the parameters of Equations (5) and (4) (except for the σ^j). The process is similar to that described in Chen and Forsyth [2010] and Chen and Insley [2012]. Let $F^j(P, t, T)$ denote the futures price in regime j at time t with delivery at T

while the spot price resides at P. (This will be shortened to F^j when there is no ambiguity.) On each observation day, t, there are futures contracts with a variety of different maturity dates, T. The futures price equals the expected value of the spot price in the risk neutral world. We set $J = 2$, assuming that there are two possible regimes in order to keep the computational complexity to a manageable level.

$$F^j(p, t, T) = E^Q[P(T)|P(t) = p, J_t = j]$$
$$j = 1, 2.$$

where E^Q refers to the expectation in the risk neutral world and J_t refers to the regime in period t. Applying Ito's lemma results in two coupled partial differential equations for the futures price, one for each regime:

$$(F^j)_t + \eta^j(\bar{P}^j - P)(F^j)_P + \frac{1}{2}(\sigma^j)^2 P^2 (F^j)_{PP} + \lambda_{jl}(F^l - F^j) = 0, \; j = 1, 2. \tag{19}$$

with boundary condition $F^j(P, T, T) = P$. The solution of these coupled pde's is known to have the form

$$F^j(P, t, T) = a^j(t, T) + b^j(t, T)P \tag{20}$$

Substituting this solution into Equation (19) yields a system of ordinary differential equations:

$$(a^j)_t + \lambda^{jl}(a^l - a^j) + \eta^j \bar{P}^j b^j = 0$$
$$(b^j)_t - (\eta^j + \lambda^{jl})(b^j) + \lambda^{jl} b^l = 0, \; j = 1, 2. \tag{21}$$

$(a^j)_t \equiv \partial(a^j)/\partial t$ and $b(s_t)_t \equiv \partial b(s, t)/\partial t$. Substituting boundary condition $F^j(P, T, T) = P$ into Equation (20) gives $a^j(t = T, T) = 0$; $b^j(t = T, T) = 1$.

Taking the matrix differential of Equation (21) gives:

$$\frac{d}{dt}[a^1 \; a^2 \; b^1 \; b^2]' = A[a^1 \; a^2 \; b^1 \; b^2]' \tag{22}$$

The solution to Equation (22) is:

$$[a^1 \ a^2 \ b^1 \ b^2]' = e^{At}[0 \ 1 \ 0 \ 1]' \tag{23}$$

where, e^{At} is the matrix exponential, and

$$A = \begin{bmatrix} -\lambda^{12} & \eta^1 \bar{P}^1 & \lambda^{12} & 0 \\ 0 & -(\eta^1 + \lambda^{12}) & 0 & \lambda^{12} \\ \lambda^{12} & 0 & -\lambda^{12} & \eta^2 \bar{P}^2 \\ 0 & \lambda^{12} & 0 & -(\eta^2 + \lambda^{21}) \end{bmatrix}. \tag{24}$$

Let θ denote the suite of parameters to be estimated: $\theta = \{\eta^j, \bar{P}^j, \lambda^{jl} \mid j, l \in \{0,1\}\}$. In addition the current regime, $j(t)$, must be estimated. σ^j is not included in θ as it must be estimated separately from the other parameters. This follows from the observation that σ^j does not appear in Equation (21), implying that for this particular price process the futures price at any time t does not depend on spot price volatilities. Determination of the σ^j for each regime is discussed below.

The calibration is carried out by finding the parameter values which minimize the ℓ_2 norm error (root mean square error) between model-implied futures prices and actual futures prices.

$$\min_{\theta, j(t)} \sum_t \sum_T (\hat{F}(J(t), P(t), t, T; \theta) - F(t, T))^2 \tag{25}$$

$$\text{subject to } \eta_{\min} \leq \eta \leq \eta_{\max}, \ \bar{P}_{\min} \leq \bar{P} \leq \bar{P}_{\max}, \ \lambda^{ij}_{\min} \leq \lambda^{ij} \leq \lambda^{ij}_{\max} \tag{26}$$

where $F(t, T)$ is the market futures price on observation day t with maturity T and $\hat{F}(J(t), P(t), t, T; \theta)$ is the corresponding model implied futures price calculated from Equations (20), (23), and (24) above. Equation (26) is a constrained non-linear optimization problem with possibly many local minimums. In order to get meaningful results we must impose economically sensible limits on the ranges of possible parameter values.

It is known that for Ito processes such as Equation (5), the volatilities, σ^j, are the same under the $\mathbb{P}$-measure as under the $\mathbb{Q}$-measure. It is therefore possible to use spot prices to

	Regime 1	Regime 2	lower bound	upper bound
η^j	0.66	0.02	.010	1
$\bar{P}^j(\$/bbl)$	96	200	0	200
λ^{jl}	0.05	0.64	0.05	0.95
σ	0.29	0.26		

Table 1: Case 1 (base case parameter estimates). $dP = \eta^j(\bar{P}^j - P)dt + \sigma^j P dz, j = 1, 2$. Note that $\bar{P}$ is in U.S.\$ and refers to West Texas Intermediate. Parameters have been annualized.

estimate values for the σ^j. For this paper we use the methodoloty of Perlin [2012] to estimate Markov state switching models.[6]

4.2 Data description and results

The calibration was carried out using monthly data for WTI futures contracts on the New York Mercantile exchange. (Data is obtained from Datastream.) Crude oil futures are available for nine years forward: consecutive months are listed for the current year and the next five years; in addition, the June and December contract months are listed beyond the sixth year. The calibration is done for all available contract maturities from 2 months through 9 years which amounts to 107 different contract maturities. Monthly observations are used, representing the first day of the month or the nearest day following the first day of the month which is a trading day. Data used is from January 2003 to April 2013.

