Economics 659: Real Options and Investment under Uncertainty
Course Outline, Winter 2022

Instructor: Margaret Insley
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Office Hours: By appointment, online using Teams, or in-person if permitted.
Class time and location: T/Th, 11:30 – 12:50 pm, On-line until at least Jan 24; If we are able to resume in-person classes the location will be in QNC 1507.
Class Number: 8225, Lec 001

Course description

This course considers the application of option concepts from finance to valuing real assets and investment opportunities. The focus is on using real options theory and methodology to determine to the optimal timing of a firm's investment expenditures when investment outcomes are characterized by uncertainty and irreversibility. The course begins with an introduction to stochastic processes, Ito's Lemma, the Black-Scholes equation, contingent claims analysis and dynamic programming. Numerical methods to solve simple option value problems will be presented, such as binomial trees and Monte Carlo simulation. Data driven decision making without parametric model assumptions will be introduced. Applications will include problems in natural resource and environmental economics, such as the optimal timing to develop a mining asset, install pollution control equipment, and implement policies to curtail greenhouse gas emissions.

Learning outcomes

Upon completion of the course, students should be able to:

- Explain intuitively why and how uncertainty and irreversibility affect a firm's optimal investment decisions.
- Explain the analogy between financial options and a firm's decision to undertake capital investments, and describe how real options analysis fits into the broader topic of investment under uncertainty.
- Explain key building blocks for valuing investments under uncertainty including stochastic differential equations, dynamic programming, Hamilton-Jacobi-Bellman equations, and risk neutral valuation.
- Analyze simple problems of investment under uncertainty, determining a closed form solution if available or using a simple numerical technique.
- Read and critique a relevant academic paper, and present a summary of the paper's motivation and results to an audience of the student's peers.

Course delivery

Course materials (lecture slides, announcements, etc) will be available on LEARN. The first portion of the class will be delivered online. You will find a link to online lectures using Zoom on LEARN. Classes will move to in-person delivery when university regulations allow.

Background required

Knowledge of microeconomic theory, basic calculus and linear algebra and some experience with differential equations are required. Assignments require some programming in Matlab, which students are expected to learn on their own. Alternatively, with the approval of the instructor, students may use an alternative programming language of their choice, such as R or Python. Please contact the instructor if you wish to use something other than Matlab.

Course Assessment

	Weighting in final grade	Important dates:
Assignment 1	10%	Due on Tues Feb 1
Midterm	20%	Tuesday Feb 15
Assignment 2	10%	Due Thurs March 10
Project	20%	Paper choice due: Tues March 1; Project write up due: Tues April 5 Presentations will be done in the last three or four classes.
Final exam	40%	Scheduled by the registrar

Midterm

- A student who misses the midterm due to illness or other extenuating circumstances should seek approval from me to write a make-up midterm. Approval will be granted only if appropriate documentation is submitted. If a student does not receive my approval, then a mark of zero will be assigned for the missed midterm. **Students must submit their documentation within one week of the missed midterm.** Note that students with flu-like symptoms can self declare their illness by filling out a form on Quest. Information is provided here: https://uwaterloo.ca/arts/undergraduate/student-support/accommodations-illness-or-extenuating-circumstances

- Students with a concern about the marking of a midterm must consult with me **within two weeks** of the date that it is returned to the class. After two weeks, I will not make any adjustments to a midterm mark.

Assignments

- It is permitted to consult with other students regarding the assignment questions, however the final work submitted must be your own. Students who submit identical (or nearly identical) assignments will receive a grade of zero.

- Assignments are to be submitted to the designated Dropbox on LEARN. Assignments may be handwritten or typewritten.

- Students with a concern about the marking of an assignment must consult with me **within two weeks** of the date that it is returned to the class. After two weeks I will not make any adjustments to a assignment mark.

- **Policy on Late Assignments:** A deduction of 5% per day will be applied to an assignment handed in late.

Project

The goal of the project is for students to read and evaluate a journal article that applies real options theory and methodology, and to summarize the article in a class presentation. Students will choose a paper from a list provided early in the term. Each student will present a summary and analysis of their chosen paper to the class. A brief report (approximately 5-7 pages, double spaced) discussing the paper will be handed in by the deadline specified above. Paper selections must be approved by the instructor by March 1.

The project will be marked out of 50, with 25 for the written discussion, 20 marks awarded for the presentation, and 5 marks for questions asked during class presentations of your classmates. (You will be responsible for asking questions after one or two your classmates' presentations.) The oral presentations will be scheduled in the last few classes of the term. Some presentations may need to be scheduled outside of class time. More instructions regarding the project will be handed out during the term.

Policy on Late Projects: A deduction of 5% per day will be applied to a project if the write up is handed in late.

Economics Department Deferred Final Exam Policy

The Deferred Final Exam Policy is found at https://uwaterloo.ca/economics/undergraduate/resources-and-policies/deferred-final-exam-policy

Course Materials

Some useful books and other resources are listed below. This list may be updated during the term. When possible, I will put copies on reserve in the library, and where available, links to on-line editions.

