Canadian Stock Market Volatility under COVID-19

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Abstract

This paper focuses on investigating the impacts of the novel coronavirus (COVID-19) on the Canadian stock market volatility from a time-varying parameter volatility model point of view.

Keywords: COVID-19; Canadian S&P/TSX Composite Index; Volatility Structural Break; Time-Varying Parameter Model; News Impact Curve.

JEL: C22; C58; G18

1 Introduction

The novel coronavirus (COVID-19) is known to be contagious from human to human. As a matter of fact, such virus is also "contagious" from human to all human-related activities. The COVID-19 has aggressively "invaded" into almost every corner on earth. We never expect the financial market to be exempt from such serious global pandemic. The financial markets have been severely impacted by the COVID-19 across the globe. This paper focuses on investigating the impacts of the COVID-19 on the Canadian stock market. The stock market could provide a unique view of the expected future of the economy, as Wagner (2020) mentioned, in essence, the stock market can be viewed as an incentivized survey of expectations of future outcomes from investors' points of view. Through studying the stock market reactions to the COVID-19 shock, we hope to have some useful information/results available in policy making for the government, regulation designing for companies and investment guidance for investors to avoid/minimize further negative impact of propagations of the COVID-19 invasion.

In this paper, we propose to investigate the Canadian stock market volatility dynamics through a Generalized AutoRegressive Conditional Heteroskedasticity (GARCH) setting with realized measures of volatility. Following Hansen and Huang (2016), we adopt the Realized Exponential GARCH (REGARCH) as the baseline model. The REGARCH can accommodate and update the new information at a faster speed than the conventional volatility models since it introduces the realized measures of the volatility constructed from the high frequency transaction data into the volatility process. Furthermore, the REGARCH assumes a flexible leverage structure to better capture the dependence between the returns and volatility.

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The parameters estimation procedure is straightforward. It can be done via the quasi Maximum Likelihood estimation. We observe many good applications of the REGARCH to the real data. Despite the empirical success of the REGARCH, the model falls short in modelling the dynamics of the data with structural changes. In this paper, the main purpose is to study the dynamics of the volatility under the global COVID-19 pandemic. Therefore, the basic framework of the REGARCH is not sufficient in describing the dynamic properties of the recent stock market data. Introducing the structural changes into the model is a natural extension in order to better characterize such data. In the literature, there are many excellent ways to incorporate structural breaks into the model (see Perron (2006)). In this paper, we follow the recent nice work of Amado and Terasvirta (2013) to introduce a time-varying structure in model parameters to characterize the structural changes. In particular, rather than modelling a sharp change on parameter values at the break-points, we allow for a smoothing transition over time. As Amado and Terasvirta (2013) mentioned, such smoothing transition not only makes the model structure to be more flexible but also is more reasonable and practical in explaining the data with structural changes. We use the benchmark Canadian index (S&P/TSX Composite Index) from the Toronto Stock Exchange (TSX). The index represents about 70% of the total market capitalization consisting of around 230 companies listed on the TSX.

The remainder of the paper is organized as follows. Section 2 discusses the proposed model specification and property. Section 3 provides the empirical analysis based on the Canadian S&P/TSX Composite Index. Section 4 concludes.

2 The Model

In general, we propose a more generalized model framework by extending the REGARCH with time-varying parameters. We refer to the Time-Varying REGARCH (TV-REGARCH). There are three processes specified in the proposed model. Let x_t be the continuously compounded return time series, defined by the logarithm of the ratio of two consecutive prices. Denote h_t as the latent logarithmic volatility of x_t at time t. y_t is the corresponding realized measure for h_t. It can be constructed using high frequency trading price data. The TV-REGARCH is presented as follows,

$$
\begin{aligned}
x_t &= \mu_t + \exp(h_t/2)\epsilon_t && (1)\\
h_t &= h_{1,t} + h_{2,t} && (2)\\
h_{1,t} &= \lambda + \alpha h_{t-1} + \delta(\epsilon_{t-1}) + \gamma u_{t-1} \\
h_{2,t} &= \sum_{s=1}^{S} (\lambda_s + \alpha_s h_{t-1} + \delta_s(\epsilon_{t-1}) + \gamma_s u_{t-1})\, G_s(t/T; \phi_s, c_s) \\
y_t &= \beta_1 + \beta_2 h_t + \tau(\epsilon_t) + u_t && (3)
\end{aligned}
$$