Results of the calibration are given in Table 1. For 10167 data points the minimized ℓ_2 norm error is 399,850, implying an average error of \$6.27 [7]. The table also reports the upper and lower bounds imposed on the optimization. The same bounds were imposed in each regime. There is no expectation that there is a unique solution to the least squares minimization procedure. The bounds are imposed to help ensure an economically reasonable solution is obtained from the optimization process.

The results show two distinct regimes. These parameter estimates are in the $\mathbb{Q}$-measure and reflect the expectations of the market including a market price of risk. Regime 1 has a long run mean of \$96 but a fairly high mean reversion speed of 0.66, which roughly implies

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[6]A alternative approach would be to estimate σ^j using data for the prices of options on oil futures as is done in Chen and Forsyth [2010].
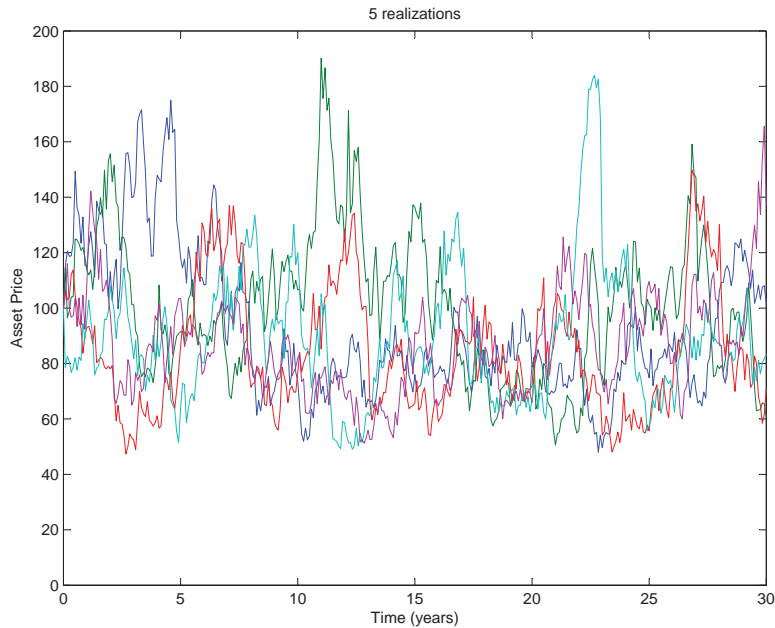
[7]$\sqrt{(399,850/10167)} = 6.27$

Figure 4: Simulation of base case regime switching price process, U.S. $/barrel for WTI

an expected time to revert to the mean of 1.5 years. Regime 2 has a much lower speed of mean reversion of .02 to a long run mean of $200 per barrel. The λ's indicate that price is expected to reside mostly in regime 1 which we will call the dominant regime. The expected time to remain in regime j is $1/\lambda_j$, which is 20 years for regime 1 and 1.6 years for regime 2.

The volatility estimates, obtained using Perlin [2012], show similar volatilities in both regimes. One complication is in assigning the volatilities estimated in the $\mathbb{P}$-measure to the regimes determined through the $\mathbb{Q}$-measure calibration, which amounts to a simple string matching problem. This is done by observing which of the volatilities is assigned to each of the months of the estimation period by the $\mathbb{P}$-measure calibration. We then compare this to the $\mathbb{Q}$-measure estimate of regimes by month, and assign the volatility to each regime that best matches the time series profile.

As an aid to visualization, we show in Figure 4 a simulation of 5 realizations of the process which is observed to hover around the regime 1mean of $96. The starting price is $100 per barrel.

Project type	In situ, SAGD
Production capacity	30,000 bbl/day
Reserves	250 million barrels
Production life length	30 years
Construction cost	$960 million over three years
Variable costs (non-energy)	$5.06 per barrel
Variable costs (energy)	5.28 % of WTI price
Fixed cost (operating)	$34 million per year
Fixed cost (sustaining capital)	$21.9 million per year
Abandonment and reclamation	2 % of total capital plus $1 million per annum
Cost to mothball and reactivate	$ 5 million
Federal corporate income tax	15%
Provincial corporate income tax	10%
Carbon tax	$40 per tonne

Table 2: Details of the prototype *in situ* project. Mothball and reactivation costs as well as on-going monitoring cost are assumed by the author. Other costs are based on technical reports from the Canadian Energy Research institute [Millington et al., 2012, Millington and Mei, 2011, McColl and Slagorsky, 2008] and Plourde [2009]

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5   Oil extraction decision problem

5.1   Project specification

We examine the decision to build and operate an oil sands *in situ* extraction project. Mining and *in situ* are the two methods currently used to extract bitumen from Alberta's oil sands with *in situ* used for deposits too deep to be mined. It is estimated that 80% of Alberta's remaining recoverable bitumen is suited to *in situ* extraction involving steam or solvent injection through horizontal or vertical wells [Millington et al., 2012].