- I will rely on the following book to introduce key concepts such as stochastic processes, Ito's lemma, and dynamic programming.
    - Dixit and R. Pindyck, (1994) *Investment under Uncertainty,* Princeton University Press. I have put a copy on reserve in the library. There is also a link to an on-line copy through Course Reserves in LEARN.

- Here is a list of some other useful references.
    - Forsyth P., "An introduction to computational finance without agonizing pain", Available at Introduction to Computational Finance
    - Hull, John C. (2006) *Options, Futures, and Other Derivatives*, Pearson, Prentice, Hall. Many editions of this book are available
    - Neftci, Salih N. (2000), An Introduction to the Mathematics of Financial Derivatives, second edition, Academic Press.
    - Ross, Sheldon M. (1999) An Introduction to Mathematical Finance: Options and Other Topics, Cambridge University Press.
    - Schwartz, Edwardo and Lenos Trigeorgis (2001) Real options and investment under uncertainty: Classical readings and recent contributions, MIT press
    - Trigeorgis, Lenos (1996) Real Options, *Managerial Flexibility and Strategy in Resource Allocation,* MIT Press. (available online through the library)

Tentative list of topics and schedule

This list of topics and readings may be adjusted during the term depending on interest and timing. Additional readings may be assigned throughout the term. A * indicates a required reading.

1. Introduction

Week 1 (Jan 6) and Week 2 (Jan 11 and 13)

1.1. Traditional investment theory versus the options approach
1.2. Introduction to financial options
1.3. A two period real options example
1.4. Extending the example to more periods

Readings

- Dixit & Pindyck, Ch 1 and 2
- Trigeorgis Ch1
- Hull, Chapter on the mechanics of options markets

2. Stochastic processes and Ito's lemma

Week 3 (Jan 18 and 20)

2.1. Introduction to stochastic processes
2.2. The Wiener process
2.3. Random walk representation of Brownian motion
2.4. Ito processes

Readings

- *Hull, Chapters on Wiener Processes and Itos's Lemma, the Black Scholes Merton Model
- *Dixit and Pindyck, Chapter 3
- *Forsyth, Sections 2.5 and 2.6
- Various chapters in Neftci
- Various chapters in Ross

Week 4 (Jan 25 and 27)

2.5. Ito's Lemma
2.6. Jump processes

3. Dynamic optimization under uncertainty

Week 5 (Feb 1 and 3) (Assignment 1 due on Feb 1)

3.1. Dynamic programming and the Bellman equation

3.2. Contingent claims approach to valuing a risky asset, Black Scholes equation and risk neutral valuation

Week 6 (Feb 8 and 10)

3.3. Capital asset pricing model

3.4. Valuing a forward contract

Readings

- * Dixit and Pindyck, Chapters 4
- Trigeorgis ,Chapters 2 and 3
- Any introductory corporate finance text

4. Simple models of investment valuation and optimal investment timing

Week 7 (Feb 15 and 17), Midterm on Feb 15

4.1. A basic investment problem when the value of the project follows GBM

Feb 21 – 25: Reading Week

Week 8 (March 1 and 3)

4.2. Comparative statics for the stochastic case

4.3. A more realistic investment problem

Week 9 (March 8 and 10)

4.4. Extensions of these basic investment problems

Readings

- *Dixit and Pindyck, Chapters 5 and 6

5. Introduction to numerical methods for solving real option problems

Week 10 (March 15 and 17)

5.1. Introduction
5.2. Binomial trees
5.3. Monte Carlo methods

Week 11 (March 22 and 24)

5.4. Data driven approaches without parametric model assumptions

Readings

- *Forsyth
  - Section 5, The Binomial Model, Intro, Sections 5.1, 5.2, 5.5
  - Section 4, Monte Carlo Methods, Intro and Sections 4.1, 4.5, 4.7, 4.9
  - Section 12.5 Bootstrap resampling
- *Hull, Chapter on Basic Numerical Procedures

6. Presentations, finishing up lecture material as needed

Week 12 (March 29 and 31, April 5)

Paper write-up due April 5

Statements and links included on all course outlines

Academic integrity: In order to maintain a culture of academic integrity, members of the University of Waterloo community are expected to promote honesty, trust, fairness, respect and responsibility. [Check the Office of Academic Integrity for more information.]

Grievance: A student who believes that a decision affecting some aspect of their university life has been unfair or unreasonable may have grounds for initiating a grievance. Read Policy 70, Student Petitions and Grievances, Section 4. When in doubt, please be certain to contact the department's administrative assistant who will provide further assistance.