(1) is referred as the return process, in which μ_t is the conditional mean component[1] and $\exp(h_t)$ models the conditional variance. ϵ_t is assumed to be an N.I.D disturbance term with zero mean and unit standard deviation. The dynamic process of h_t is specified in (2), which is referred as the general "GARCH" equation. The baseline process of h_t is specified in $h_{1,t}$. λ is the constant term in the GARCH

[1]We set μ_t to be zero in this paper as Hansen and Huang (2016) found that imposing such restriction results in a better empirical performance of the model.

process. α is the so-called the volatility persistence parameter, which measures the correlation of the last period volatility and the current period volatility. $\delta(\epsilon_{t-1})$ is the leverage effect function. We adopt the quadratic form based on the second-order of Hermite polynomial. That is, $\delta(\epsilon) = \delta_1\epsilon + \delta_2(\epsilon^2 - 1)$. γ reveals the impact of the realized measure of volatility on future latent volatility. As mentioned earlier, we propose to accommodate structural breaks in modelling the volatility dynamics. Hence, $h_{2,t}$ is introduced in modelling h_t. Following Amado and Terasvirta (2013), we adopt the general logistic transition function to allow for smooth transition in modelling parameter change, where $G = \Big(1 + \exp(\phi \prod_{k=1}^{K}(t/T - c_k))\Big)^{-1}$. ϕ is the smoothness parameter, which controls the rate of the parameter switch. c is the threshold, which indicates the breakpoint location in a [0,1] interval. In general, we assume that there exist S structural breakpoints. For every breakpoint, each parameter in the baseline $h_{1,t}$ are smoothingly switched to a new level. Since G is function of time, all parameters in (2) are all time-varying. As a note, this paper focuses on investigating the impact of the COVID-19 on the Canadian stock market volatility. Hence, in the empirical section, we set S to be one. In other words, we could see the dynamic response of the volatility to the outbreak of the COVID-19 via examining the parameter transitions. Lastly, (3) serves the role as a measurement equation linking the overall log conditional variance (h_t) and its corresponding realized measure (y_t). If the realized measure is a good approximate for the conditional variance, then we expect β_1 to be zero and β_2 to be one.[2] Following the REGARCH in Hansen and Huang (2016), $\tau(\epsilon)$ is also a leverage function, which has a form of the second-order of Hermite polynomial. That is, $\tau(\epsilon) = \tau_1\epsilon + \tau_2(\epsilon^2 - 1)$. It is worth mentioning that the two leverage functions $\tau(.)$ and $\delta(.)$ make the independence assumption between ϵ_t and u_t empirically realistic according to Hansen and Huang (2016). We assume $u_t \sim N(0, \sigma_u^2)$.

In summary, the model used in our empirical section has 14 parameters, $\theta = (\lambda, \alpha, \delta_{11}, \delta_{12}, \gamma, \lambda_1, \alpha_1, \delta_{21}, \delta_{22}, \gamma_1, \beta_2, \tau_1, \tau_2, \sigma_u)$.[3] Following Amado and Terasvirta (2013) and Hansen and Huang (2016), we implement the quasi MLE estimation. The standard errors can be constructed using the standard sandwich form in terms of score and Hessian of the log-likelihood function.