The characteristics of the prototype oil sands project are summarized in Table 2. Production capacity, production life, and construction and operating costs are taken from a report produced by the Canadian Energy Research Institute (CERI) [Millington et al., 2012]. Total construction costs of $960 million are assumed to be spread over three years. Energy represents a significant component of variable costs. The CERI assumption is that a project of this magnitude will require 321,000 GJ per day of natural gas and 300 MWH per day of electricity. In this paper it is assumed that the prices of energy inputs remains unchanged in

WTI price \$C/barrel	Gross revenue royalty rate	Net revenue royalty rate
$P < 55$	1 %	25 %
$55 \leq P \leq 120$	Increases linearly	Increases linearly
$P > 120$	9 %	40 %

Table 3: Alberta's royalty rates for oil sands production, [Government of Alberta, 2007]

relation to the price of WTI. Specifically, the cost of natural gas and electricity to produce bitumen is assumed to remain at 5.28% of the price of WTI on a \$/barrel basis.[8] Of course both electricity and natural gas prices could be modelled as separate stochastic factors. However this is not the focus of the paper, and so we make this simplifying assumption.[9] In recent years the price of natural gas has fallen significantly as supplies for shale gas deposits have augmented North American gas supplies. This has reduced input costs for oil sands operations, but also provides competition on the demand side.

Firms producing Alberta oil must pay royalties to the provincial government. Royalty rates differ depending on whether or not a decision maker has recovered the allowed project costs. Prior to the payout date royalties are paid on gross revenues[10] at the gross revenue royalty rates shown in Table 3. After payout has been achieved royalties are the greater of the gross revenue royalty or the net revenue royalty based on the net revenue royalty rate shown in the same table. The implication is that the royalty rate is a path dependent variable. This implies that the date of payout is dependent on the stochastic oil price, making the calculation of the post-payout royalty non-trivial. For simplicity, we have used the pre-payout royalty rate in our analysis.

We include a \$40 per tonne tax on carbon, which reflects a proposal from the Alberta government made in April 2013. Currently large industrial operations in Alberta have a

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[8]This is based on a price for WTI of \$94 per barrel, an electricity price of \$51 per MWh [Millington et al., 2012], and a wholesale natural gas price in Alberta (AECO C) of \$3.00/GJ. The \$/GJ price of electricity plus natural gas is equal to 5.28 percent of the \$/bbl price of WTI, and we assume this relationship remains constant.

[9]See Almansour and Insley [2013] for a study of the relationship between oil and natural gas prices and the impact of this relationship on the economics of an oil sands operation.

[10]Gross revenue is defined as the revenue collected from the sale of oil sands products (or the equivalent fair market value) less costs of any diluents contained in any blended bitumen sold. Allowed costs are those incurred by the project operator to carry out operations, and to recover, obtain, process, transport, or market oil sands products recovered, as well as the costs of compliance with environmental regulations and with termination of a project, abandonment and reclamation of a project site. [Millington et al., 2012]

target of reducing emissions per unit of of production by 12% relative to a baseline. If a firm does not meet this target it must pay $15 per tonne into a technology fund. The current proposal is to increase the target by 40% and the price per tonne to $40. This price per tonne of carbon converts to $3.64 per barrel of bitumen assuming that each barrel of bitumen produces 91 kg of greenhouse gasses.[11]

The price of Alberta bitumen is at a considerable discount to the price of WTI. In this paper we follow the assumption of Plourde [2009] and fix the price of bitumen in the field at 55% of the price of WTI crude. In reality, the price discount for bitumen fluctuates from month to month depending on markets and refinery and pipeline capacity. We assume that this is a reasonable average.

We model our prototype project as having the option to proceed through five stages, with the decision maker choosing the optimal time to move from one stage to the next. The stages are as follows.

- Stage 1: Before construction begins

- Stage 2: Project 1/3 complete

- Stage 3: Project 2/3 complete

- Stage 4: Project 100 % complete and in full operation

- Stage 5: Project is temporarily mothballed

- Stage 6: Project abandoned

The decisions to move from Stages 1 to 2, 2 to 3, or 3 to 4 each require spending 1/3 of the total constructions costs. The decision maker has the option to postpone moving through these construction stages, but staying longer than a year in any phase is assumed to incur extra costs of $50 million per year. Moving to Stage 4 also requires spending fixed and variable operating costs. If price gets too low the decision maker has the option to temporarily mothball production at an assumed cost of $5 million. It is further assumed that it costs $5 million to reactivate the operation. When the project is mothballed only the

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[11]This estimate is from [Moorhoulse et al., 2010, page 54].

sustaining capital of $21.9 million is incurred. In addition there is the option to abandon the project at a cost of 2% of construction costs[12] plus a $1 million per year cost for monitoring and maintenance.