Discipline: A student is expected to know what constitutes academic integrity to avoid committing an academic offence, and to take responsibility for their actions. [Check the Office of Academic Integrity for more information.] A student who is unsure whether an action constitutes an offence, or who needs help in learning how to avoid offences (e.g., plagiarism, cheating) or about "rules" for group work/collaboration should seek guidance from the course instructor, academic advisor, or the undergraduate associate dean. For information on categories of offences and types of penalties, students should

refer to Policy 71, Student Discipline. For typical penalties, check Guidelines for the Assessment of Penalties.
Appeals: A decision made or penalty imposed under Policy 70, Student Petitions and Grievances (other than a petition) or Policy 71, Student Discipline may be appealed if there is a ground. A student who believes they have a ground for an appeal should refer to Policy 72, Student Appeals.
Note for students with disabilities: AccessAbility Services, located in Needles Hall, Room 1401, collaborates with all academic departments to arrange appropriate accommodations for students with disabilities without compromising the academic integrity of the curriculum. If you require academic accommodations to lessen the impact of your disability, please register with AccessAbility Services at the beginning of each academic term.

Other Information from the Faculty of Arts

1. Intellectual Property

Students should be aware that this course contains the intellectual property of their instructor, TA, and/or the University of Waterloo.

Intellectual property includes items such as:

- Lecture content, spoken and written (and any audio/video recording thereof);
- Lecture handouts, presentations, and other materials prepared for the course (e.g., PowerPoint slides);
- Questions or solution sets from various types of assessments (e.g., assignments, quizzes, tests, final exams); and
- Work protected by copyright (e.g., any work authored by the instructor or TA or used by the instructor or TA with permission of the copyright owner).

Course materials and the intellectual property contained therein, are used to enhance a student's educational experience. However, sharing this intellectual property without the intellectual property owner's permission is a violation of intellectual property rights. For this reason, it is necessary to ask the instructor, TA and/or the University of Waterloo for permission before uploading and sharing the intellectual property of others online (e.g., to an online repository).

Permission from an instructor, TA or the University is also necessary before sharing the intellectual property of others from completed courses with students taking the same/similar courses in subsequent terms/years. In many cases, instructors might be happy to allow distribution of certain materials. However, doing so without expressed permission is considered a violation of intellectual property rights.

Please alert the instructor if you become aware of intellectual property belonging to others (past or present) circulating, either through the student body or online. The intellectual property rights owner deserves to know (and may have already given their consent).

2. Chosen/Preferred First Name

Do you want professors and interviewers to call you by a different first name? Take a minute now to verify or tell us your chosen/preferred first name by logging into WatIAM.

Why? Starting in winter 2020, your chosen/preferred first name listed in WatIAM will be used broadly across campus (e.g., LEARN, Quest, WaterlooWorks, WatCard, etc). Note: Your legal first name will always be used on certain official documents. For more details, visit Updating Personal Information.

Important notes

- If you included a preferred name on your OUAC application, it will be used as your chosen/preferred name unless you make a change now.
- If you don't provide a chosen/preferred name, your legal first name will continue to be used.

3. Mental Health Support

All of us need a support system. The faculty and staff in Arts encourage students to seek out mental health support if they are needed.

On Campus

Due to COVID-19 and campus closures, services are available only online or by phone.

- Counselling Services: counselling.services@uwaterloo.ca / 519-888-4567 ext. 32655
- MATES: one-to-one peer support program offered by the Waterloo Undergraduate Student Association (WUSA) and Counselling Services

Off campus, 24/7

- Good2Talk: Free confidential help line for post-secondary students. Phone: 1-866-925-5454
- Grand River Hospital: Emergency care for mental health crisis. Phone: 519-749-4300 ext. 6880
- Here 24/7: Mental Health and Crisis Service Team. Phone: 1-844-437-3247
- OK2BME: set of support services for lesbian, gay, bisexual, transgender or questioning teens in Waterloo. Phone: 519-884-0000 extension 213

Full details can be found online on the Faculty of Arts website

Download UWaterloo and regional mental health resources (PDF)

Download the WatSafe app to your phone to quickly access mental health support information.

4. Territorial Acknowledgement

We acknowledge that we are living and working on the traditional territory of the Attawandaron (also known as Neutral), Anishinaabe and Haudenosaunee peoples. The

University of Waterloo is situated on the Haldimand Tract, the land promised to the Six Nations that includes ten kilometres on each side of the Grand River.
For more information about the purpose of territorial acknowledgements, please see the CAUT Guide to Acknowledging Traditional Territory.

5. Academic freedom at the University of Waterloo

Policy 33, Ethical Behaviour states, as one of its general principles (Section 1), "The University supports academic freedom for all members of the University community. Academic freedom carries with it the duty to use that freedom in a manner consistent with the scholarly obligation to base teaching and research on an honest and ethical quest for knowledge. In the context of this policy, 'academic freedom' refers to academic activities, including teaching and scholarship, as is articulated in the principles set out in the Memorandum of Agreement between the FAUW and the University of Waterloo, 1998 (Article 6). The academic environment which fosters free debate may from time to time include the presentation or discussion of unpopular opinions or controversial material. Such material shall be dealt with as openly, respectfully and sensitively as possible." This definition is repeated in Policies 70 and 71, and in the Memorandum of Agreement, Section 6