3 Empirical Analysis

The empirical data used in this section is the Canadian S&P/TSX Composite Index. In particular, we have two time series (return and realized kernel) as the inputs for estimation of our proposed model.[4] As of May 1, 2020, we use the most updated data downloaded from the Realized Library of the Oxford-Man Institute of Quantitative Finance.[5] Both the returns and realized kernels are at daily frequency. The sample

[2]For identification purpose, we restrict the intercept term in (3) to be zero in this paper.

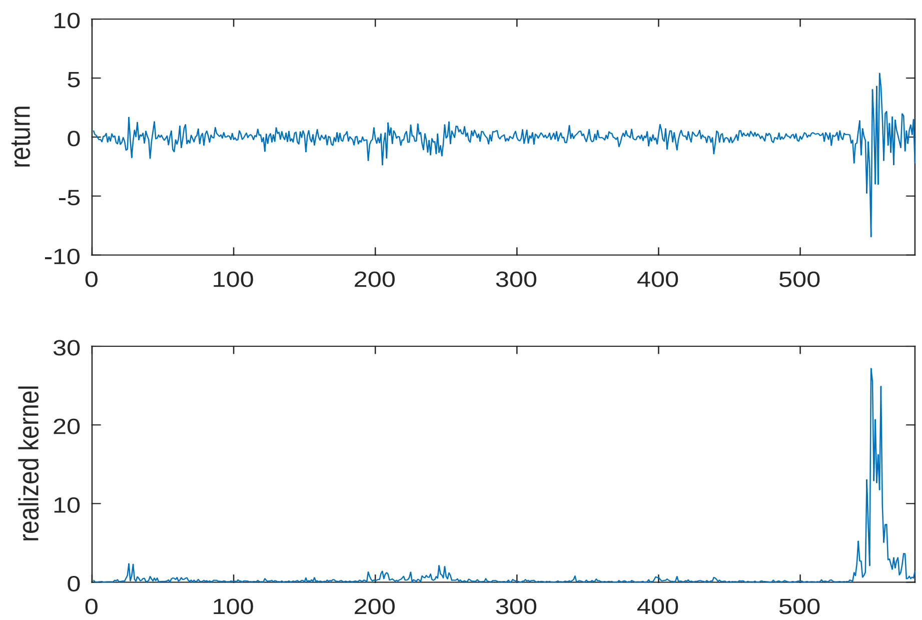
[3]As a note, we also estimate the smoothness parameters, ϕ, for the transitions of (λ, α, δ_{11}, δ_{12}, γ) in G function. The estimation can be done via an iterative optimization procedure. In essence, we initialize the estimation with ϕ_0. We get the optimized θ_0. Then with θ_0, we repeat the optimization process to get the next step ϕ_1. We iterate the whole process until convergence.

[4]In this paper, we use the open-to-close daily return and the realized kernel volatility using the non-flat Parzen smoothing scheme. We scale the return by 100 and the realized kernel volatility by 10000 for better representation.

[5]We would like to thank the Oxford-Man Institute of Quantitative Finance to publicly share the most updated data for research.

period is from January 02, 2018 to April 30, 2020, which amounts to 581 observations. To visualize the data, we plot the return (upper panel) and the corresponding realized kernel volatility series (bottom panel) in Figure 1.

Figure 1: Daily Return and Realized Kernel



At the end of both sample series we observe big turbulence under both return and realized kernel time-series plots. This is consistent with our expectation. Since the outbreak of the COVID-19 pandemic, the Canadian stock market has responded to virus with dramatic movements. For instance, according to the headline of the Bloomberg that highlighted "Investors Flee Canada's Stock Market on Worst Day in 80 Years" on March 12, 2020, the S&P/TSX Composite Index got the biggest one-day drop since May 1940. The 230 companies in the benchmark lost approximately CAD \$ 265 billion in market value. The Toronto Stock Exchange halted all tradings as the circuit-breaker was triggered. Simliarly, we also observe that the COVID-19 drove the tremendous surge on the daily market realized kernel volatility that is constructed from intra-daily high frequency transcation data.