5.2 Results Analysis

### 5.2.1 Case 1 based on calibrated parameter values

We determine the value of the project and optimal decisions by solving the HJB equation (Equation 18) using the project specifications and costs detailed in Table 2 and 3 and parameter values of Table 1. Figure 5 shows the value of the project in each regime for the base case for different starting prices and different resource stock levels prior to any construction expenditures. We observe the project's value rising with price and reserve level in both regimes, as expected. In regime 2 value rises more quickly with price and reaches a higher value. This reflects the low speed of mean reversion, so that asset value is more dependent on the current price than in regime 1 where the speed of mean reversion is much higher. This is easier to see in the 2-D diagram shown in the left panel of Figure 6. Here we see for each regime, the value of the project versus price, prior to beginning construction (stage I) and once construction has started (stage 2) less the cost of construction to reach stage 2. We observe that in regime 2 value increases more rapidly with price. It is optimal to begin construction when the value in stage 2 less construction cost exceeds the value in stage I. The right hand panel of Figure 6 shows a similar diagram for moving from stage III to stage IV when production begins. The optimal price at which production begins is where the two lines cross for a given regime. We observe that the shapes of the curves are similar as in the left panel, but the project is more valuable now that it is nearly or fully completed.

Table 4 shows the critical prices at which it is optimal to move from one stage to the next for two different reserve amounts. For example for the base case with $S = 250$ million barrels and if we are in regime 1, construction would begin at $27.50 (stage 1 to 2) and the project would be completed (stage 3 to 4) if price is at least $54. Some interesting observations emerge.

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[12]This is the assumption used by CERI [Millington et al., 2012].
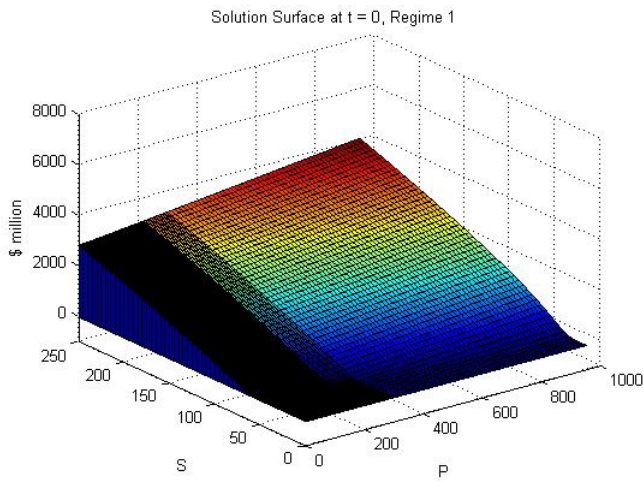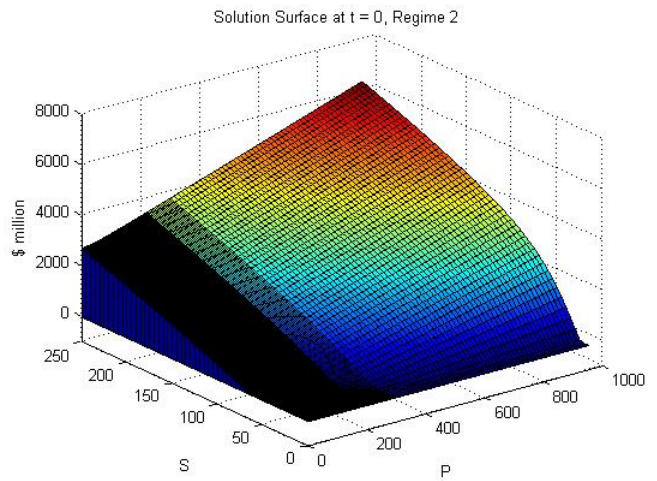
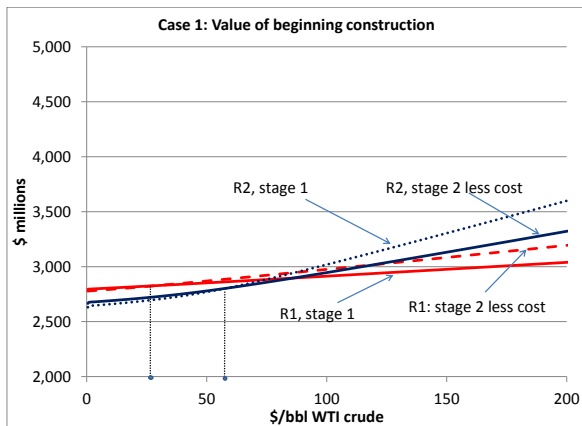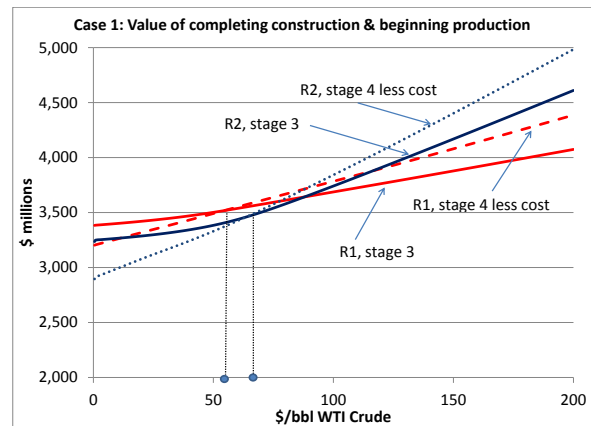(a) Regime 1   (b) Regime 2

Figure 5: Project value in each regime, $ millions, versus resource stock size (S) in barrels of bitumen and price (P) in $/barrel for WTI.