We present the data summary statistics in Table 1. In this paper, in order to determine the breakpoint in the data, we trace back the timeline of the outbreak of COVID-19 in Canada. According to the Globalnews, the first "presumptive" case of the coronavirus in Canada was detected on January 25th, 2020 in Toronto. As January 25th, 2020 is a Saturday, we set the following Monday (January 27th, 2020) as the breakpoint date in our data.[6]

Table 1 presents the first four moments (mean, variance, skewness and kurtosis) of the data. We also report the corresponding moments of the sub-samples based on the breakpoint (Pre-COVID-19 and COVID-19). In summary, the moment values are consistent with our expectation. Although we only have 63 data points in the

[6]As our model uses the smoothing transition in parameter switch, the choice of the breakpoint does not change our general results significantly in this paper. We have also experimented with different breakpoints around late January and early February, our analysis remains similar. As an interesting future research direction, if more COVID-19 data are available, one could adopt a data-driven algorithm for detection of the breakpoint.

Table 1: Summary Statistics

	Mean	Var	Skew	Kurt	Length	JBtest
Return–(whole sample)	-0.0129	0.6788	-1.5477	31.1503	581	19416*
Log(RK)–(whole sample)	-1.7936	1.5960	1.2076	5.2658	581	265.4912*
Return–(Pre-COVID-19)	-0.0149	0.2244	-0.7478	6.0868	518	253.9338*
Log(RK)–(Pre-COVID-19)	-2.0174	0.8849	0.2999	3.0239	518	7.7775*
Return–(COVID-19)	0.0035	4.4785	-0.8123	6.6861	63	42.5959*
Log(RK)–(COVID-19)	0.0462	3.6937	-0.1406	1.9219	63	3.2589

*Note: JB test stands for the Jarque-Bera Normality test at 5% level. * means that the null hypothesis of "the sample follows a normal distribution" is rejected.*

COVID-19 subsample, the COVID-19 increases the variances of both return and realized kernel volatility of the whole sample significantly. For instance, the variance of the Pre-COVID-19 returns is 0.2244, while when we include the COVID-19 subsample into the whole sample, we observe a 200% increase in the variance (to 0.6788). In addition, the COVID-19 makes the return distribution to be more left-skewed as the skewness coefficient changes from -0.7478 to -1.5477. As expected, the big increases in kurtosis values for both return and log realized kernel indicate that we observe more extreme values after the outbreak of COVID-19. We also perform the Jarque-Bera (JB) Normality test and report the JB test statistics in the last column of Table 1. All the samples are not normally distributed, except the log(RK)-COVID-19.[7]

We apply our proposed model to the data. The empirical results are provided in Table 2.

Due to the flexible structure of the model with time varying parameters, we are able to visualize the coefficients' transitions under the COVID-19. We present the evolution of each parameter in the conditional volatility process in our model through time in Figure 2.

From Figure 2, we could see the impacts of the COVID-19 on our model coefficients. What are the economic implications? We will translate the model interpretations of the COVID-19 impacts on the Canadian Stock Market in several aspects as follows.

First, in terms of the expectation of the level of the market volatility, we derive the volatility moment based on the model. As the moment is function of model parameters which are time-varying, we are able to plot changes on expectations of the market volatility level over time. We present it in Figure 3.

One can see that the expected market volatility is relatively stable at a constant level till the outbreak of the COVID-19. The read dash line represents the breakpoint date, which is January 27th, 2020. We observe a sudden jump in levels around the breakpoint. Such jump is expected as the outbreak of the COVID-19 has created an unprecedented level of risk and uncertainty, owing to its deadly threat to human life, its significantly negative impact on global supply chains and capital flows, and etc. The investors on the markets became panic, which drove the tremendous surge

[7]This may be due to small sample size of 63. We may need more data points to construct reliable JB statistics.

Table 2: Estimation Results for TV-REGARCH

	Estimate	Standard Error
λ	-0.8301	0.1221
α	0.9233	0.0114
δ_{11}	-0.1536	0.0385
δ_{12}	0.0259	0.0133
γ	0.1550	0.0282
β_2	1.0328	0.0054
τ_1	-0.0370	0.0306
τ_2	0.1872	0.0195
σ_u	0.6551	0.0194
λ_1	0.2842	0.0605
α_1	0.0128	0.0060
δ_{21}	0.1375	0.1259
δ_{22}	0.1269	0.0357
γ_1	0.0579	0.1265

Note: The smoothness parameters' estimates, ϕ, for the transitions of (λ, α, δ_{11}, δ_{12}, γ) in G function are (70.9077 8.3094 2.9034 26.7586 315.3483).

Figure 2: Parameter Transition Plots

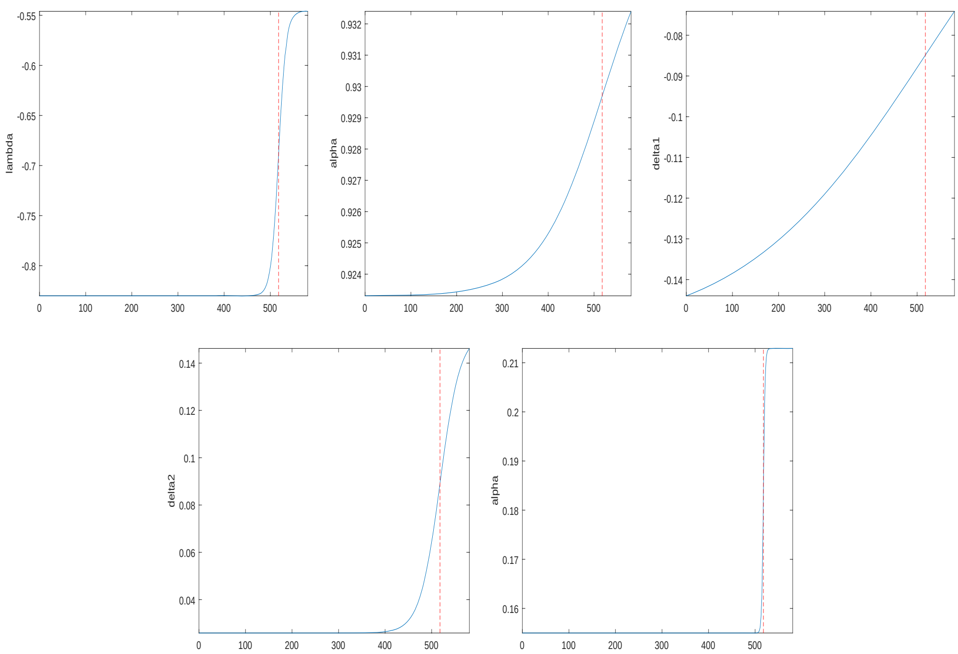
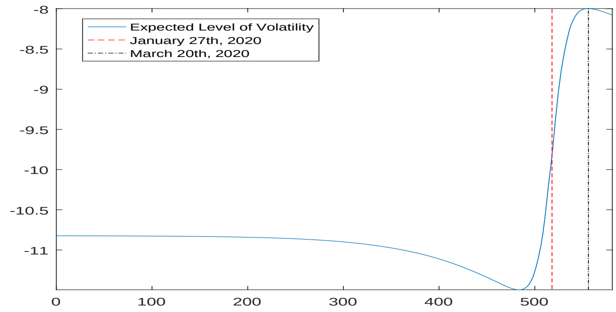