(a) From Stage 1 to 2   (b) From Stage 3 to 4

Figure 6: Value of beginning construction (Stage 1 to 2) and value of finishing the project and beginning production (Stage 3 to 4)

		$S = 250$ million bbls		$S = 125$ million bbls	
Transition from :		Regime 1	Regime 2	Regime 1	Regime 2
Stage 1 to Stage 2: Begin construction		27.5	57	69	77
Stage 2 to Stage 3: Continue		55	68	78	82
Stage 3 to Stage 4: Finish, Begin production		54	63	65	72
Stage 4 to Stage 5: Mothball		42.5	56	56	66
Stage 5 to Stage 4: Reactivate		45	59	59	68
Stage 4 or 5 to Stage 6: Abandon		NA	NA	NA	NA

Table 4: Critical prices for moving between stages, Case 1, U.S. \$/barrel, WTI, Full reserve level at $S = 250$ million barrels and half reserves at $S = 125$ million barrels.

First, it will be noted that critical prices to begin and continue construction (stages 1 through 3) are lower for regime 1 than regime 2. This follows from the differing speed of mean reversion, η, in the two regimes as well as the differing long run equilibrium price values, $\bar{P}$. In regime 1, there is the expectation of fairly speedy reversion to the equilibrium price of \$96. For regime 2 price is drifting up only very slowly to a long run mean of \$200, which makes it optimal to delay beginning construction until a higher threshold price is reached.

We also observe in Table 4 a significantly lower price at which it is optimal to mothball the operation when in regime 1 versus regime 2 (\$42.5 versus \$56). We are more willing to tolerate low prices while operating in regime 1, with the expectation that a period of low prices will be of short duration. Once mothballed the critical prices for reactivation are higher than the prices that caused the decision maker to shut down in the first place. This result implies some persistence in the mothballed state, reflecting the value of the option to delay the irreversible costs of reactivation and production. This phenomenon was highlighted in Dixit [1992] and Mason [2001].

It is interesting to note that there are no critical prices shown for abandonment for either reserve level shown. We also observe that the critical prices are all higher when the stock of reserved is reduced by half. For a producing project when half of the reserves have been used up, mothballing would happen at a significantly higher price, as would reactivation, compared to when the reserves are fully stocked. This reflects the increasing value of the resource as the stock is depleted - i.e. a larger $\partial V/\partial S$ term in Equation (8).

	Case 1	Case 1	Case 2	Case 3	Case 4
	Regime 1	Regime 2	Weighted Average	Mean Reverting	GBM
η	0.66	0.02	0.6	1	-0.01
$\bar{P}$	96	200	104	96	0
λ^{jl}	.05	0.64	NA	NA	NA
σ	0.29	0.26	0.287	0.29	0.29

Table 5: Cases 1, 2, 3, and 4 parameter values. $dP = \eta^j(\bar{P}^j - P)dt + \sigma^j P dz, j = 1, 2.$

In both regimes, the critical prices behave in a non-monotonic fashion as construction proceeds, initially rising when going from stages 1 to 2 and then falling in going from stages 2 to 3. This is not an intuitive result. The economics of moving from one stage of construction to the next depends on benefits versus the costs of delay. The benefits of delay include the delay in the spending of construction costs for the next stage. The costs of delay include the delay in receiving revenue from production plus any maintenance costs incurred when construction is paused. The cost of delayed revenue depends on the expected oil price when the project will be completed. This cost is higher when the project is at an earlier stage of construction, as this implies the decision maker cannot quickly finish construction to take advantage of a surge in oil prices. Getting construction underway is like exercising an option that allows the decision maker to move one step closer to a producing project. This is not a general result, and depends on the specifics of each case. In particular, the cost of delaying revenue will depend on the nature of the price process involved. We examine this issue further in Section 5.2.3 with a sensitivity case.

5.2.2 Case 2: Sensitivity based on weighted average parameters of the two regimes from Case 1

To further investigate the extent to which regime switching impacts the results, we will consider a single regime case in which the parameters are a weighted average of the two regime case. The weights used reflect the expected length of stay in each regime. The parameters used for case 2 are given in the third column of Table 5.

The value of the project in this second case compared to the first case is shown in Figure 7. We observe that project value for case 2 prior to construction is higher than for case 1 in
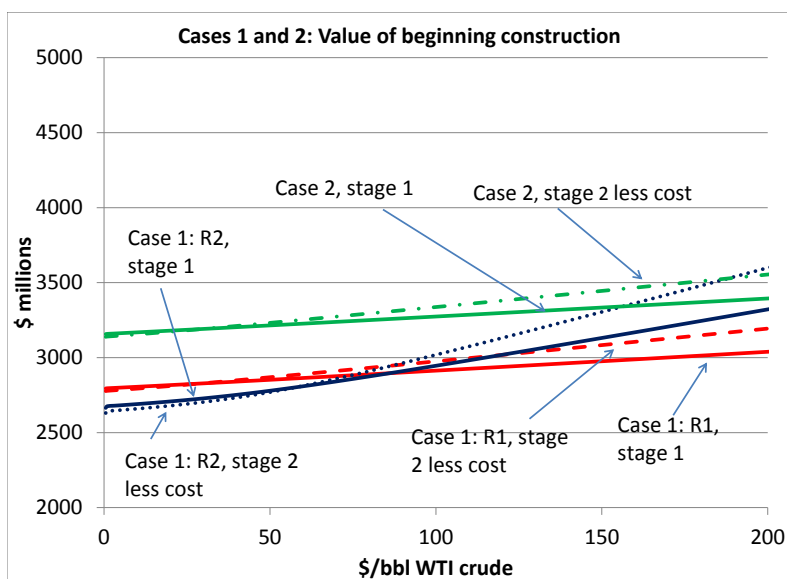
Figure 7: Comparing values of cases 1 (regime switching) and 2 (single regime) prior to beginning construction

either regime, at least for prices of WTI below $200 per barrel. This makes sense intuitively as Case 2 has an long run equilibrium price that is 8% higher than for case 1, regime 1 and also has a relatively high speed of mean reversion. Beyond a WTI price of $200, project value in case 1, regime 2 exceeds that of case 2. However even for prices up to $500/barrel, project value for case 1, regime 1, which is the dominant regime in case 1, remains below that of case 2. Critical prices for cases 1 and 2 are compared in Figure 8. We observe that for the intial investment (stage 1), case 2 has the same critical price as case 1, regime 1, which is not surprizing given that the weighted average parameter values are close to those of regime 1. In subsequent stages, case 2 critical prices fall between those of the two regimes in case 1. Even though in the regime switching case we expect price to reside in regime 1 for 93% of the time, we still observe a difference in the optimal decision between the case 1 and case 2. Use of the weighted average single regime process rather than taking account of the two regimes would result in sub-optimal decisions.
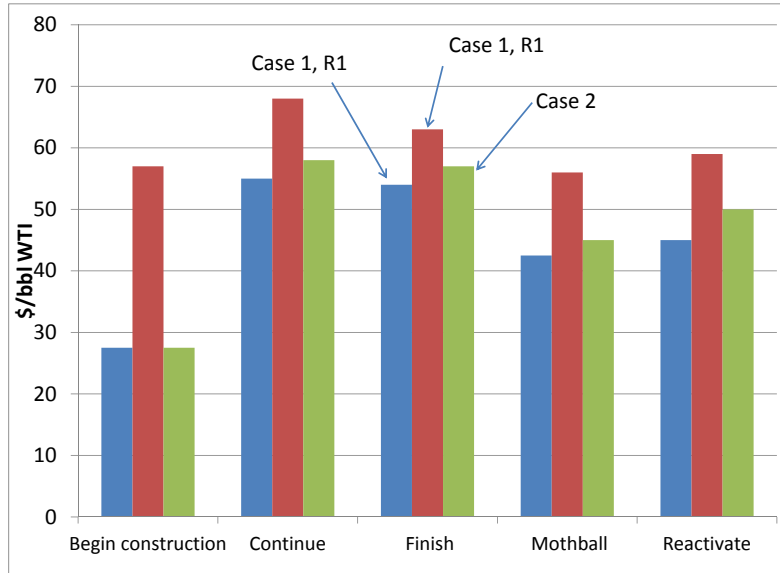
Figure 8: Comparing critical prices for various stages, cases 1 and 2

5.2.3 Cases 3 and 4: Examining critical prices for mean reversion versus geometric Brownian motion

The non-monotonic nature of the critical prices through the construction phases shown in the base case is somewhat puzzling. Some illimination is provided by comparing how critical prices behave for a single regime price process characterized by mean reversion in the drift term compared to a single regime process of geometric Brownian motion. In Table 5, parameter values are given for case 3 which is a mean reverting process with a long run equilibrium value of $96 and a fairly strong mean reversion speed of 1. We will contrast this with a single regime which follows a geometric Brownian motion process: $dP = 0.1Pdt + 0.29\sigma Pdz$.