Figure 3: Expectation of the Market Volatility Level



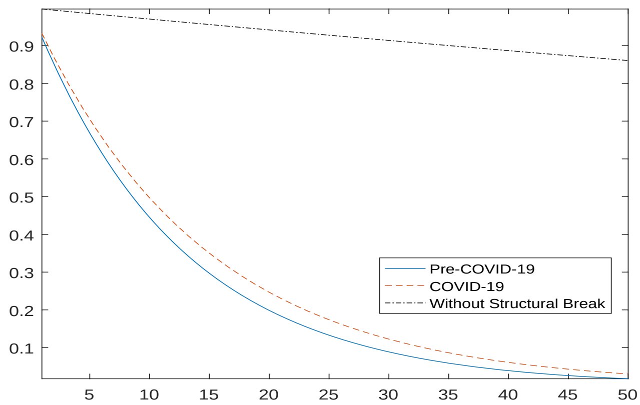
in the market volatility level. However, as an interesting and positive observation in Figure 3, we see a slight decline in the curve around March 20th, 2020. We believe that this decline is potentially related to the Bank of Canada announcement on that date. In particular, on March 20th, 2020, the Bank of Canada announced four specific additional important measures to support market functioning. One of them was to activate the Contingent Term Repo Facility (CTRF) by April 3rd, 2020 to counter any severe market-wide liquidity stresses and further support the stability of the Canadian financial system.[8] We believe that the decline in the expectation of the market volatility level is the market response to such positive policy announcement. On the other hand, we have to admit that due to the lack of the observations, we could not draw a clear conclusion whether such decline is actually a meaningful "decline" or not. This certainly leaves an interesting question for future research when more data become available.

Secondly, we would like to comment on the market volatility persistence under the COVID-19. As is well known, the GARCH family models can well capture the persistence behavior of the conditional variances. For instance, in the standard GARCH specification, the sum of the ARCH and GARCH coefficients is typically found to be close to one for most of the financial return data. However, in the structural break literature, many researchers have questioned and challenged such "stylized" phenomenon, see for example Hillebrand (2005). The main concern is that a simple structural break in the data could severely bias the overall persistence estimation toward one. This is known as the so-called "spurious almost integration" effect in volatility persistence. In order to examine the volatility persistence under the COVID-19, we adopt the well-known "half-life" measure. The half-life measure was originally proposed in the nuclear physics to measure the survival time period of stable atoms. It becomes popular in the financial econometrics as it could capture the shock effects, see Lamoureux and Lastrapes (1990). In our context, this measure essentially presents the time span over which a shock to the current volatility level reduces to half of its original size. In our proposed framework, α represents the volatility persistence. Therefore, the half-life measure, depending solely on persistence, can be computed by solving k from $\frac{1}{2} = \alpha^k$. For comparison, we also perform the estimation without considering the COVID-19 structural break in the model. The α estimate is 0.9979, which is almost one. The half-life measure based on this

[8]For details on other measures that the Bank of Canada announced, please refer to the official policy announcement webpage of the Bank of Canada.

persistence estimate implies that a shock to the current volatility on the Canadian stock market would take about 330 days to diminish to half of its original size. However, based on our proposed framework with the COVID-19 structural break, the same half-life measure indicates it would only around 10 days to shrink the shock to its half size. We believe that this is more reasonable in practice. In addition, we plot the Dynamic Impulse Response (DIR) functions over 50 lags with and without the COVID-19 structural break in Figure 4.

Figure 4: Dynamic Impulse Response



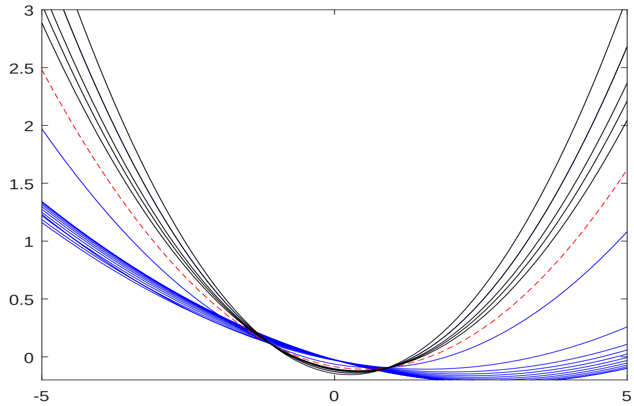
Note: The blue solid line represents the DIR in the Pre-COVID-19 period. The red dash line represents the DIR derived from the COVID-19 transition parameters. The black dotted line represents the DIR without considering the COVID-19 structural break in model.

In Figure 4, the blue solid line represents the DIR in the Pre-COVID-19 period. The red dash line represents the DIR derived from the COVID-19 transition parameters. For comparison, we also plot the DIR without considering the COVID-19 structural break in the black dotted line. Consistent with general findings in the literature, if the model ignores the structural break, the effect of a shock remains for an "unreasonably" long lags. In Figure 4, we observe the black dotted line is almost flat. Even after 50 days, a unit shock to the market volatility still has about 86% left. We believe that the policy based on such conclusion could be erroneous and misleading as it relies on the overestimated volatility persistence by overlooking the structural break. Now, we investigate the DIR changes under the COVID-19 using our proposed model with structural break. The COVID-19 drives up the volatility persistence. In term of the DIR, for example, after 25 days, a unit shock on the current volatility has only 13.6% left in the Pre-COVID-19 market, while 17.4% left when we consider the COVID-19 transition as of April 30th, 2020. In general, we find that the COVID-19 makes the shocks to the volatility lasting longer and stronger. We will further discuss the impacts of the positive and negative shocks in the following part.

Following Engle and Ng (1993) and Hansen and Huang (2016), we investigate the impact of news on the market volatility under the COVID-19 via the news impact curve (NIC). We present the evolution of the NIC under the transitions of the time-varying parameters of the COVID-19 structural break in Figure 5.

In Figure 5, the blue solid lines represent the NIC in the Pre-COVID-19 period. The red dash line represent the NIC at the breakpoint. The black solid lines

Figure 5: Evolution of News Impact Curve



Note: The blue solid lines represent the NIC in the Pre-COVID-19 period. The red dash line represent the NIC at the breakpoint. The black solid lines represent the NIC under the COVID-19.

represent the NIC under the COVID-19. There are two interesting and important findings. First, the NIC becomes more and more symmetric under the transition from the Pre-COVID-19 to COVID-19 period. In other words, we observe more pronounced leverage effect before the COVID-19 hit the Canadian stock market. The symmetric shape of the NIC indicates that the positive news and negative news on the Canadian stock market has similar impact on volatility under the COVID-19. Second, we find that the market volatility is more sensitive to the news (regardless of positive or negative) than before. For instance, fixing at a negative shock at $\epsilon = -3$ level, we found that such negative news has over 100% more impact on the market volatility in the current COVID-19 market than in the Pre-COVID-19 period. This provides some empirical evidence to support the possible explanation that the information about pandemic is richer and diffuses at a faster speed now, which could trigger the stress and panic at a larger scale on market especially during the current COVID-19 period. On other side of the NIC, we could also see that positive shocks have more effective impacts on the volatility during the COVID-19 period as well. As mentioned earlier, we observe a decline in market volatility level right after the Bank of Canada announcement on supporting measures to stabilize the Canadian financial market. It implies that the potential supportive policy from the government does have effective impacts to reduce the market volatility level especially during the pandemic period.