The critical prices for these two cases are depicted in Figure 9. We observe two distinct patterns. For the mean reverting process, critical prices rise as construction procedes and there is a zero critical price to begin construction. Since it is expected that price will converge fairly quickly to $96, it is optimal to begin the construction process right away, no matter what the current price. For stages 2 and 3 the critical prices increase as construction nears
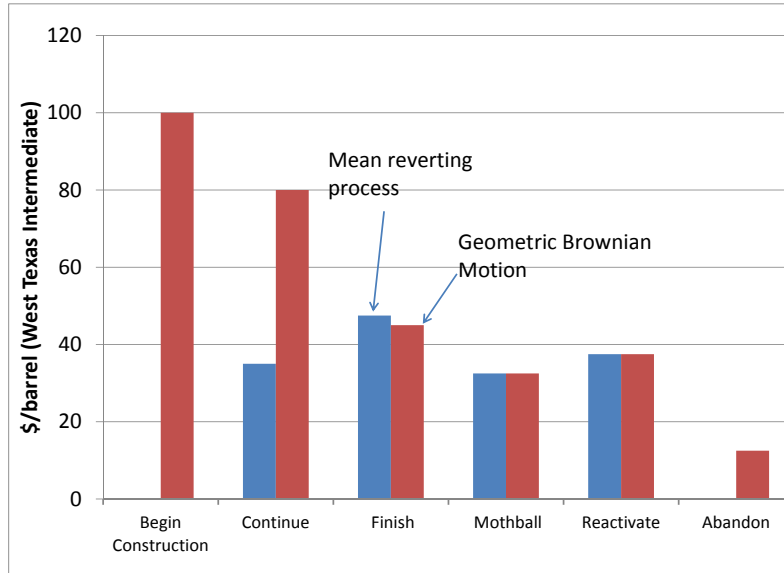
Figure 9: Comparing critical prices for various stages, case 3 (mean reverting process) and case 4 (geometric brownian motion) Note that for the MR process there are no critical prices for beginning construction or abandoning the project.

completion. In contrast for the GBM price process, we see that the critical price starts at a high level and then falls as construction proceeds. These differing patterns can be explained by considering the optimal actions under each price process. For a mean reverting process it is important to take advantage of prices that are temporarily above the long run mean, as it is expected that price will decline in the future. Hence the investor will want to get construction underway so as to be in a position to complete the project quickly if an upsurge in prices makes this desirable. For the GBM case, it is expected that prices will drift up over time, so the opportunity cost of delaying construction will be lower than for the mean reverting case. The non-monotonic nature of critical prices versus the stage of construction seen in Case I reflects the fact that there is regime switching between two distinct regimes, one with fairly strong mean reversion and the other with very weak mean reversion, so we are seeing a combination of effects.

Another interesting point regarding the GBM case shown in Figure 9 is that there is now a critical price for abandoning the project, something not observed in other cases. This

	Case 5 Regime 1	Case 5 Regime 2	Case 6
η^j	1	.05	.525
$\bar{P}^j$	60	150	105
λ^{jl}	0.1	0.1	na
σ	0.3	0.3	0.3

Table 6: Cases 5 and 6 parameter values. $dP = \eta^j(\bar{P}^j - P)dt + \sigma^j P dz, j = 1, 2.$

reflects the fact that under a GBM process, project value is more dependant on the current price.

5.2.4   Cases 5 and 6: High and low price regimes

As noted in the introduction, the prevailing mood in Alberta's oil industry, as reflected in media reports, appears to be one of concern over the long run economics of oil sands due to competition from alternate sources of fossil fuels as well as pressure from civil society groups to halt further pipeline development needed to transport bitumen to market. With these sentiments in mind we consider a case, which we call case 5, with two alternate price regimes reflecting buoyant or depressed prices for bitumen, with an equal probablity of residing in either regime. In the buoyant scenario (regime 2) the long run equilibrium price for WTI is set at \$150, but the speed of mean reversion is quite low at 0.05. In the depressed scenario, regime 1, the long run equilibrium price for WTI is \$60 and the speed of mean reversion is quite high at 1. The probability of switching out of either of these regimes in a year is 10%, implying a reasonable expectation of persistence in each regime of 10 years.. This contrasts with case 1 in which most of the time we expect to reside in regime 2. We compare case 5 to a single regime which is the average of the two regimes (which we call case 6). The chosen parameter values for cases 5 and 6 are shown in Table 6.

Figure 10 shows the values of beginning construction for cases 5 and 6. We observe that for prices below about \$120, case 6 has a larger value than either regime in case 5. The two cases have quite different critical prices, as is shown in Figure 11. For example, the critical price for beginning construction is substantially lower in case 6 than for either regime in case 5. The critical price for beginning construction in case 5, regime 1, is \$80, reflecting the depressed outlook for oil prices in this scenario. On the other had, case 6 is characterized
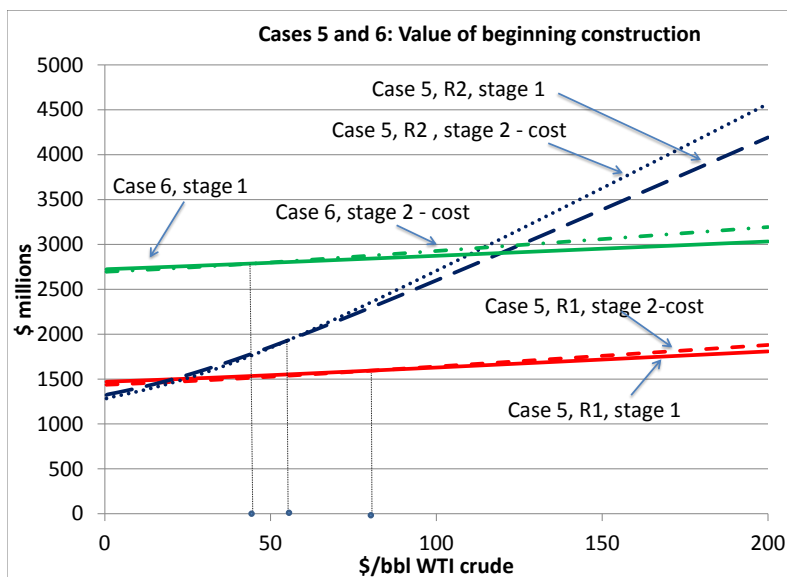
Figure 10: Comparing values of cases 5 (regime switching) and 6 (single regime) prior to beginning construction