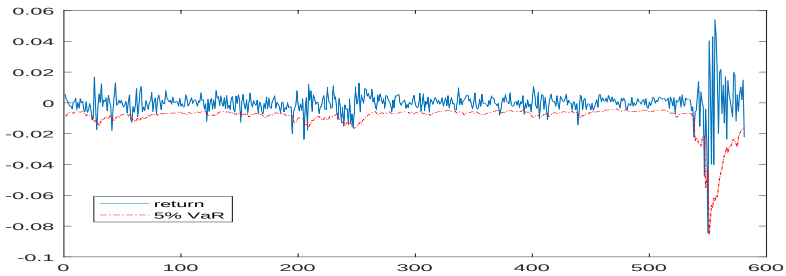
Lastly, based on the transition of γ values (from 0.1550 to 0.2129), we may interpret that the realized kernels (constructed from the intra-daily high frequency trading data) are more informative in explaining the future volatility during the COVID-19 period than before. For this reason, we utilize the realized kernels as the benchmark for evaluating our out-of-sample volatility forecasts.

As of May 1, 2020, we perform the one-day-ahead out-of-sample volatility forecast based on our proposed model. We use the week of April 27, 2020 to May 1, 2020 as our evaluation window. We also report the one-day-ahead Value-at-Risk (VaR) measure on the Canadian S&P/TSX Composite Index. These forecasting results are reported in Table 3 along with the Root of Mean Square Error (RMSE) measure.[9] In addition, we apply the popular back-testing algorithm for constructions of the VaR measures at 5% level for the whole sample data. The VaR plot is presented in Figure 6. One can see that the 5% VaR curve, in general, captures the movement in the return series. During the COVID-19 period, the VaR estimates are significantly lower than those in the pre-COVID-19 period on average. This indicates that the financial institutes would have to gauge larger amount of financial assets needed to cover possible losses during the COVID-19.

Table 3: Out-of-Sample Forecast

	April 27	April 28	April 29	April 30	May 1
log (RK)	-10.0040	-9.6433	-9.8579	-8.8389	-10.1885
Model Forecast	-8.9618	-9.1402	-9.2257	-9.1872	-8.4124
RMSE	1.0422	0.5031	0.6322	0.3483	1.7761
VaR	-0.0186	-0.0170	-0.0163	-0.0166	-0.0245

Figure 6: Value at Risk at 5%



4 Conclusion

Canada is under attack by the COVID-19. As of May 4th, 2020, the number of confirmed cases in Canada surpassed 60,000 (with deaths about 4000). Ranked by the number of confirmed cases of the COVID-19, Canada is currently the top 12th country across the globe. We are seriously concerned about the situation in Canada. In this paper, we adopt a time-varying coefficient volatility model to investigate the potential COVID-19 impact on the Canadian stock market using the most updated S&P/TSX Composite Index data. Here are the highlighted general conclusions:

- The global pandemic of the COVID-19 drove the tremendous surge in expected market volatility level of the Canadian S&P/TSX Composite Index around

[9] As a note, the main purpose of this paper is to understand the current COVID-19 situation on the Canadian stock market. We believe that the model used in this paper could benefit from adding more components in the volatility dynamic process, such as macroeconomic variables. For this reason, we do not horse-race alternative model specifications for evaluation of the forecasting performance. We will leave this for further study on model development.

January 25th, 2020 when the very first "presumptive" case of the coronavirus was detected in Canada.

- After March 20th, 2020, the model observes a slight decline pattern in the market volatility, which is potentially related to the Bank of Canada positive announcement on taking additional specific measures for stabilizing the financial market in Canada.

- From the research perspective, the model for studying the COVID-19 data should have a feature of accommodating regime-switch or structural break. This paper illustrates an example that overlooking the structural break of the COVID-19 in a conventional volatility model could result in seriously biased results.

- Our study shows that during the COVID-19 period, the negative news could be more negative, and similarly the positive news could be more positive. As the information nowadays in general becomes richer and diffuses much more rapidly. Our study shows that the market volatility becomes more sensitive to the news (regardless of positive or negative) during the COVID-19. Combining the conclusion point 2 above, we believe that the Canadian government's supportive policies to the public should have effective impacts (at least) to stabilize the financial market.

- We do FIRMLY believe that Canada will win the battle against the COVID-19 although it is hard!

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