by fairly quick mean reversion to a reasonable price of \$105, making it worthwhile to begin construction when prices are only \$45. It is clear that the project value and optimal actions are very different depending on in which regime one resides for case 5, and are very different from the weighted average regime of case 6.

Cases 5 and 6 demonstrate the potentially significant impact on values and critical prices of a regime switching price process. This impact is not captured by a weighted average of the regimes.

6 Concluding Remarks

We argue that a regime switching price process is a logical choice for capturing the boom and bust cycles of commodity prices and the associated non-constant drift and volatility parameters. We have calibrated a regime switching price process for crude oil with two regimes and considered the implications for optimal development of a prototype resource extraction project. These results provide intuition about the optimal actions of decision
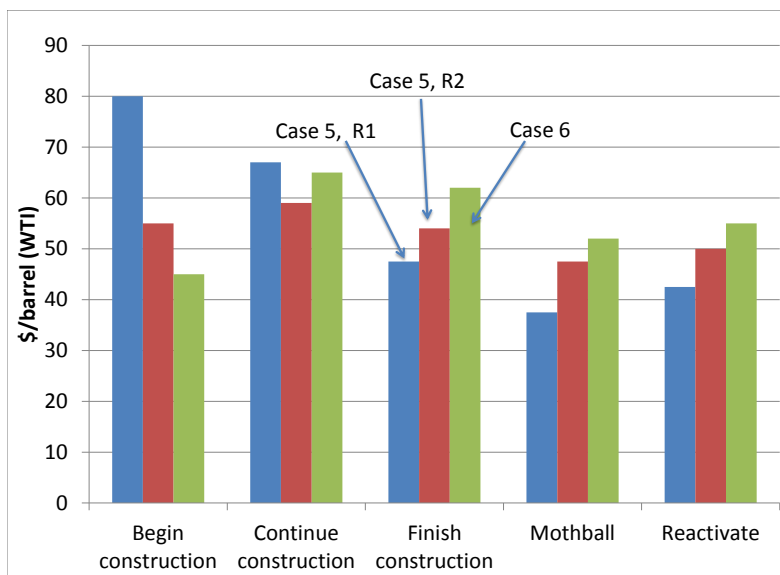

Figure 11: Comparing critical prices for various stages, cases 5 (regime switching, high and low price regimes) and 6 (single weighted average regime)

makers in the resource extraction business, as well as suggest implications for regulators charged with mitigating the environmental consequences of such projects.

The calibrated parameter values show two regimes: a high price regime with a low speed of mean reversion and a moderate price regime with a rapid speed of mean reversion. Although optimal actions differed between the two regimes, most of the time (in the risk neutral world) we expect to reside in the moderate price regime, which we called the dominant regime. In contrast a single regime case which was a weighted average of the two regimes, provided a higher project value and different optimal decisions than in either regime.

We observe an interesting pattern in critical prices as construction proceeds. In the base case critical prices increased going from stage 1 to stage 2 of construction and then fell for the final stage 3. The critical price of proceeding to the next stage of construction depends on the costs versus benefits of delay. For further intuition we consider two single regime cases, one with a GBM price process and the other with a mean reverting price process. For the former critical prices increased as construction proceeded from one stage to the next, whereas

for the latter case critical prices fell as construction progressed. For the mean reverting price case, the optimal action is to spend the construction dollars early so that the decision maker is ready to take advantage of favourable price changes should they occur. Spending for each stage of construction is similar to purchasing an option to take the next step towards to having a producing property. For a price process that is strongly mean reverting it is optimal to purchase those early options, so as to have the ability to get to the production stage given favourable prices. For the base case (case 1) we observe a non-monotonic pattern of critical prices that reflects the two different regimes, and the critical price to begin construction is lower than for subsequent stages. If some of the most serious environmental consequences of resource extraction come from this construction phase, then regulators should be aware of this effect and the potential for a surge in investment. Awareness would imply being prepared in terms of having an adequate regulatory framework in place to deal with a sudden ramp up in activity.

The main conclusion of this paper is that optimal decisions are affected when we explicitly model different regimes for resource prices. Ignoring regime switching would result in non-optimal desions for private firms. On the other hand, a general expectation among resource firms that the future will be characterized by boom and bust cycles for commodity prices will affect the timing of their investments. This phenomenon is something that policy makers should be aware of, particularly if initial phases of investment in resource extraction operations involves significant loss of environmental amenities. Future research will attempt to include key environmental amenities, such as water, in the optimal extraction decision, and will examine needed regulations to protect the public good during the different phases of development and extraction.

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